ECE 498SM: Principles of Safe Autonomy

Practice problems

1. The exam will be a closed book exam on CBTF
2. You may use 1 sheet of notes
3. You may use a calculator
4. If something in the question looks ambiguous, make and state your assumptions, and then proceed to solve the problem with those assumptions
5. Absolutely no interaction between students is allowed
6. Please write clearly

Problem 1. State feedback control design

Consider the linear system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u = Ax + Bu$$

(a) Is the open loop system stable?

(b) Define full state feedback control law:

$$u = -[k_1 \; k_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -Kx$$

Then what is the closed loop system $A_{cl} = A - BK = ?$

(c) Can the values of k_1 k_2 be chosen to make the system stable? Write down the characteristic equation of A_{cl} (the equation that needs to be solved to find the eigenvalues of A_{cl}).

(d) Suppose we want the eigenvalues to be precisely at $\lambda_1 = -5$ and $\lambda_2 - 6$. What values of k_1 k_2 should be chosen? [Hint: Compare the coefficients of the equation with solutions at -5 and -6 with those of the equation in part (c)]

Problem 2. Perception

Part a. kNN classifier where k=5 creates zones of ambiguity where points could be classified to either class. Illustrate how this can happen with two simple examples.

Part b: Degraded lane markings like in the figure below is a serious problem in many highways. Your autonomous car's brand new lane detection system uses various filters and a *Canny edge detector* to detect lane markings. You notice that on roads like this, the detector is breaking long edges into short segments separated by gaps. Recall, *t_high* is used to start an edge and *t_low* is used to continue in Canny. Explain how you might adjust the thresholds to address this breakup problem.

Problem 4. Invariance and stability

Consider a dynamical system described by $\dot{x} = f(x)$ where the state vector $x \in \mathbb{R}^n$ and $f: \mathbb{R}^n \to \mathbb{R}^n$ is Lipschitz continuous. As usual, the solutions of the system are written as $\xi: \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$. Suppose the initial state of the system is the set $\Theta \subseteq \mathbb{R}^n$.

Suppose the system is Lyapunov stable. Given a set $B_\epsilon \subseteq \mathbb{R}^n$ that is a ball of radius $\epsilon > 0$, under what additional conditions on Θ is the set B_ϵ an invariant of the system?

Problem 5. Continuity

Show that sin(x) is Lipschitz a continuous function.
[*Hint. |sin(z)| <= |z|, sin(x) – sin(y) =* 2cos($\frac{x+y}{2}$)sin($\frac{x-y}{2}$) *]*

Problem 6. Inductive proofs

Consider a discretized model of a ball dropped from height h bouncing on a table. The model has three positive parameters h: initial height

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Ball(d, h, g > 0)
        Initially: v(0) = 0, x(0) = h
        v(t+1) = v(t) – g.d
        x(t+1) = x(t) + v(t).d – ½ g.d²
        if x(t+1) < 0
            x(t+1) = 0
            v(t+1) = sqrt(v²(t) + 2g.x(t))
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(a) Consider the candidate invariant Inv1: for all t, x(t) <= h, which says that the ball never exceeds the initial drop height. Can this invariant be proved inductively? Why and why not?

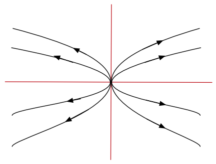
(b) Consider another candidate invariant Inv2: for all t, $v^2(t) - 2g(h-x(t)) = 0$. Prove this invariant inductively.

Problem 7. Stability and eigenvalues

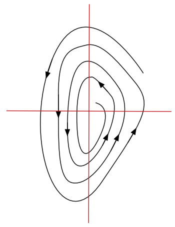
The pictures show possible trajectories of a 2-dimensional linear system $\dot{x} = Ax$ in the plane. Suppose the two eigenvalues of A are λ_1 and λ_2. We denote the real parts of these eigenvalues as $Real(\lambda_i)$. Consider the following possible configurations of these eigenvalues:

(a) $\lambda_{1,2} = a \pm b\,i, a < 0$
(b) Real positive eigenvalues, $0 < Real(\lambda_1) < Real(\lambda_2)$
(c) Real negative eigenvalues, $Real(\lambda_1) < Real(\lambda_2) < 0$

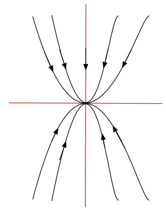
Match each picture with the corresponding eigenvalue distribution.



Diverging from point (1)



Spiral in (2)



Converging to point (3)

Problem 8. Short questions.

Question. Why is separability of a kernel useful in filtering?

Question. Write down the mathematical expression of a derivative of the Gaussian filter. Is this filter separable?

Question. In the bag of visual words representation of images, the number of distinct words in the visual dictionary equals to the
(a) number of clusters used on the image patches
(b) the number of images in the training set
(c) the value of K in K-nearest neighbor classification
(d) the length of the feature vectors
(e) none of the above

Question. Why is it important to scale feature vectors prior to classification? Can you explain with concrete examples?

Question. What is the main advantage of using a smaller sigma of Gaussian kernel when doing lane(edge) detection? What about the main disadvantage?