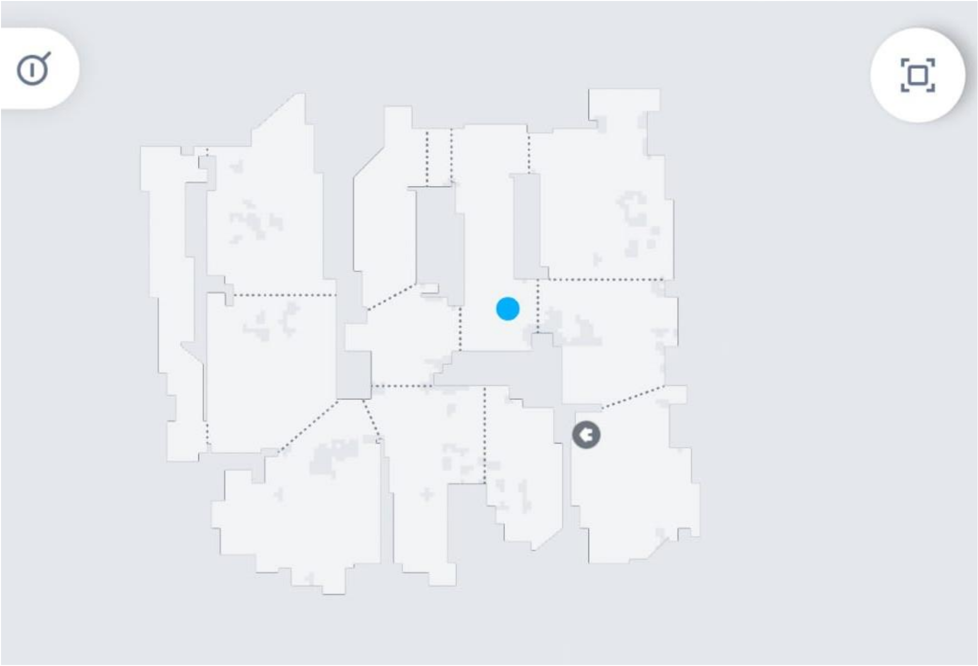
Principles of Safe Autonomy: Lecture 13: State Estimation, Filtering and Robot Localization

Sayan Mitra

Reference: Probabilistic Robotics by Sebastian Thrun, Wolfram Burgard, and Dieter Fox

Slides: From the book’s website

Roomba mapping

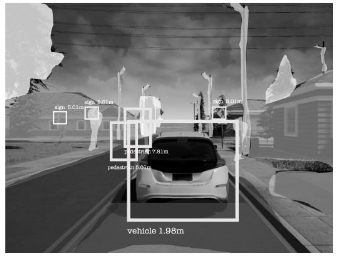


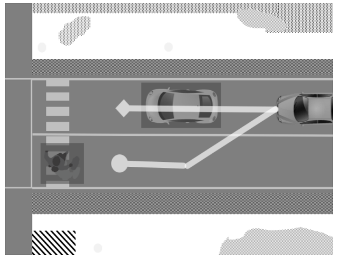
iRobot Roomba uses VSLAM algorithm to create maps for cleaning areas

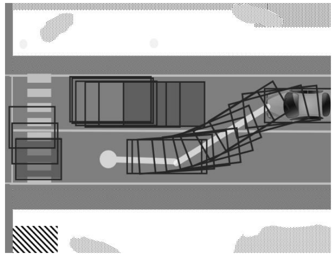
Announcements

GEM platform

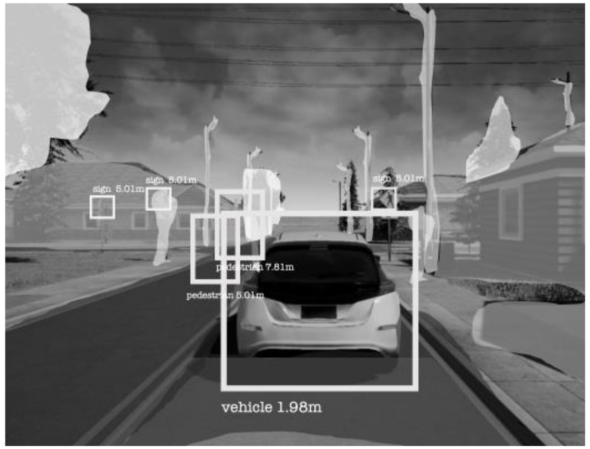
Autonomy pipeline







Sensing	Perception	Decisions and planning	Control
Physics-based models of camera, LIDAR, RADAR, GPS, etc.	Programs for object detection, lane tracking, scene understanding, etc.	Programs and multi-agent models of pedestrians, cars, etc.	Dynamical models of engine, powertrain, steering, tires, etc.



Perception

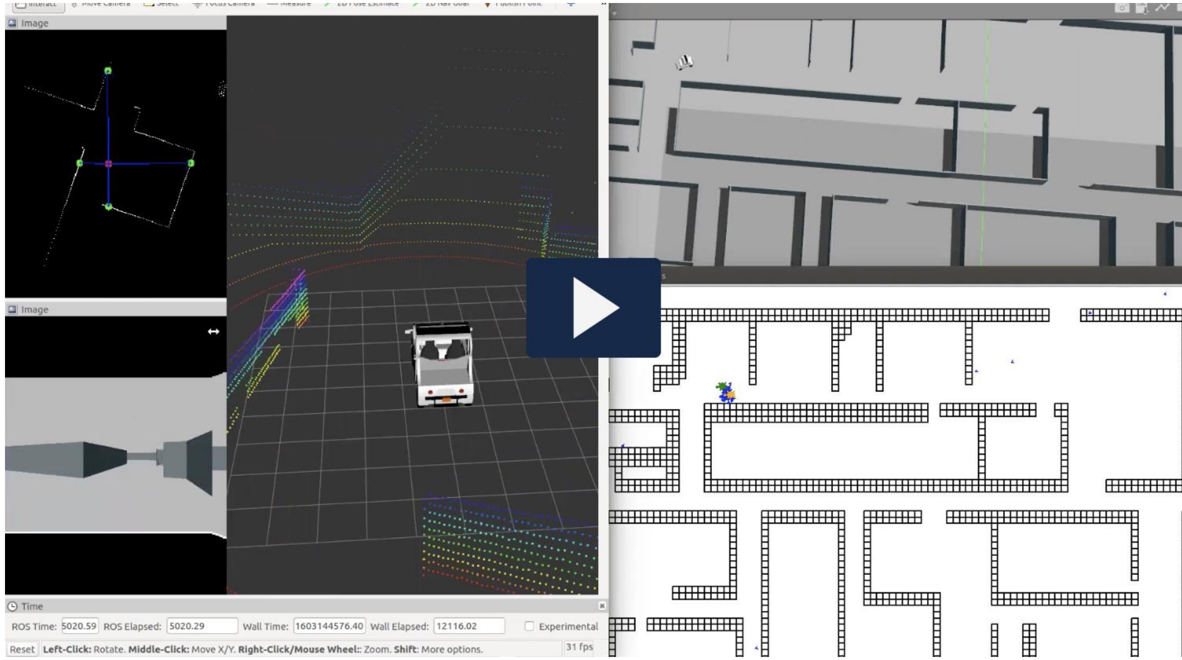
Programs for object detection, lane tracking, scene understanding, etc.

Outline of filtering and state estimation module

- Introduction: Localization problem, taxonomy
- Probabilistic models
- Discrete Bayes Filter
  - Review of Bayes rule and conditional probability
- Histogram filter
  - Grid localization
- Particle filter (next time)
  - Monte Carlo localization

Localization problem (MP4)

- Determine the pose of the robot relative to the *given map* of the environment
  - Pose: position, velocity, attitude, angles
  - Also known as position or state estimation problem



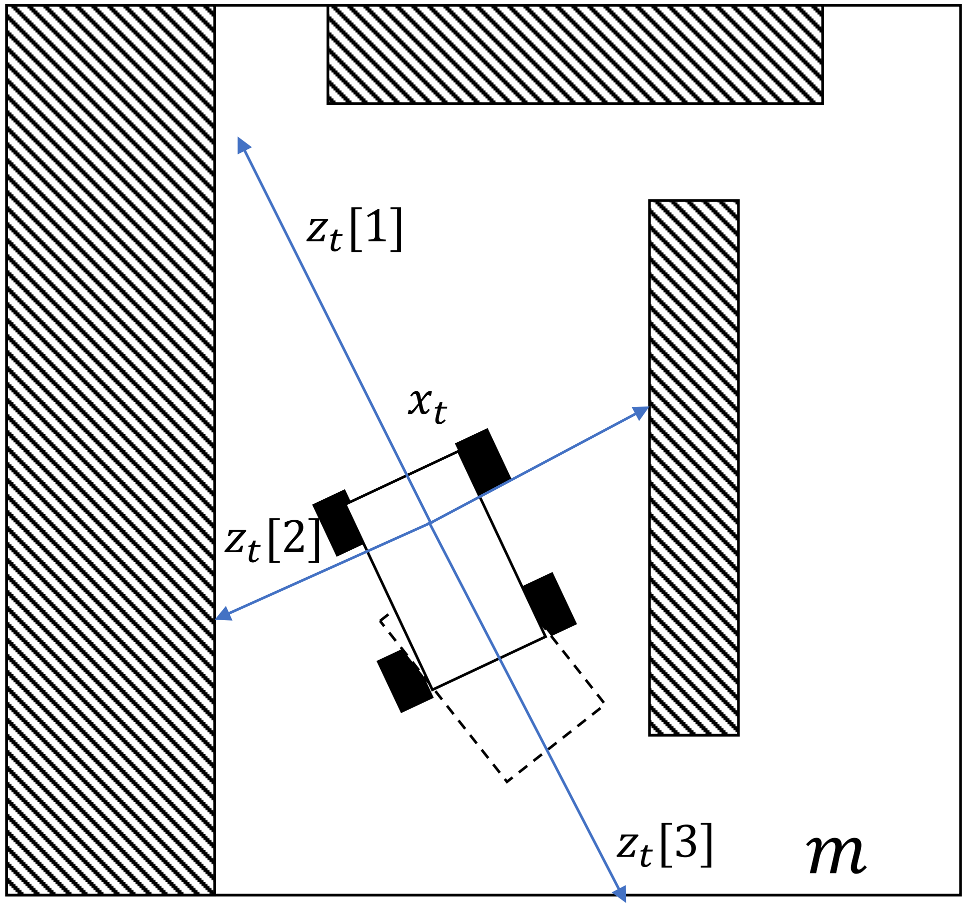
- First: why localize?
- How does your robot know its position in ECEB?
- "Localization is the biggest hack in autonomous cars" --- people drive without localization

Setup: State evolution and measurement models

- Deterministic model:

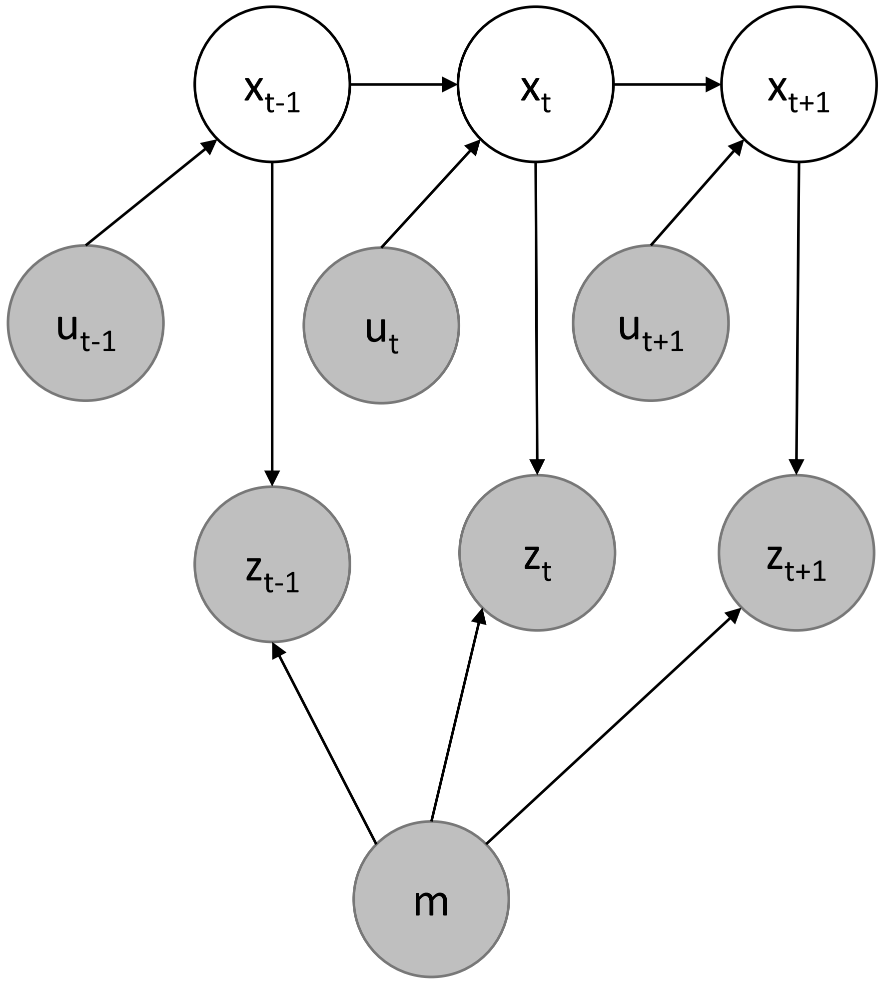
  System evolution: $x_{t+1} = f(x_t, u_t)$
  - x_t: unknown state of the system at time t
  - u_t: known control input at time t
  - f: known dynamic function, possibly stochastic

  Measurement: $z_t = g(x_t, m)$
  - z_t: known measurement of state x_t at time t
  - m: unknown underlying map
  - g: known measurement function
- We will work with probabilistic models going forward



This is not exactly the measurement model of MP4

Localization as coordinate transformation



Shaded known:
map (m), control inputs (u), measurements(z). White nodes to be determined (x)

maps (m) are described in global coordinates. Localization = establish *coord transf.* between m and robot's local coordinates

Transformation used for objects of interest (obstacles, pedestrians) for decision, planning and control

Localization taxonomy

Global vs Local

- Local: assumes initial pose is known, has to only account for the uncertainty coming from robot motion (*position tracking problem)*
- **Global**: initial pose unknown; harder and subsumes position tracking
- Kidnapped robot problem: during operation the robot can get teleported to a new unknown location (models failures)

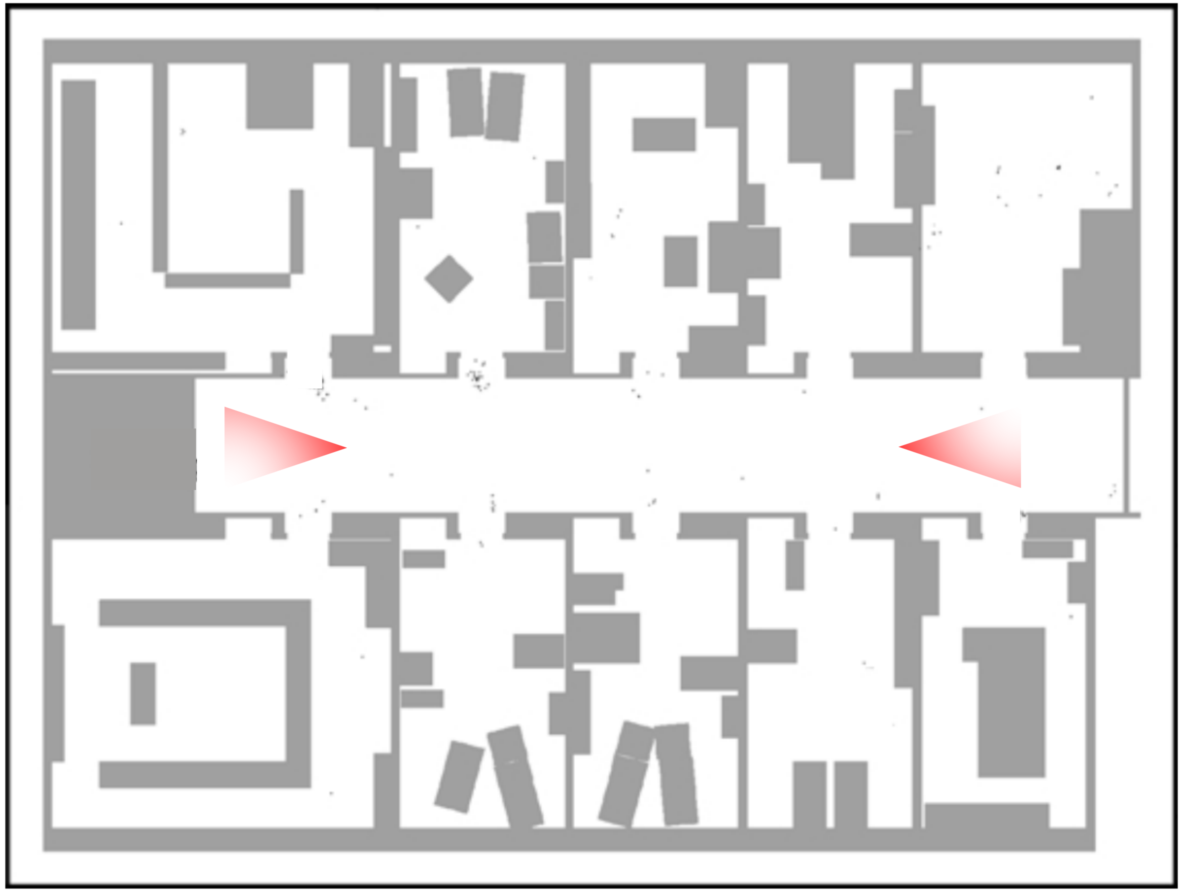
Static vs Dynamic Environments

Single vs Multi-robot localization

Passive vs Active Approaches

- **Passive**: localization module only observes and is controlled by other means; motion not designed to help localization (Filtering problem)
- Active: controls robot to improve localization

Ambiguity in global localization arising from locally symmetric environment



Discrete Bayes Filter Algorithm

- System evolution: $x_{t+1} = f(x_t, u_t)$
  - x_t: state of the system at time t
  - u_t: control input at time t
- Measurement: $z_t = g(x_t, m)$
  - z_t: measurement of state x_t at time t
  - m: unknown underlying map

Setup, notations

- Discrete time model
- $x_{t_1:t_2} = x_{t_1}, x_{t_1+1}, x_{t_1+2}, \ldots, x_{t_2}$ sequence of robot states $t_1$to t_2
- Robot takes one measurement at a time
  - $z_{t_1:t_2} = z_{t_1}, \ldots, z_{t_2}$ sequence of all measurements from $t_1$to t_2
- Control also exercised at discrete steps
  - $u_{t_1:t_2} = u_{t_1}, u_{t_1+1}, u_{t_1+2}, \ldots, u_{t_2}$ sequence control inputs

Review of conditional probabilities

Random variable X *takes values* $x_1, x_2, ..$

Example: Result of a dice roll (X*) and* $x_i = 1, \ldots, 6$

$P(X = x)$ *is written as* $P(x)$

Conditional probability: $P(x|y) = \frac{P(x,y)}{P(y)}$ provided $P(y) > 0$

$P(x, y) = P(x|y)P(y)$

$\quad = P(y|x)P(x)$

Substituting in the definition of Conditional Prob. we get Bayes Rule

$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$, provided $P(y) > 0$

Using measurements to update state estimates

$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$, provided $P(y) > 0$ ---- *Equation (*)*

X *: Robot position,* Y *: measurement,*

$P(x)$: *Prior distribution (before measurement)*

$P(x|y)$: *Posterior distribution (after measurement)*

$P(y|x)$*: Measurement model / inverse conditional / generative model*

$P(y)$: does not depend on x; normalization constant

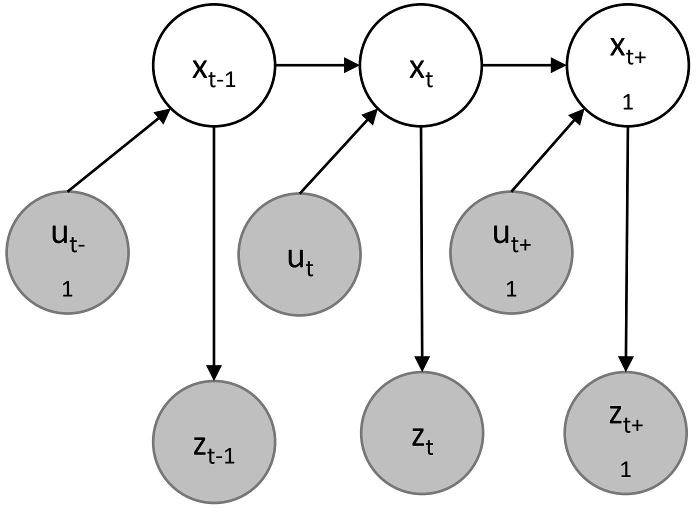
$x_{t+1} = f(x_t, u_t)$

State evolution and measurement: probabilistic models

Evolution of state and measurements governed by probabilistic laws

$p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t})$ describes motion/state evolution model

- If state is complete, sufficient summary of the history then
  - $p(x_t \mid x_{0:t-1}, z_{0:t-1}, u_{0:t-1}) = p(x_t \mid x_{t-1}, u_t)$ state transition prob.
  - $p(x' \mid x, u)$ if transition probabilities are time invariant

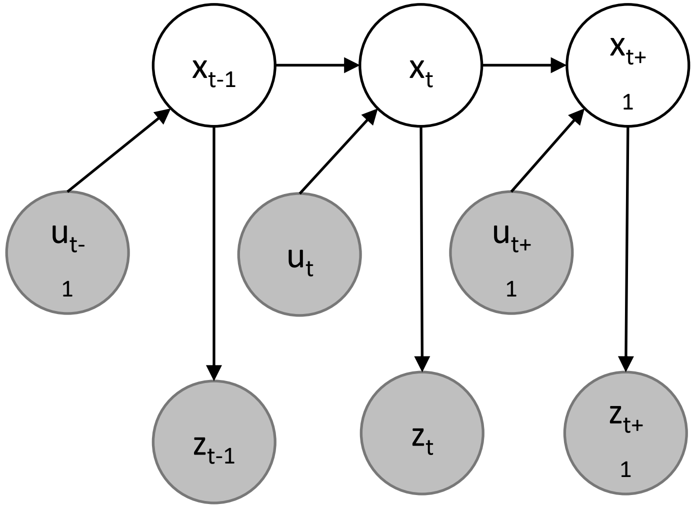


$z_t = g(x_t)$

Measurement model

Measurement process $p(z_t \mid x_{0:t}, z_{1:t-1}, u_{0:t-1})$

- Again, if state is complete
- $p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$
- $p(z_t \mid x_t)$: measurement probability
- $p(z \mid x)$: time invariant measurement probability



Beliefs

Belief: Robot's knowledge about the state of the environment

True state is unknowable / measurable typically, so, robot must infer state from data and we have to distinguish this inferred/estimated state from the actual state x_t

$bel(x_t) = p(x_t|z_{1:t}, u_{1:t})$

Posterior distribution over state at time t given all past measurements and control. This will be calculated in two steps:

1. Prediction: $\overline{bel}(x_t) = p(x_t|z_{1:t-1}, u_{1:t})$
2. Correction: Calculating $bel(x_t)$ from $\overline{bel}(x_t)$ a.k.a measurement update (will use Equation (*) from earlier)

Recursive Bayes Filter

Algorithm Bayes_filter($bel(x_{t-1}), u_t, z_t$)

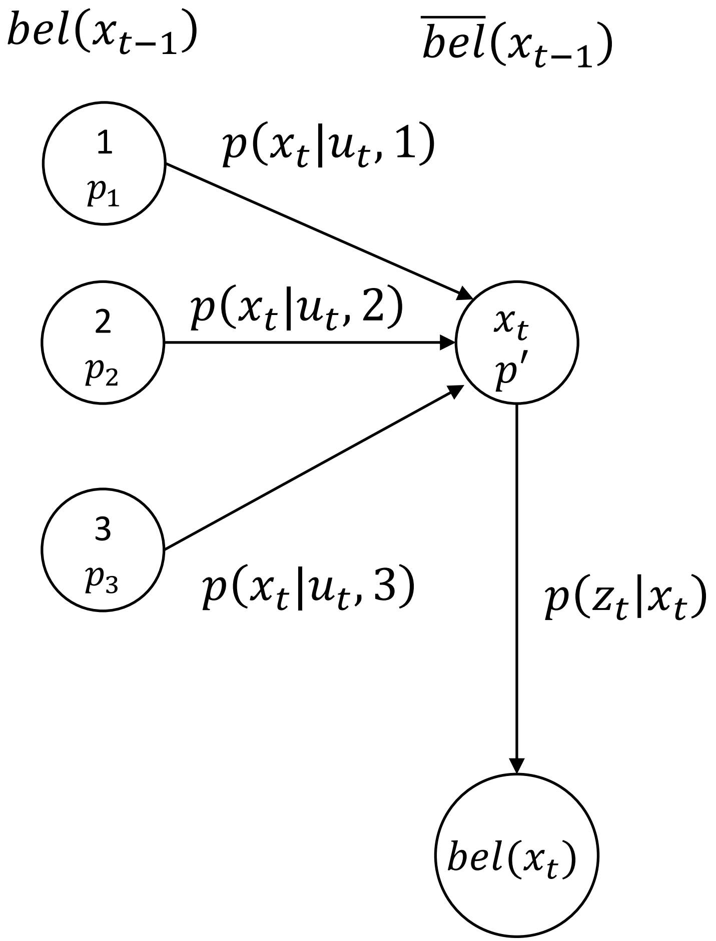
for all x_t do:

$$\overline{bel}(x_t) = \int p(x_t | u_{t,} x_{t-1}) bel(x_{t-1}) dx_{t-1}$$

$$bel(x_t) = \eta \, p(z_t | x_t) \, \overline{bel}(x_t)$$

end for

return $bel(x_t)$



Histogram Filter or Discrete Bayes Filter

Finitely many states $x_i, x_k, etc.$ Random state vector X_t

$p_{k,t}$: belief at time t for state x_k; discrete probability distribution

Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$):

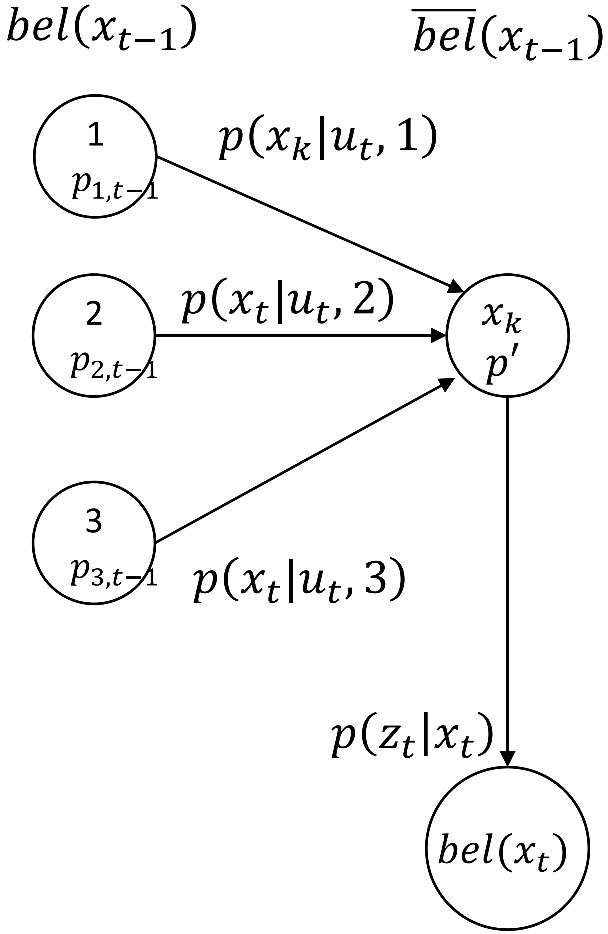
for all k do:

$$\bar{p}_{k,t} = \sum_i p(X_t = x_k | u_t, X_{t-1} = x_i) p_{i,t-1}$$

$$p_{k,t} = \eta\, p(z_t \,| X_t = x_k) \bar{p}_{k,t}$$

end for

return $\{p_{k,t}\}$



Grid Localization

- Solves global localization in some cases kidnapped robot problem
- Can process raw sensor data
  - No need for feature extraction
- Non-parametric
  - In particular, not bound to unimodal distributions (unlike Extended Kalman Filter)

Grid localization

Algorithm Grid_localization ($\{p_{k,t-1}\}, u_t, z_t, m$)

for all k do:

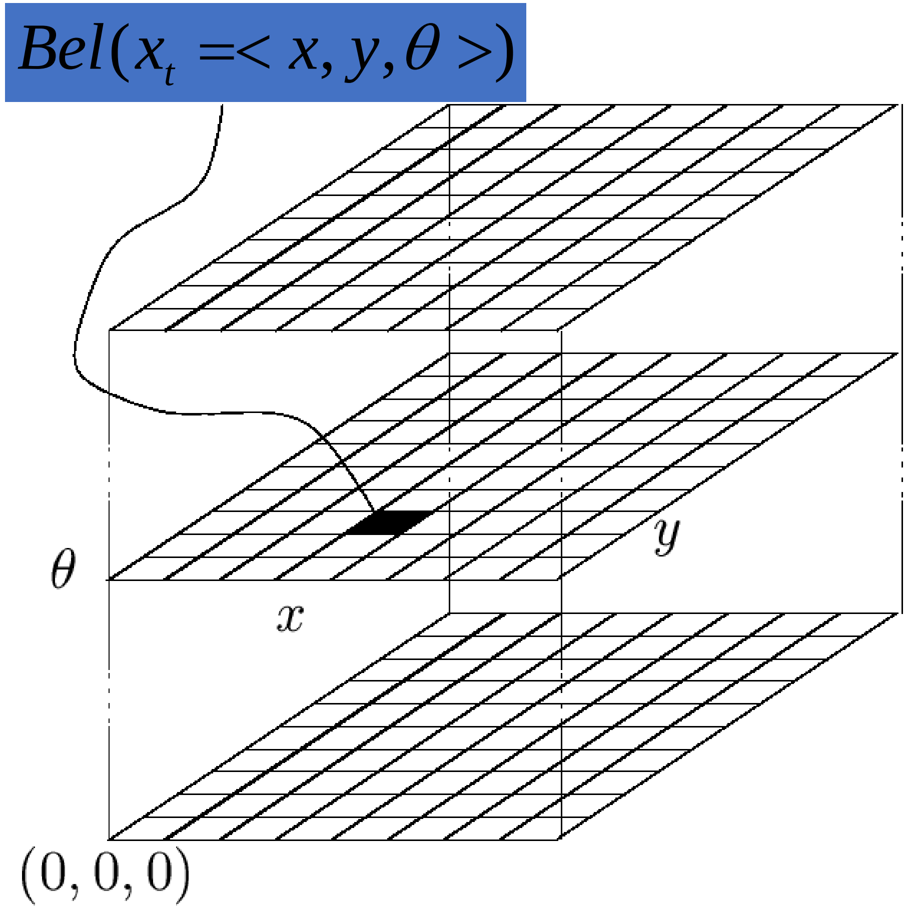
$$\bar{p}_{k,t} = \sum_i p_{i,t-1}\, \boldsymbol{motion\_model}(mean(x_k), u_t, mean(x_i))$$

$$p_{k,t} = \eta\, \bar{p}_{k,t} \boldsymbol{measurement\_model}(z_t, mean(x_k), m)$$

end for

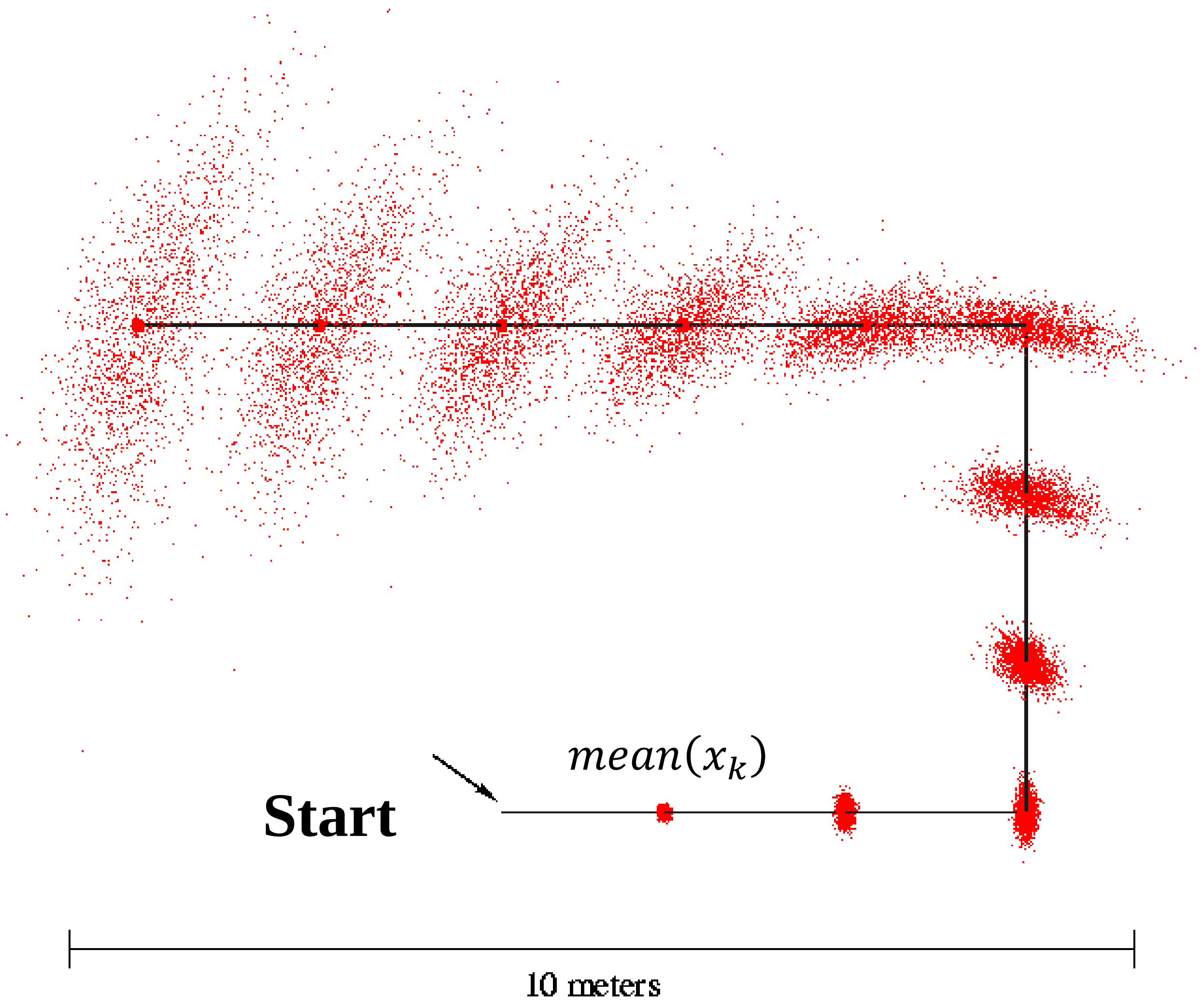
return $bel(x_t)$

Piecewise Constant Representation

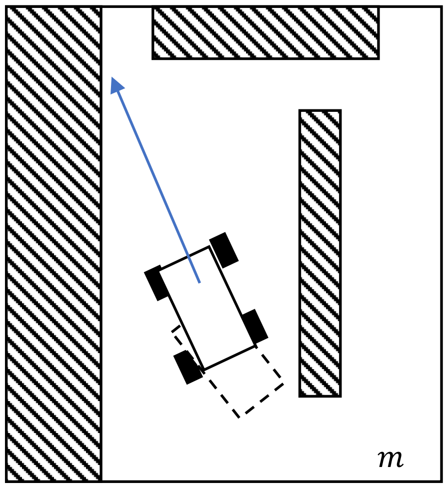


Fixing an input u_t we can compute the new belief

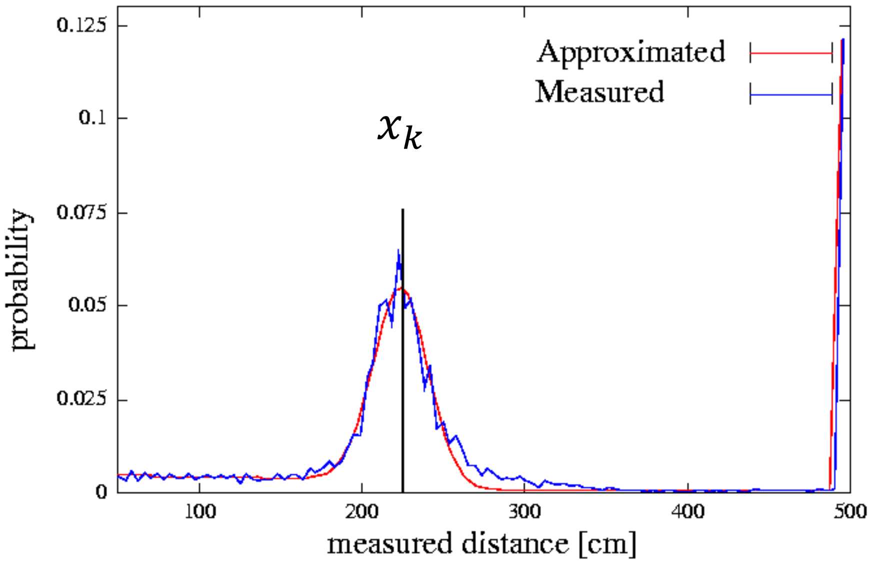
Motion Model without measurements



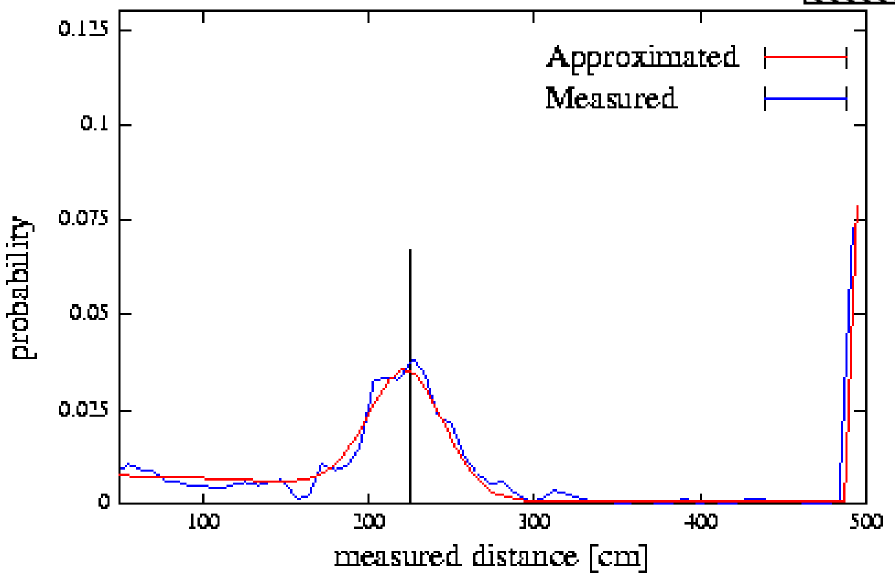
Proximity Sensor Model



$p(z_t \,|X_t = x_k)$

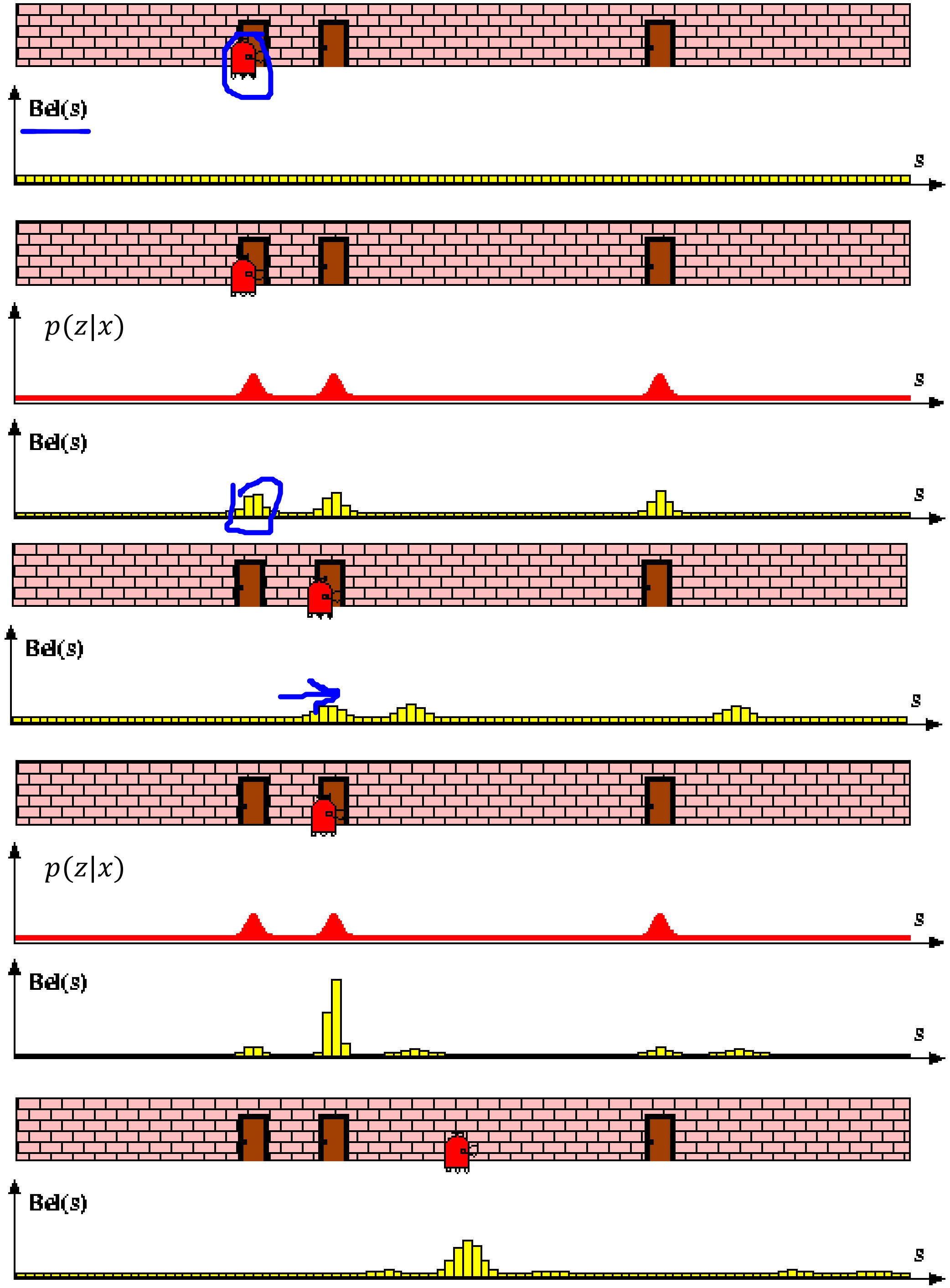


Laser sensor

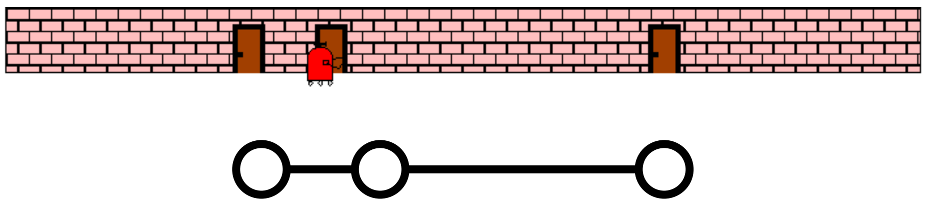


Sonar sensor

Grid localization, $bel(x_t)$represented by a histogram over grid

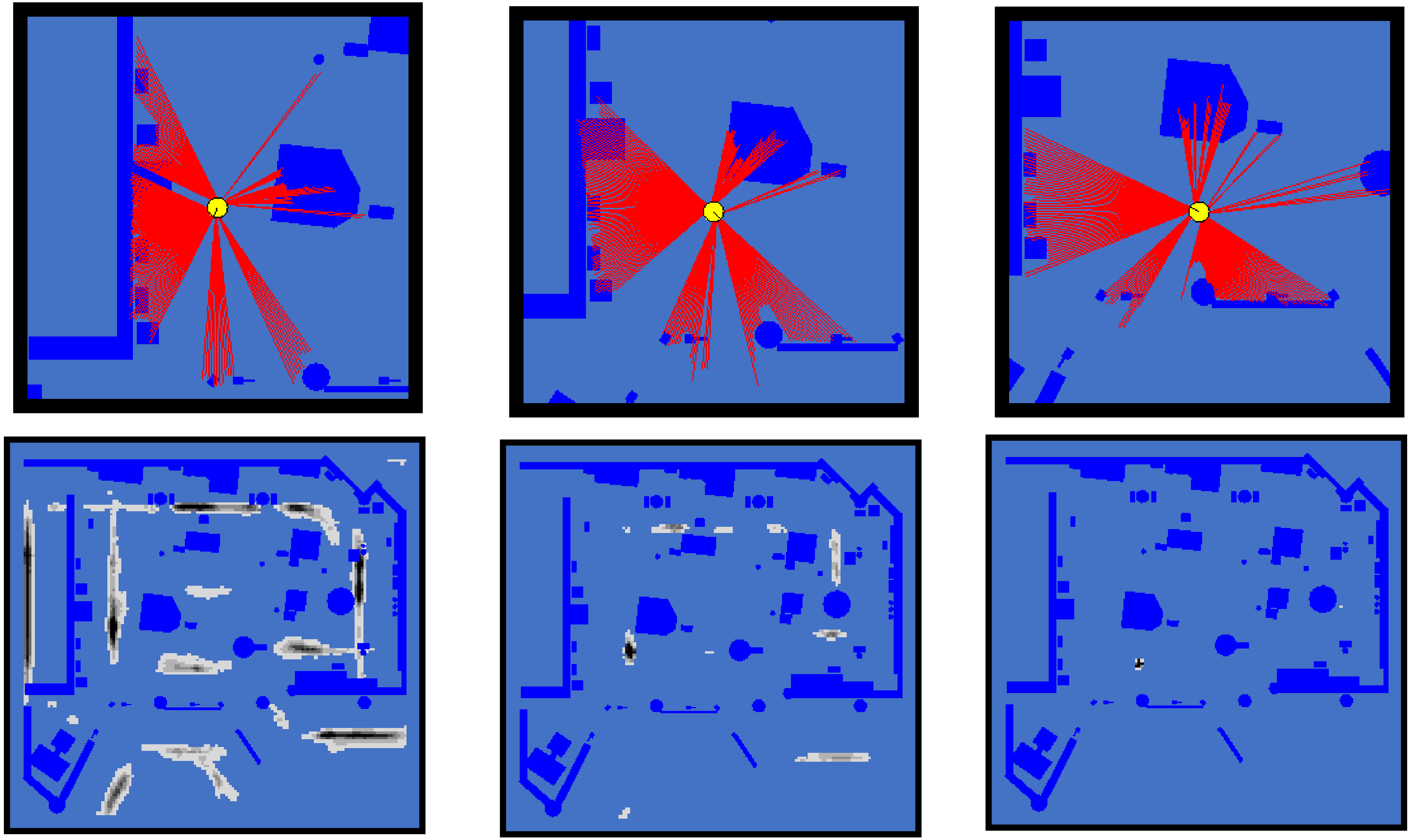


Summary

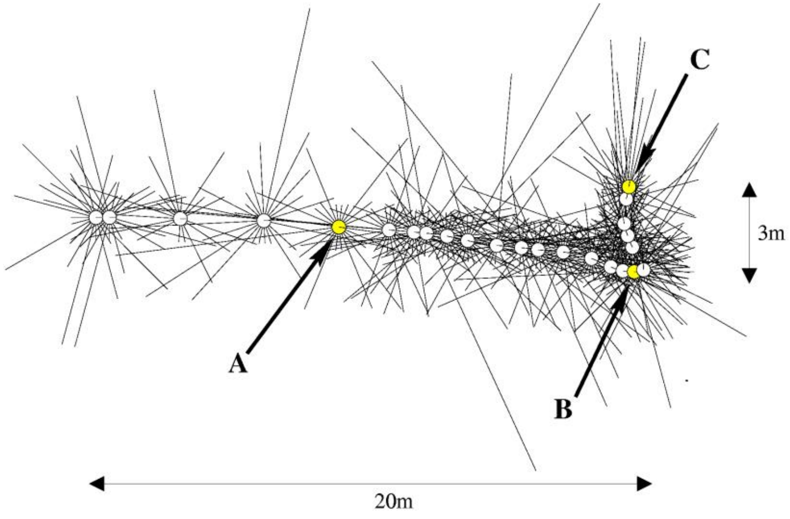


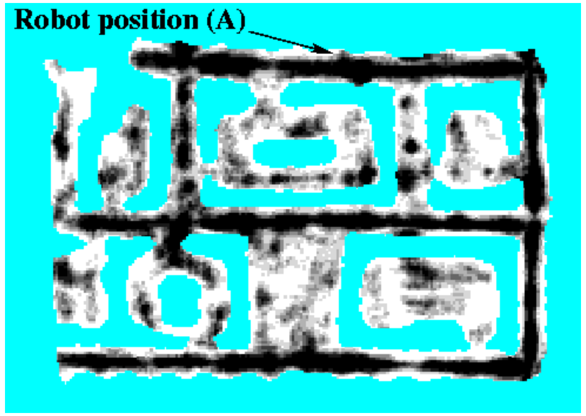
- Key variable: Grid resolution
- Two approaches
  - Topological: break-up pose space into regions of significance (landmarks)
  - Metric: fine-grained uniform partitioning; more accurate at the expense of higher computation costs
- Important to compensate for coarseness of resolution
  - Evaluating measurement/motion based on the center of the region may not be enough. *If motion is updated every 1s, robot moves at 10 cm/s, and the grid resolution is 1m, then naïve implementation will not have any state transition!*
- Computation
  - Motion model update for a 3D grid required a 6D operation, measurement update 3D
  - With fine-grained models, the algorithm cannot be run in real-time
  - Some calculations can be cached (ray-casting results)

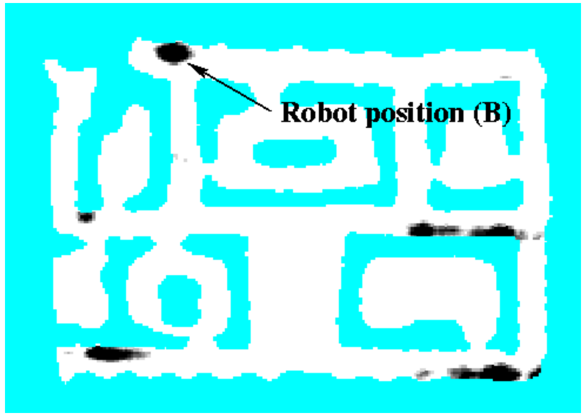
Grid-based Localization

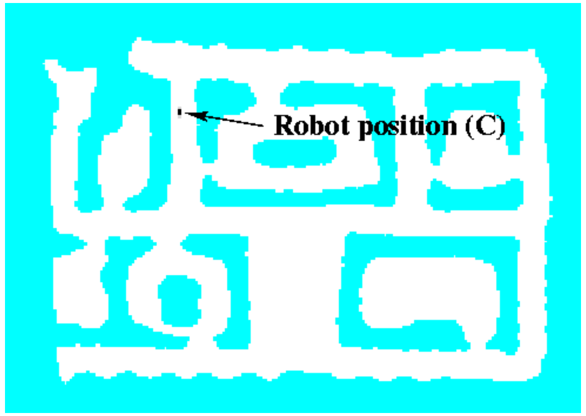


Sonars and Occupancy Grid Map









Monte Carlo Localization

- Represents beliefs by particles

Particle Filters

- Represent belief by finite number of parameters (just like histogram filter)
- But, they differ in how the parameters (particles) are generated and populate the state space
- Key idea: represent belief $bel(x_t)$ by a random set of state samples
- Advantages
  - The representation is approximate and nonparametric and therefore can represent a broader set of distributions than e.g., Gaussian
  - Can handle nonlinear tranformations
- Related ideas: Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter, Filtering: [Rubin, 88], [Gordon et al., 93], [Kitagawa 96], Dynamic Bayesian Networks: [Kanazawa et al., 95]d

Particle filtering algorithm

$X_t = x_t^{[1]}, x_t^{[2]}, \ldots x_t^{[M]}$ particles

Algorithm Particle_filter(X_{t-1}, u_t, z_t):
$\bar{X}_{t-1} = X_t = \emptyset$

for all m in [M] do:

sample $x_t^{[m]} \sim p(x_t | u_t, x_{t-1}^{[m]})$

$w_t^{[m]} = p\left(z_t \middle| x_t^{[m]}\right)$

$\bar{X}_t = \bar{X}_t + \langle\, x_t^{[m]}, w_t^{[m]} \rangle$

end for

for all m in [M] do:

draw i *with probability* $\propto w_t^{[i]}$

add $x_t^{[i]}$ *to* X_t

end for

return X_t

ideally, $x_t^{[m]}$ is selected with probability prop. to $p(x_t \mid z_{1:t}, u_{1:t})$

$\bar{X}_{t-1}$ is the temporary particle set

// sampling from state transition dist.

// calculates *importance factor* w_t or weight

// resampling or importance sampling; these are distributed according to $\eta\, p\left(z_t \middle| x_t^{[m]}\right) \overline{bel}(x_t)$

// survival of fittest: moves/adds particles to parts of the state space with higher probability

Importance Sampling

suppose we want to compute $E_f[I(x \in A)]$ but we can only sample from density g

$E_f[I(x \in A)]$

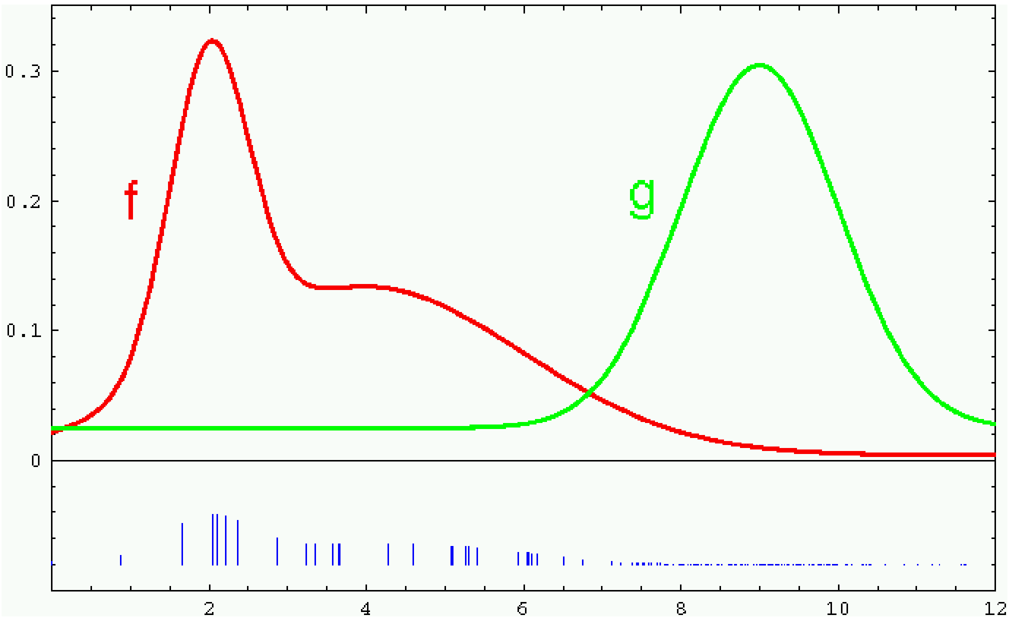
$= \int f(x) I(x \in A) dx$

$= \int \frac{f(x)}{g(x)} g(x) I(x \in A) dx$, provided $g(x) > 0$

$= \int w(x) g(x) I(x \in A) dx$

$= E_g[w(x) I(x \in A)]$

We need $f(x) > 0 \Rightarrow g(x) > 0$



Weight samples: $w = f\,/\,g$

Monte Carlo Localization (MCL)

$X_t = x_t^{[1]}, x_t^{[2]}, \dots x_t^{[M]}$ particles

Algorithm MCL(X_{t-1}, u_t, z_t,m):
$\bar{X}_{t-1} = X_t = \emptyset$

for all m in [M] do:

$x_t^{[m]} = \boldsymbol{sample\_motion\_model}(u_t\ x_{t-1}^{[m]})$

$w_t^{[m]} = \boldsymbol{measurement\_model}(z_t, x_t^{[m],m})$

$\bar{X}_t = \bar{X}_t + \langle\, x_t^{[m]}, w_t^{[m]} \rangle$

end for

for all m in [M] do:

draw i $with\ probability\ \propto w_t^{[i]}$

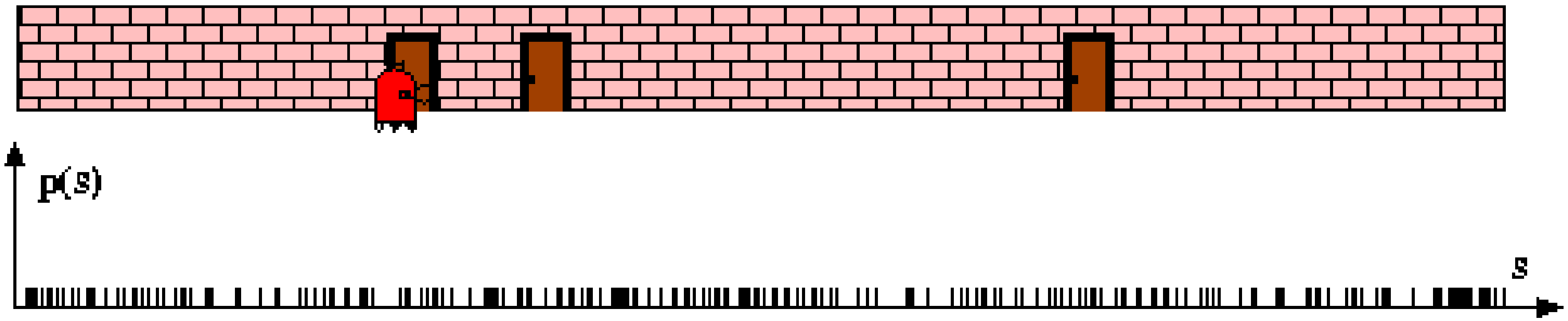
add $x_t^{[i]}$ $to\ X_t$

end for

return X_t

Plug in motion and measurement models in the particle filter

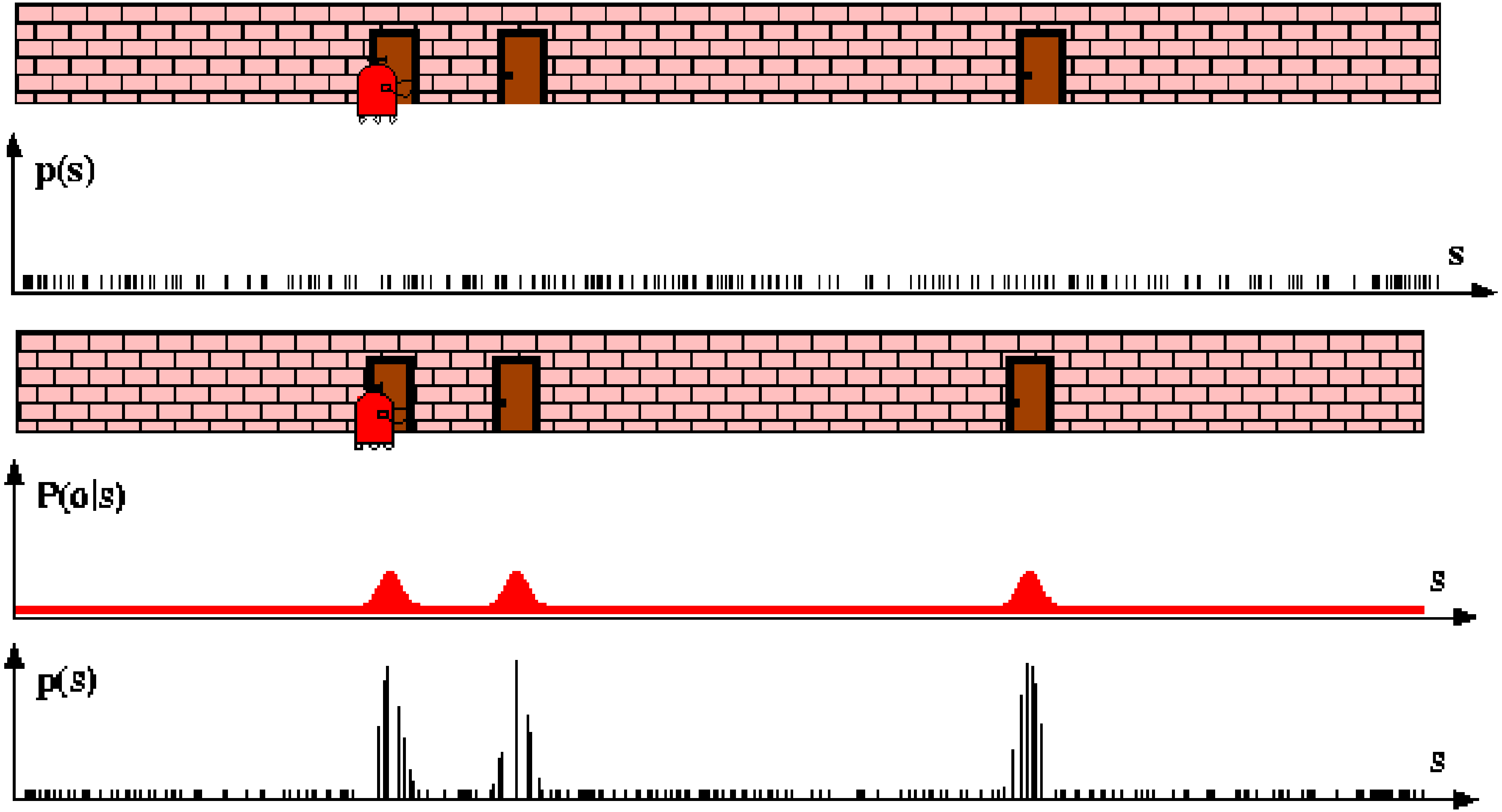
Particle Filters



Sensor Information: Importance Sampling

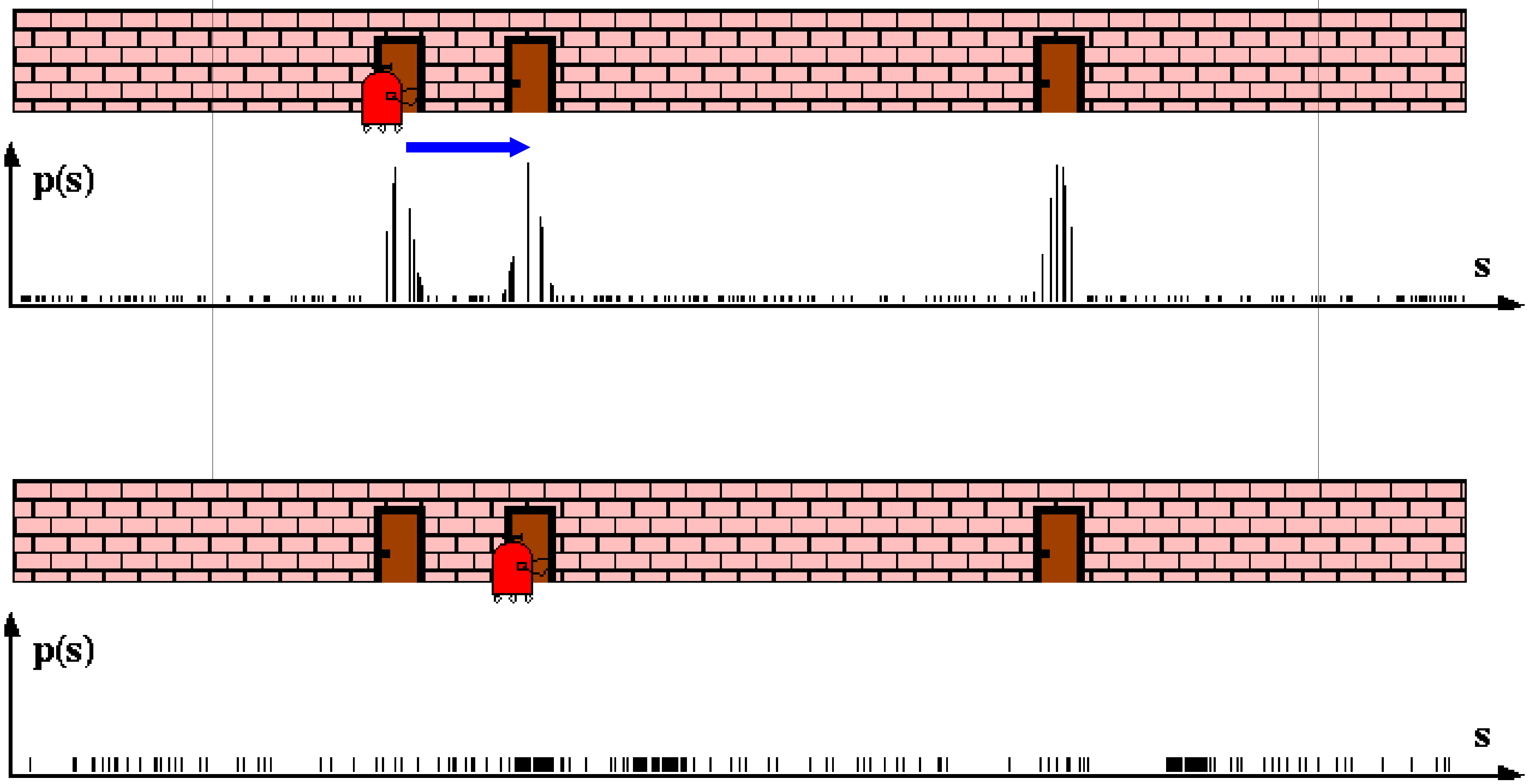
$$Bel(x) \leftarrow \alpha\ p(z \mid x)\ Bel^{-}(x)$$

$$w \leftarrow \frac{\alpha\ p(z \mid x)\ Bel^{-}(x)}{Bel^{-}(x)} = \alpha\ p(z \mid x)$$



Robot Motion

$$Bel^{-}(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\, \mathrm{d}x'$$



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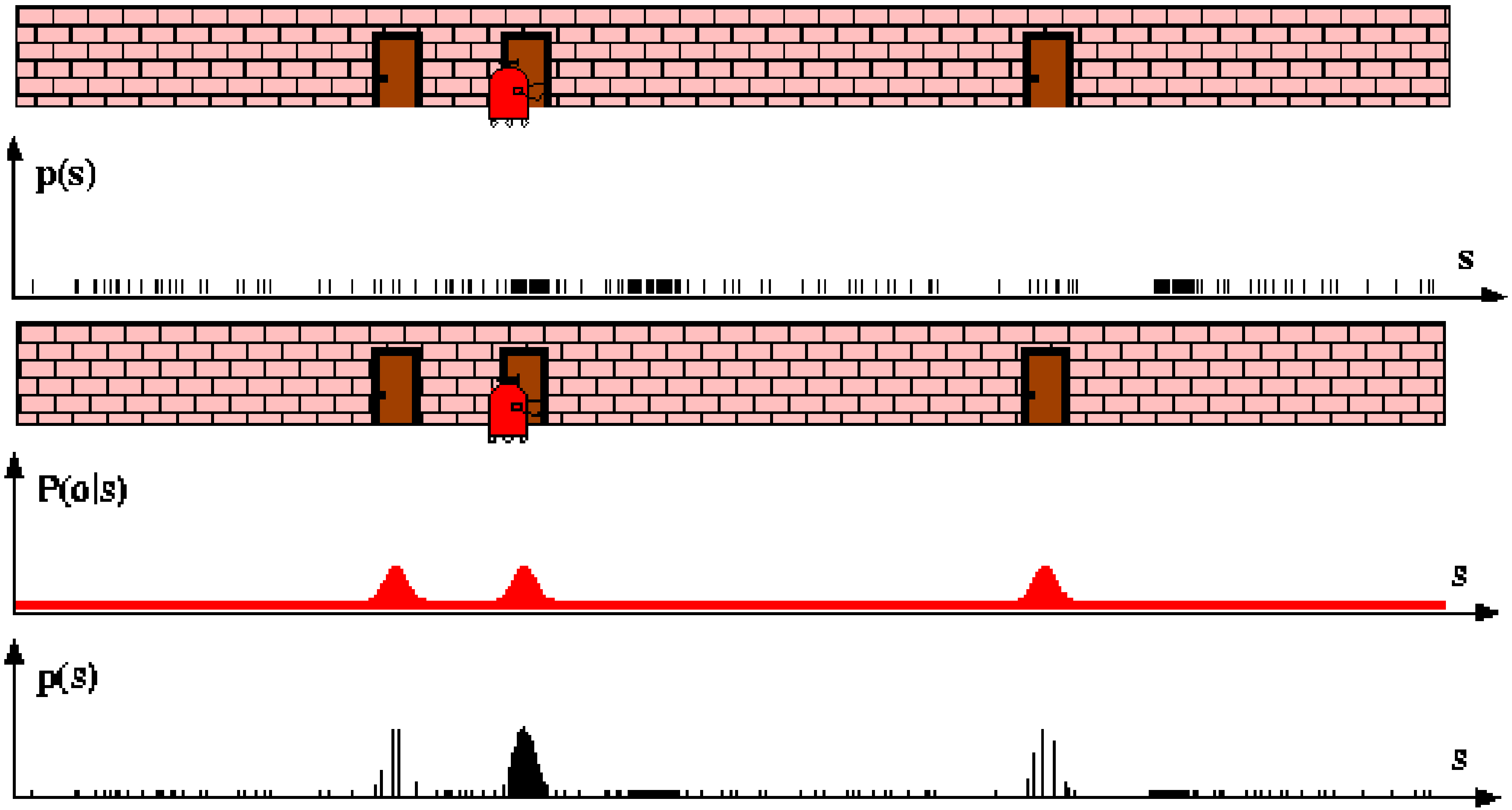
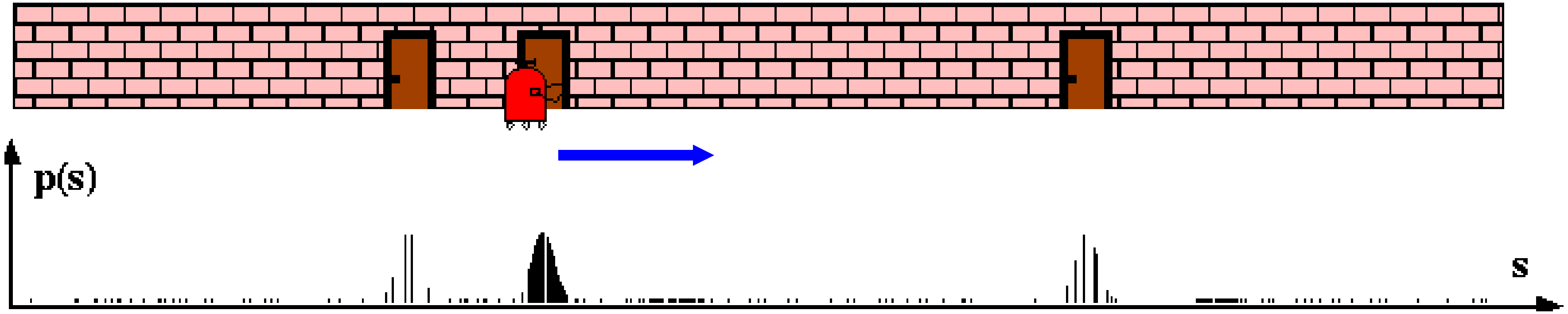
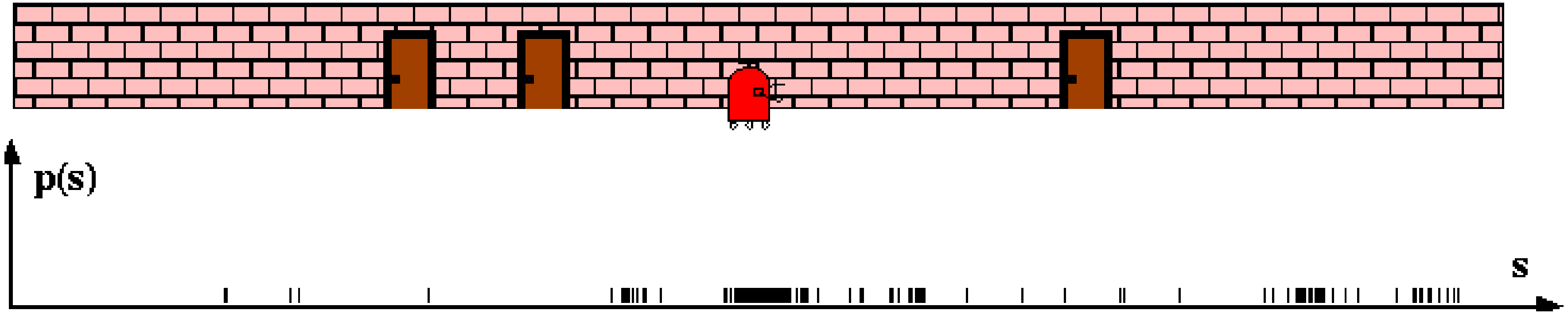
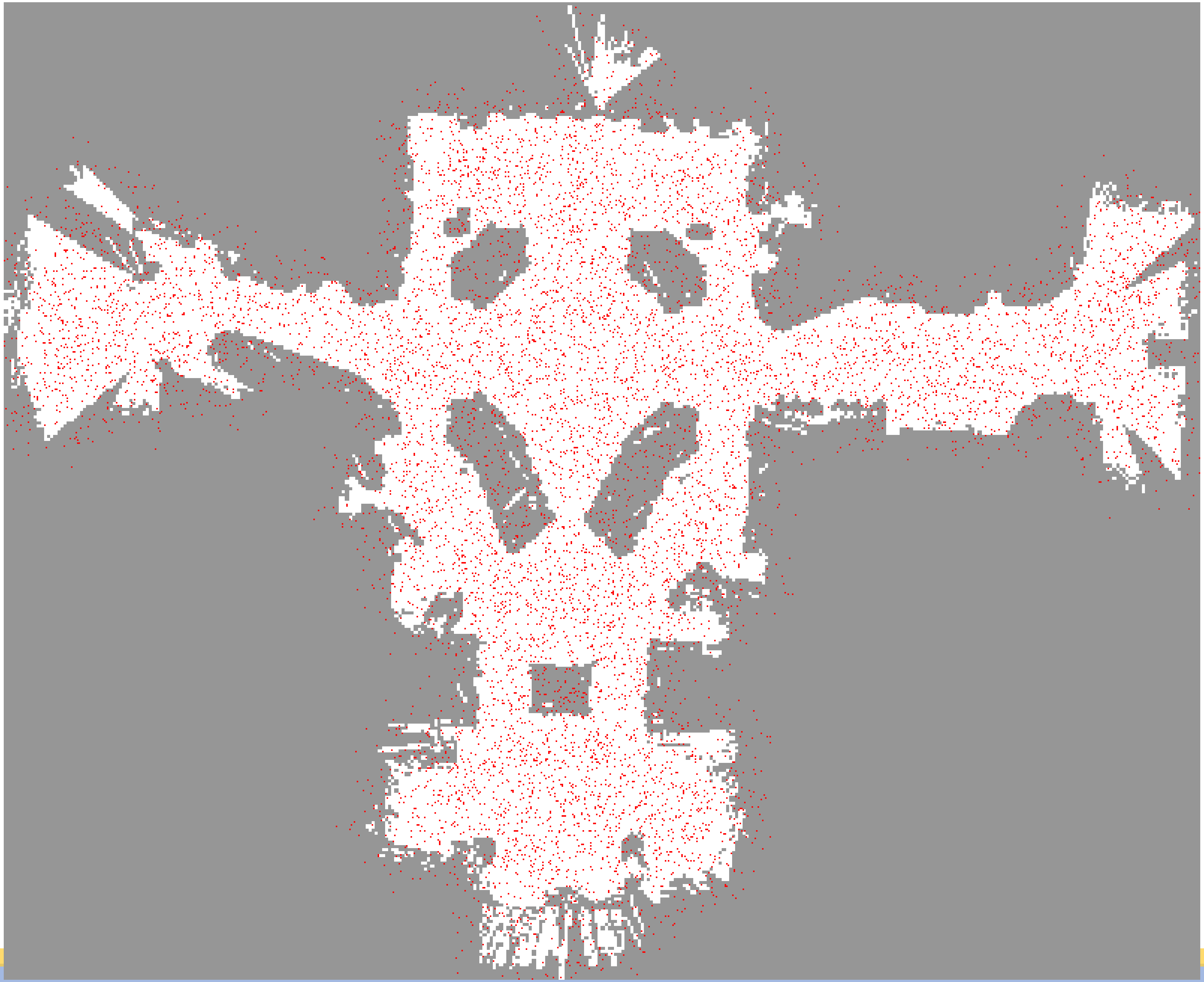


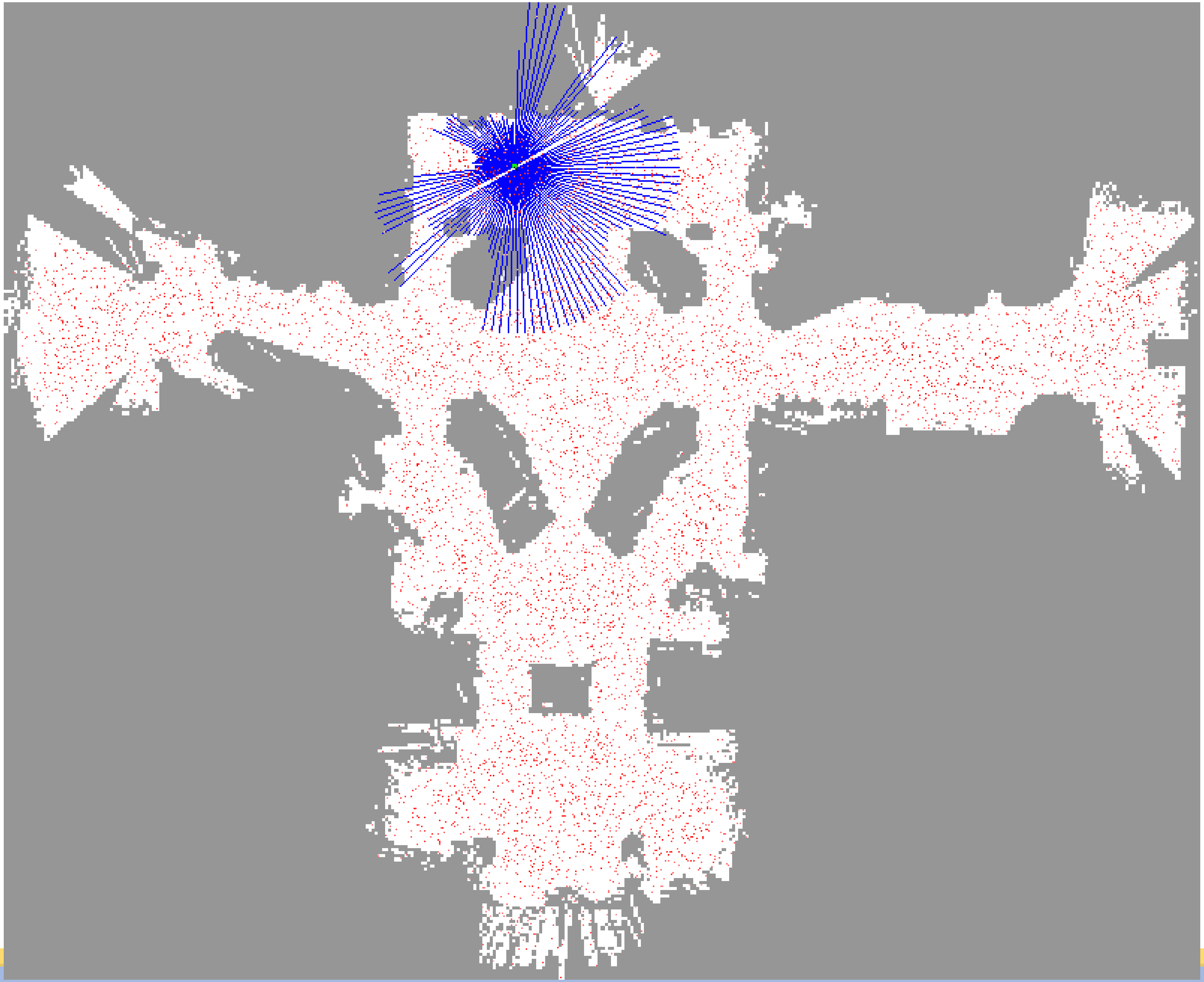
Robot Motion

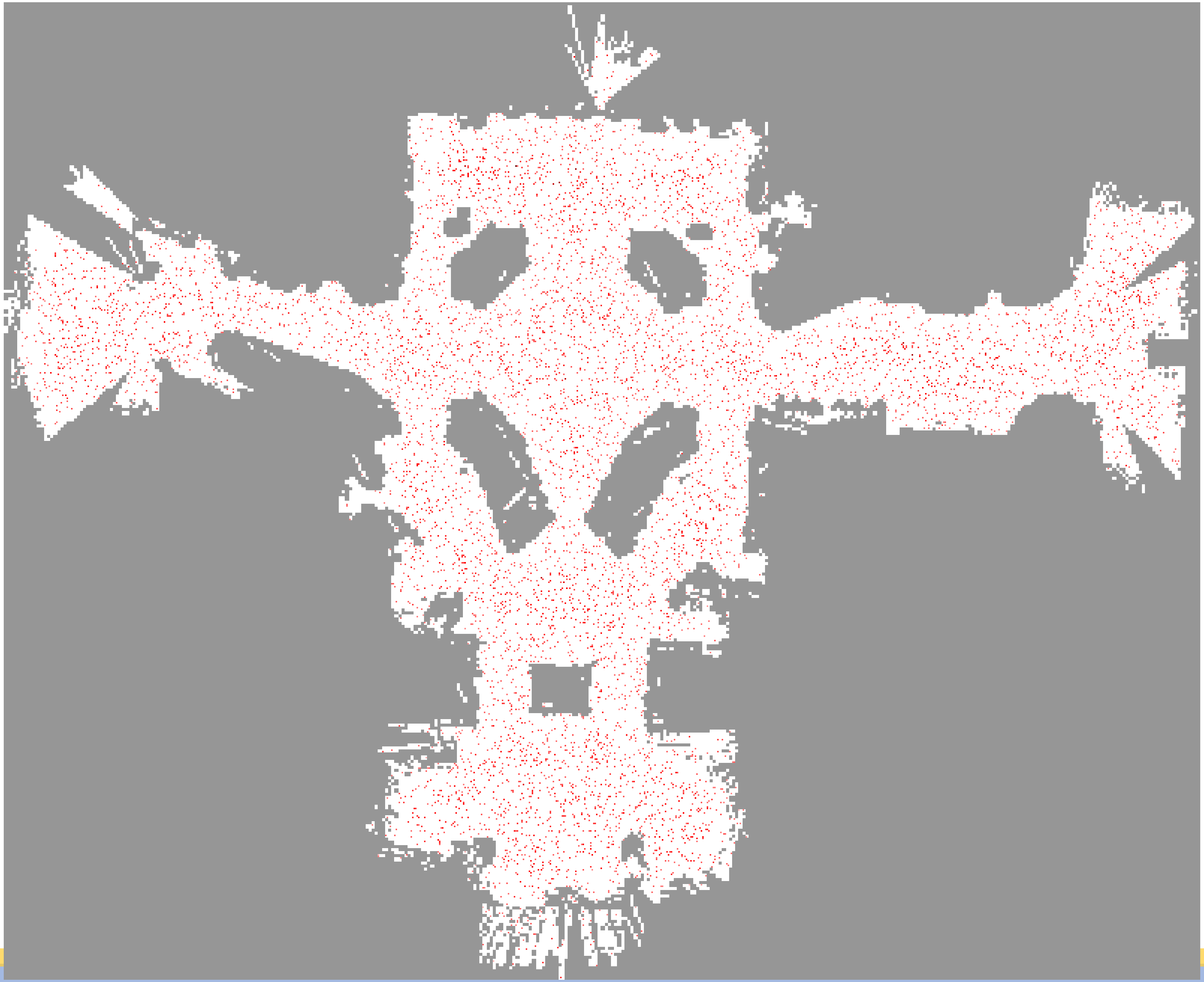
$$Bel^{-}(x) \leftarrow \int p(x \mid u, x')\, Bel(x')\ \mathrm{d}x'$$

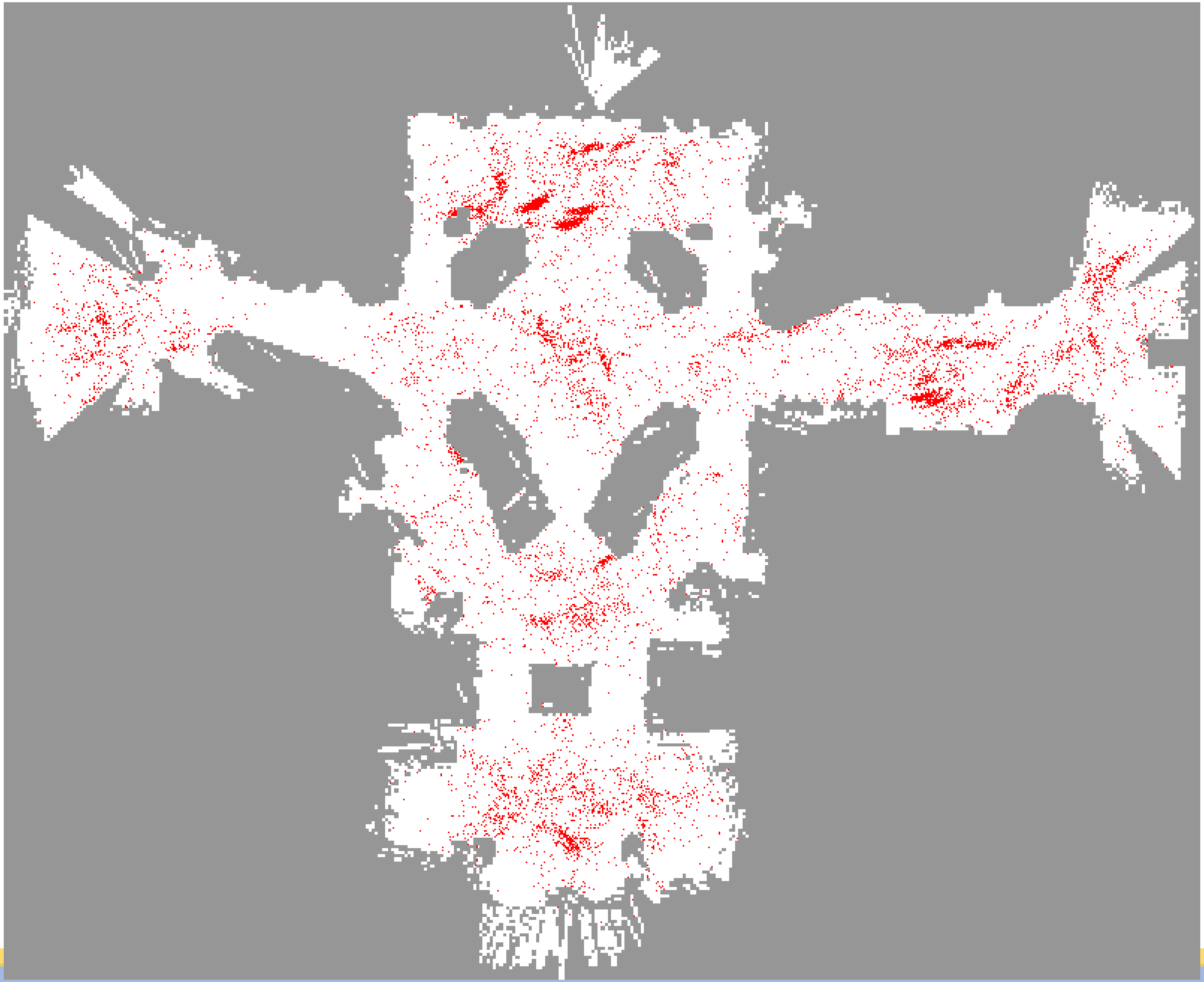


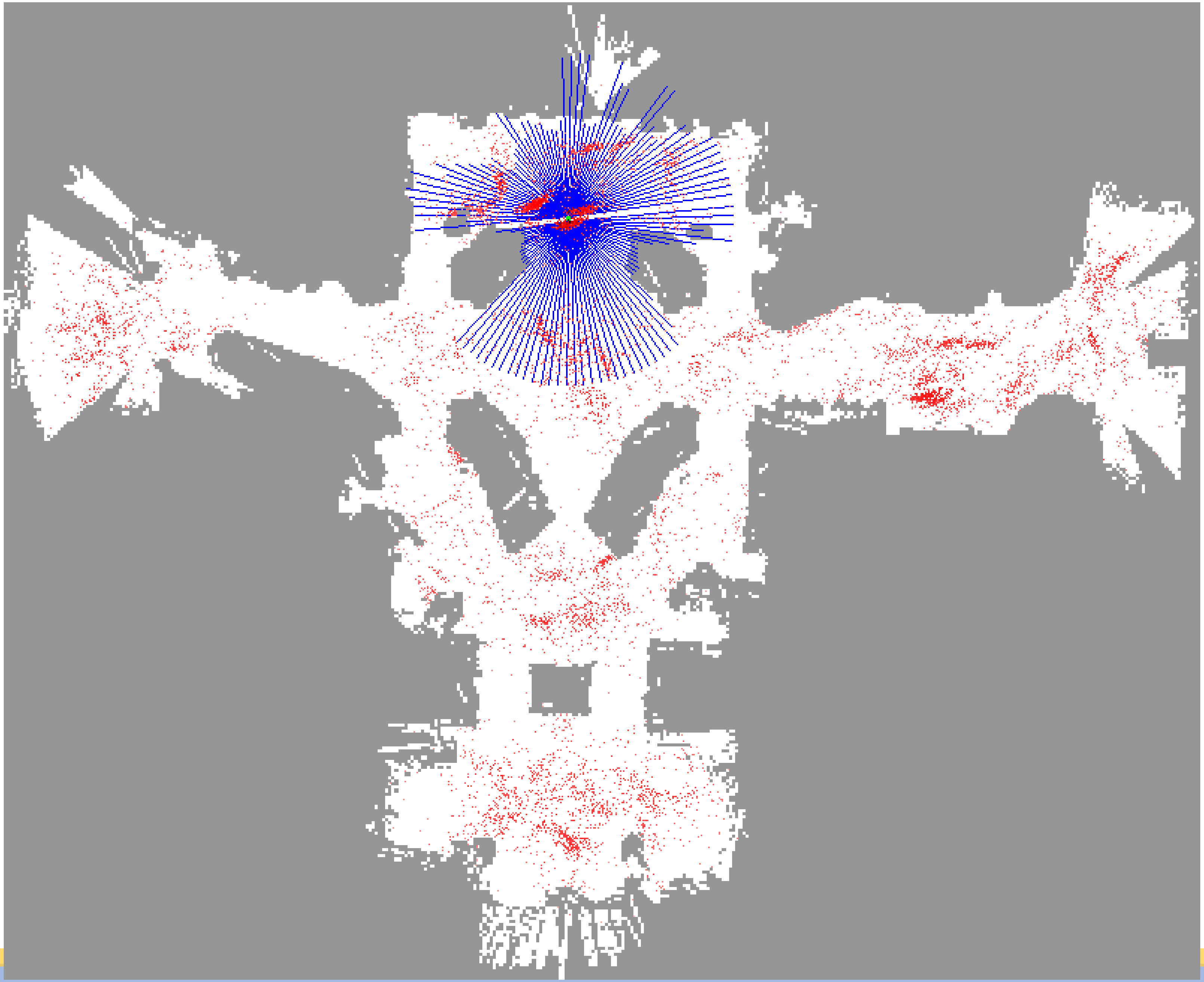


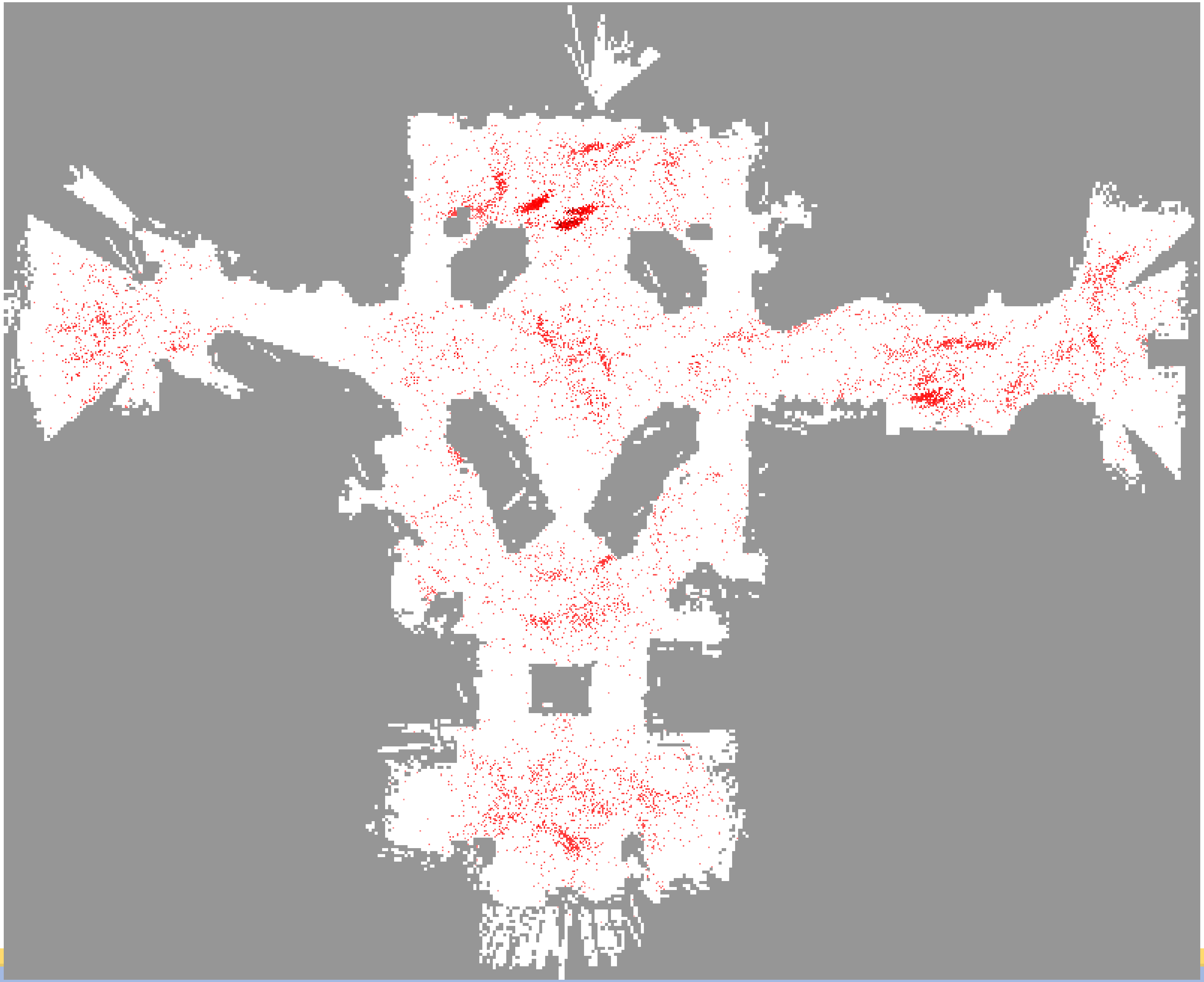


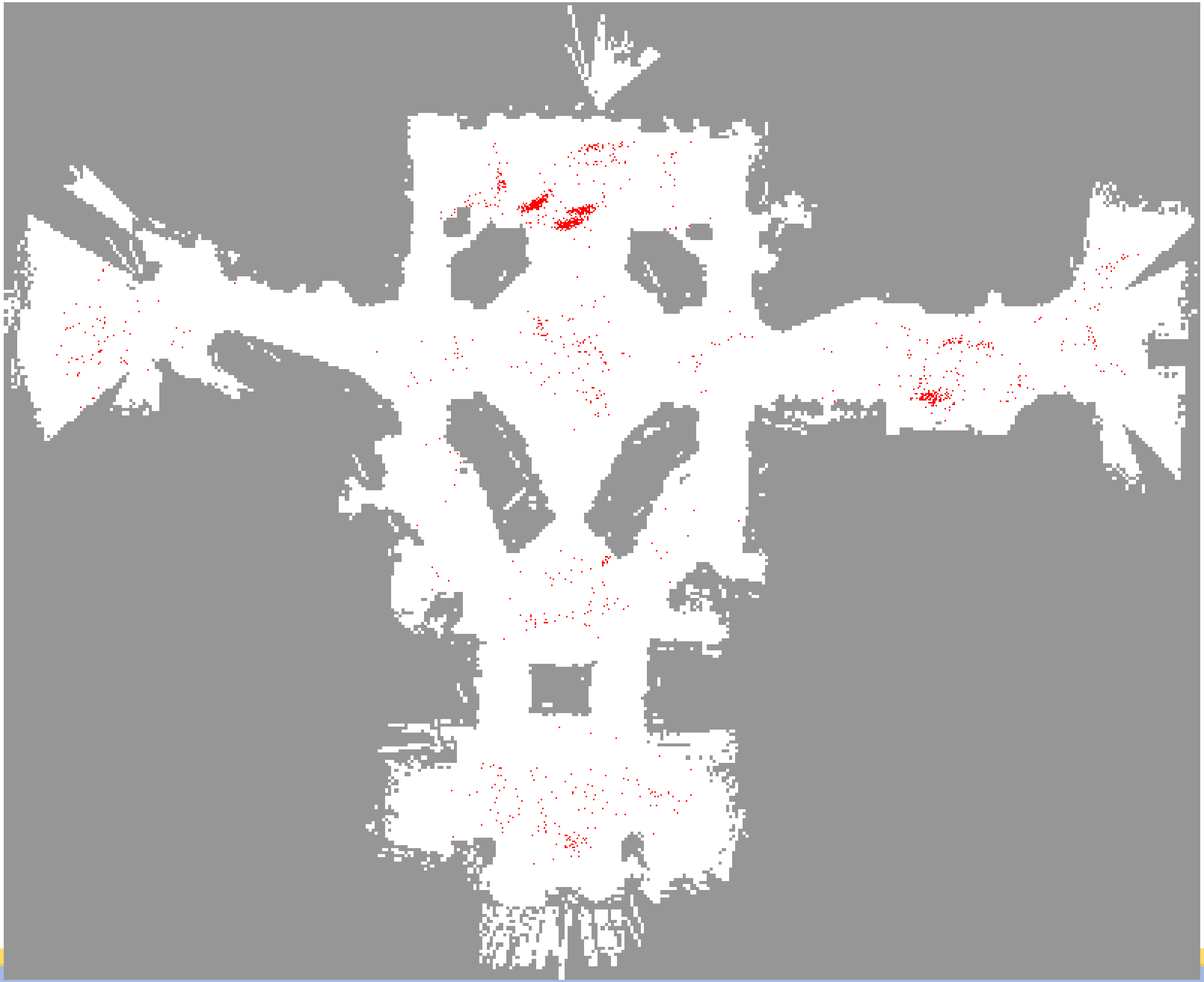


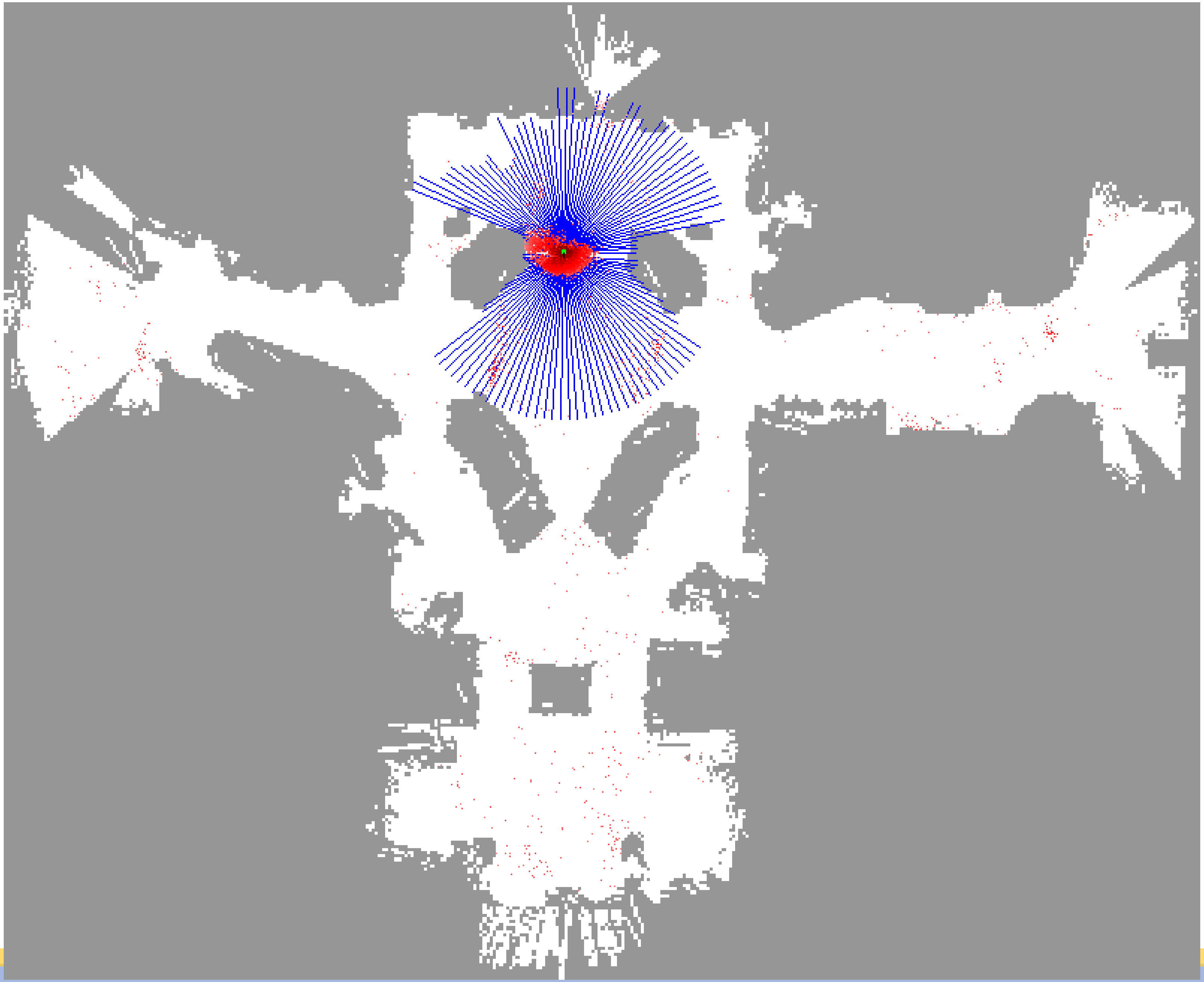


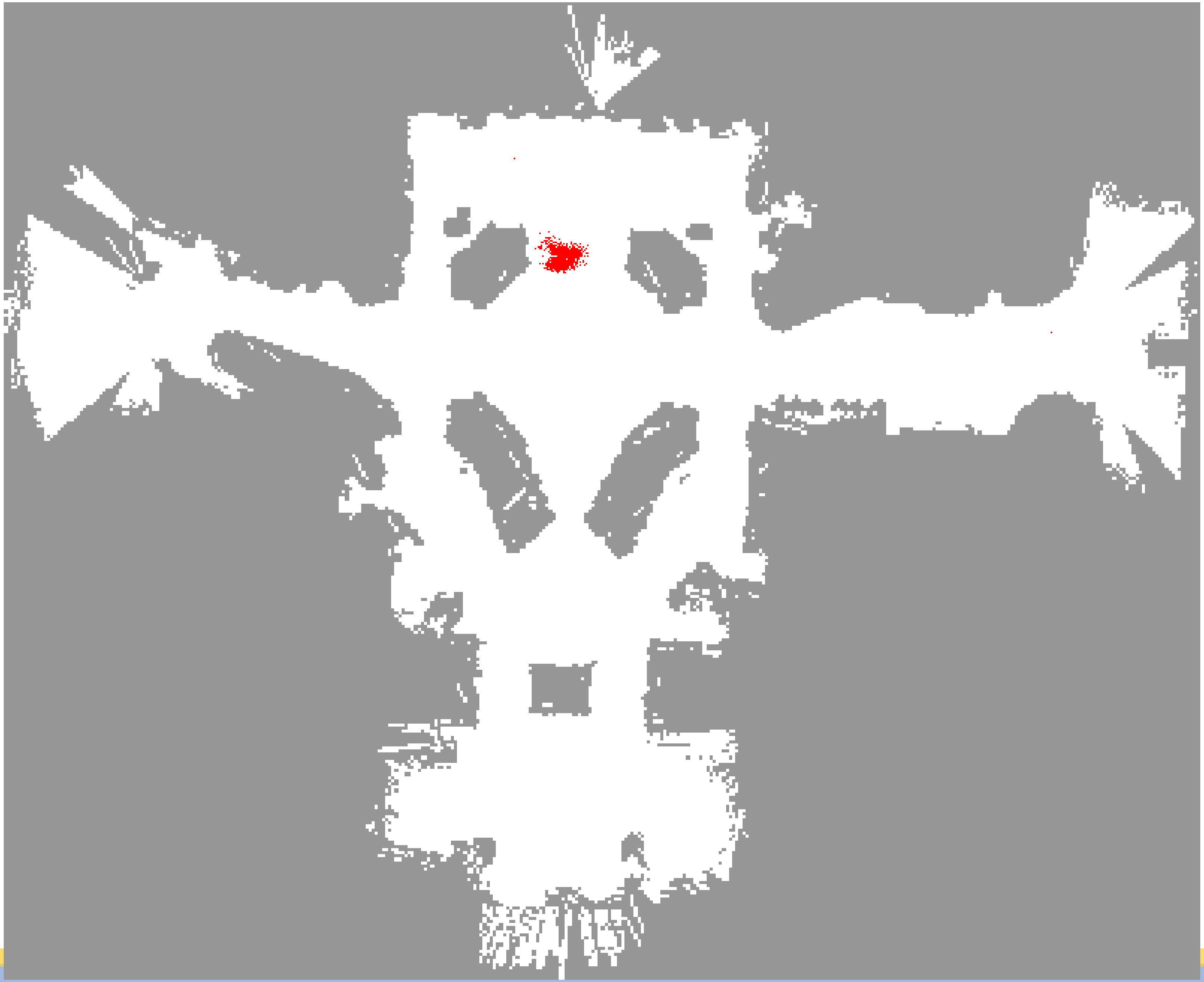


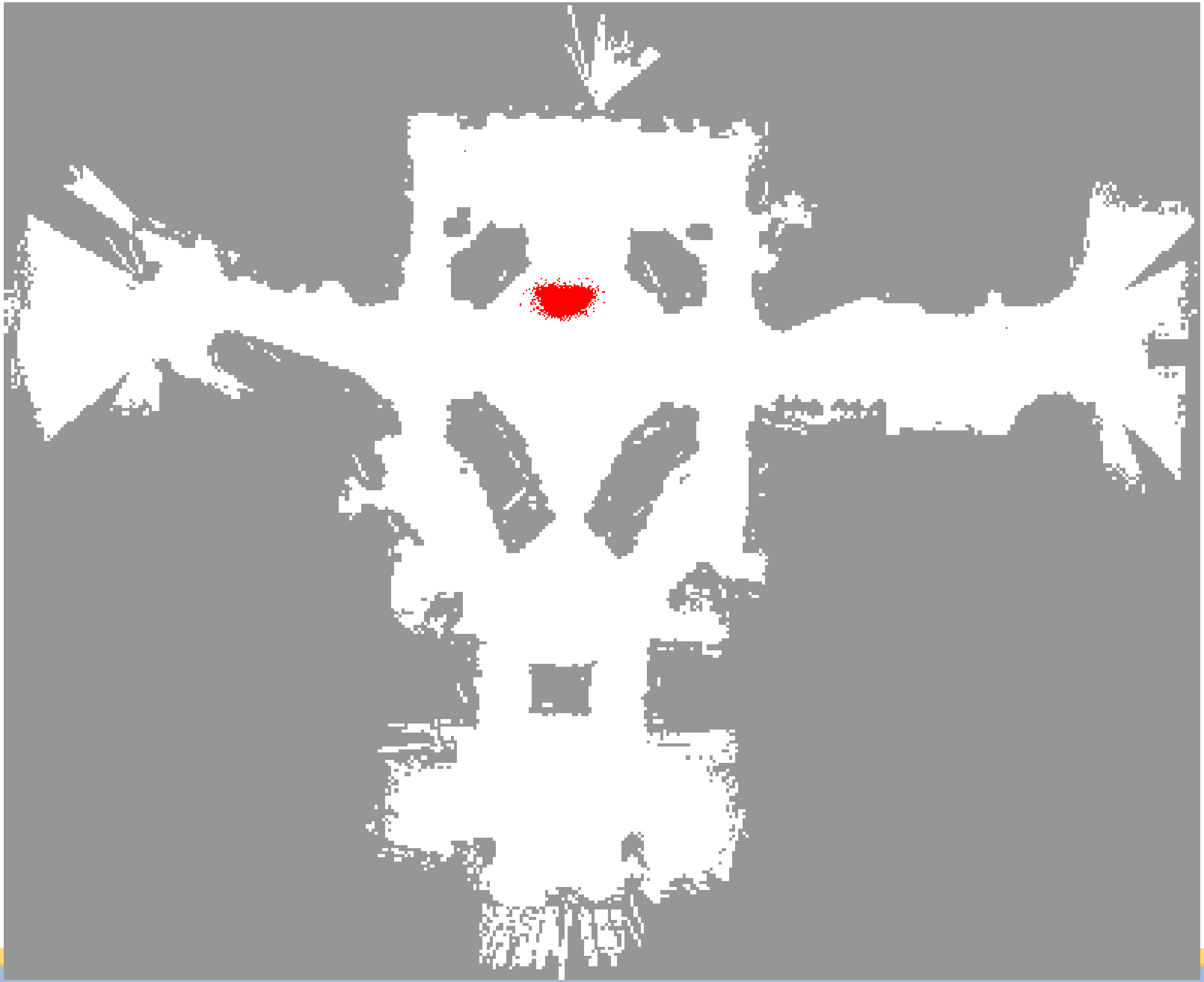


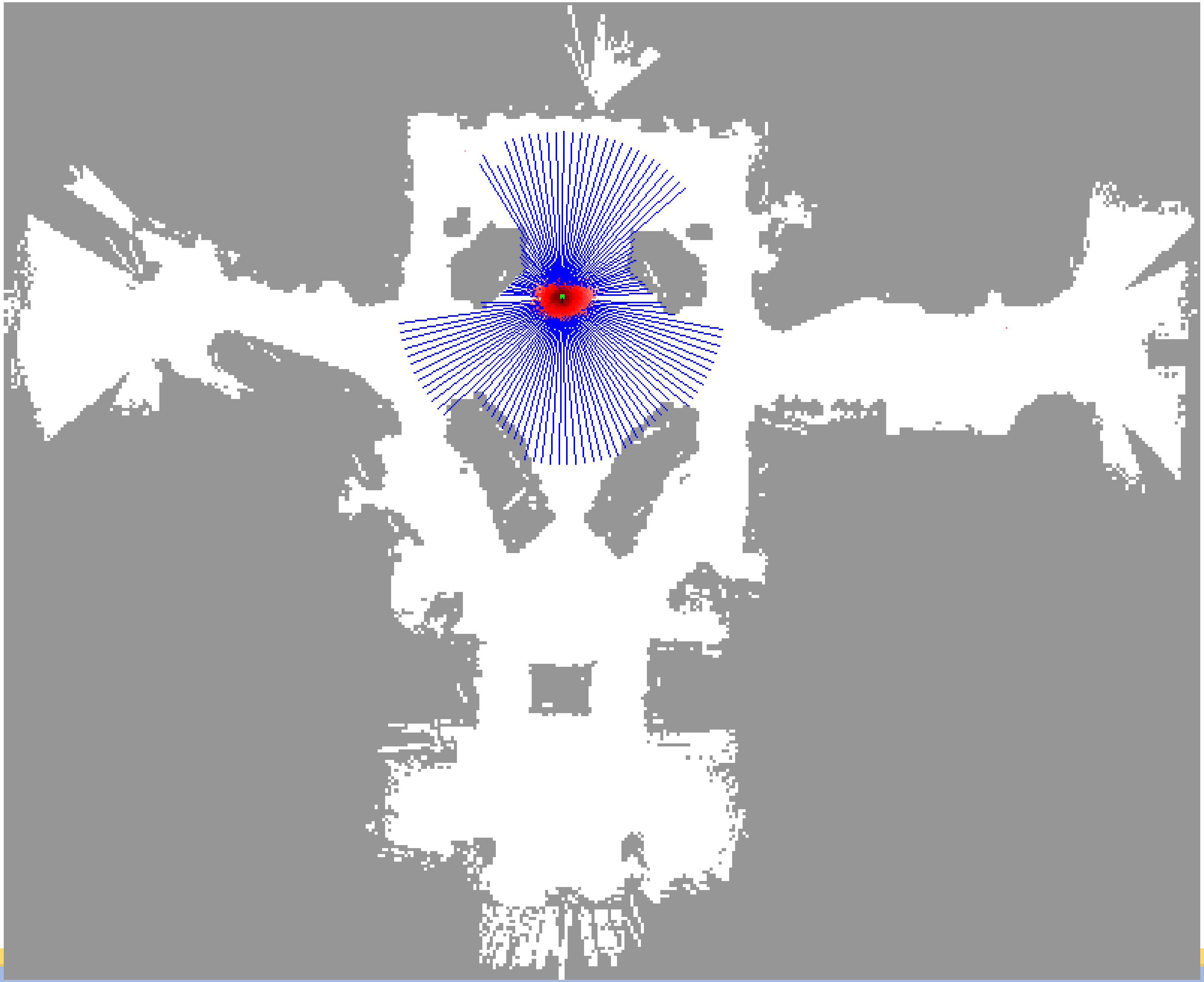


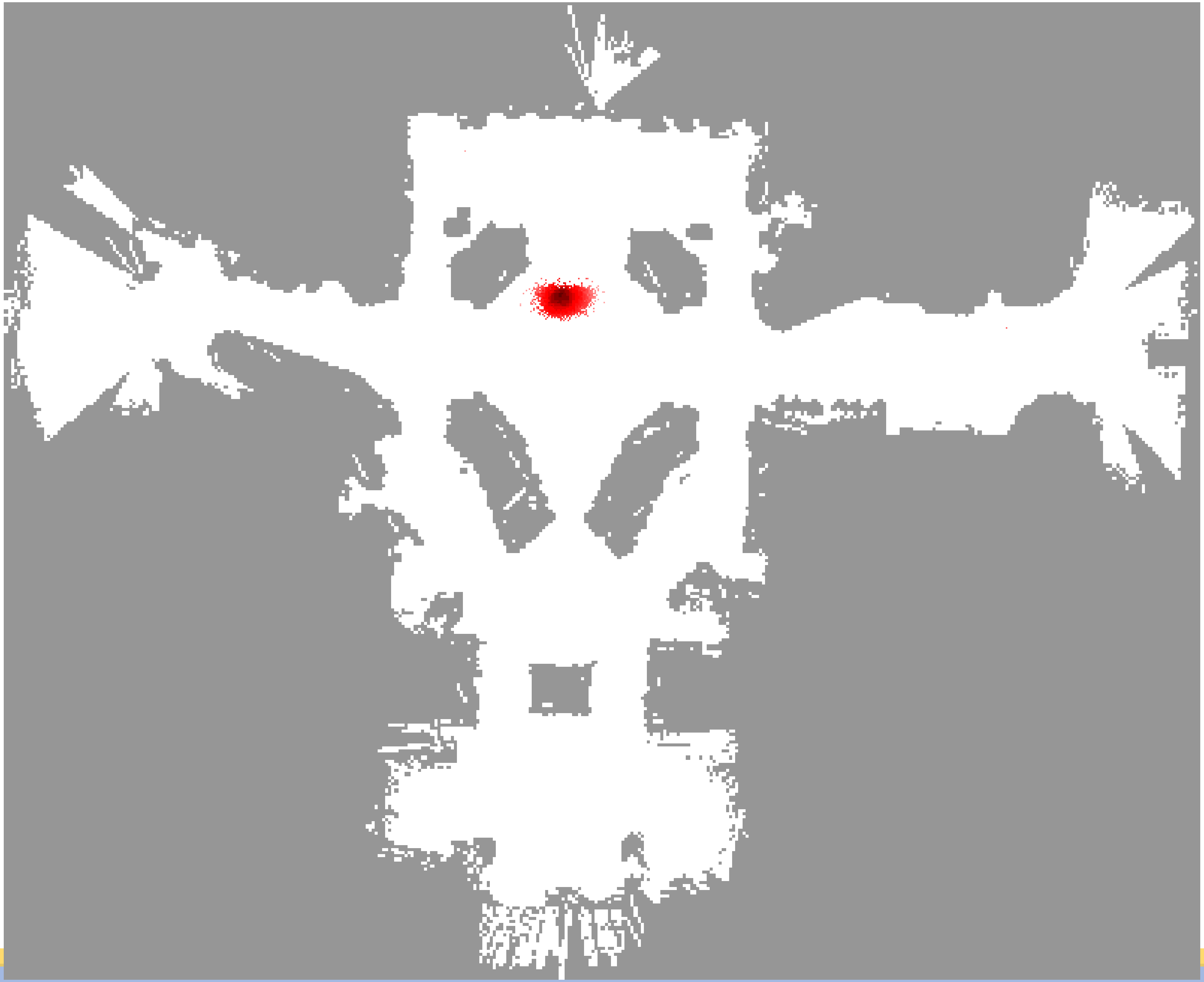


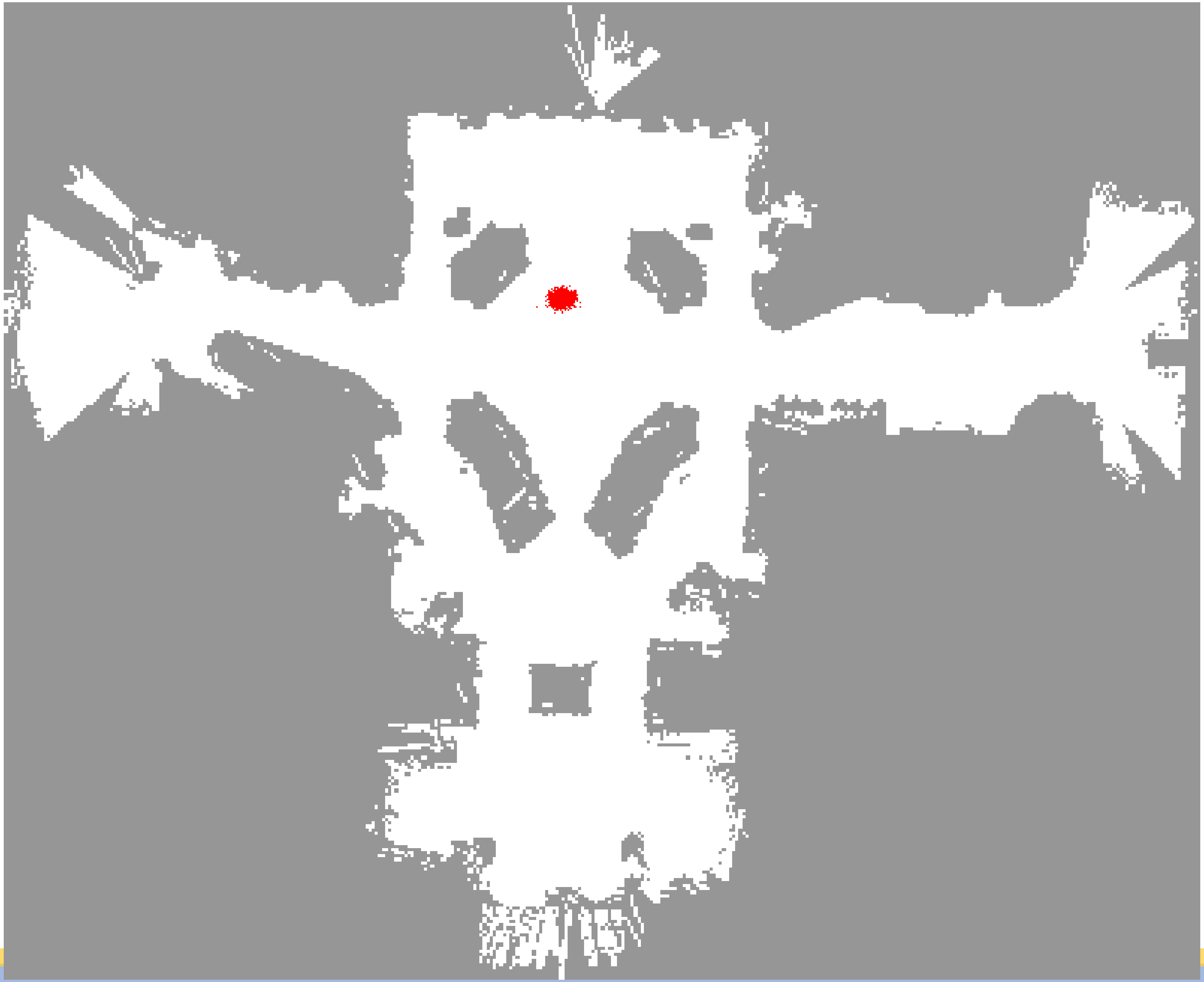


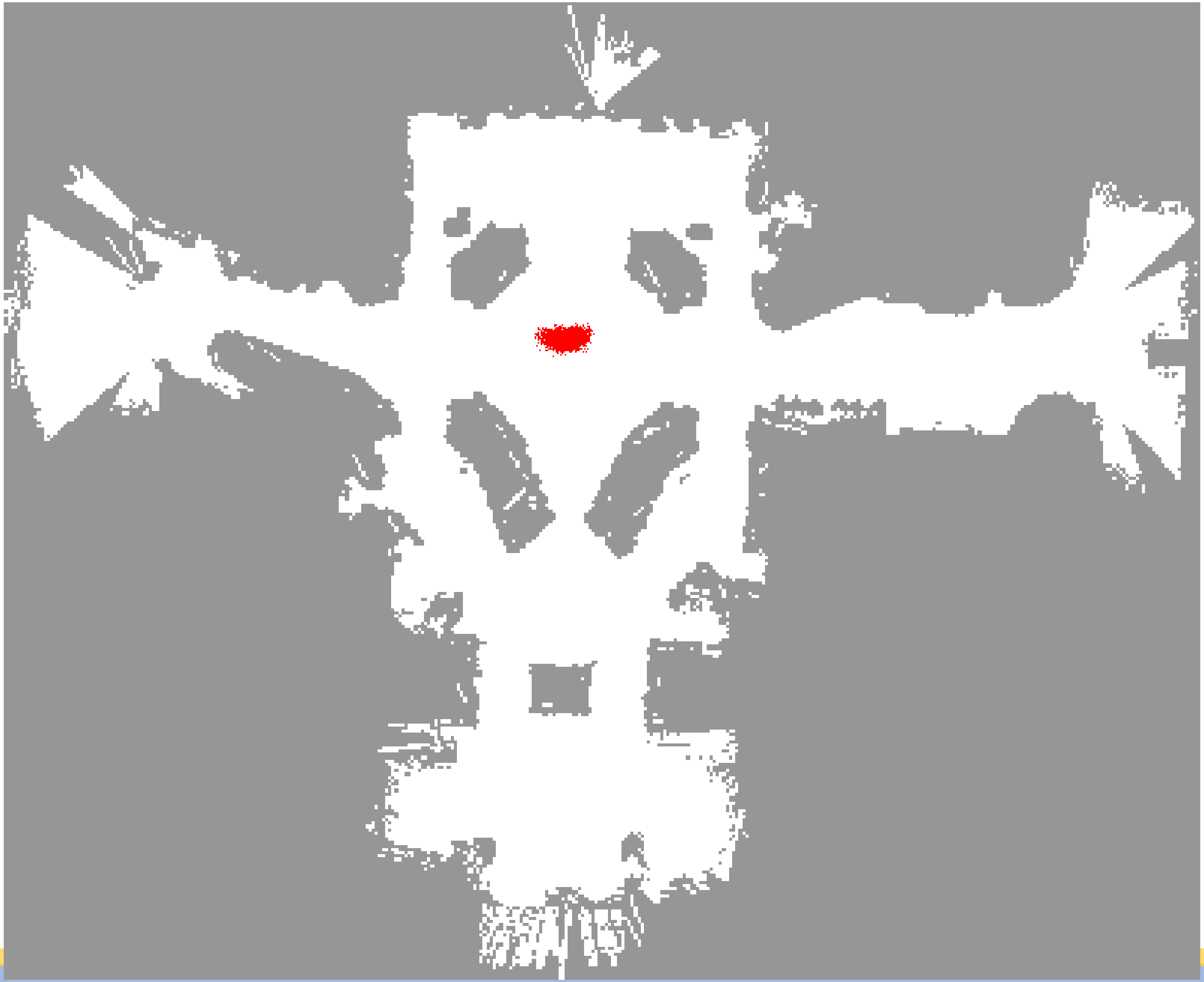


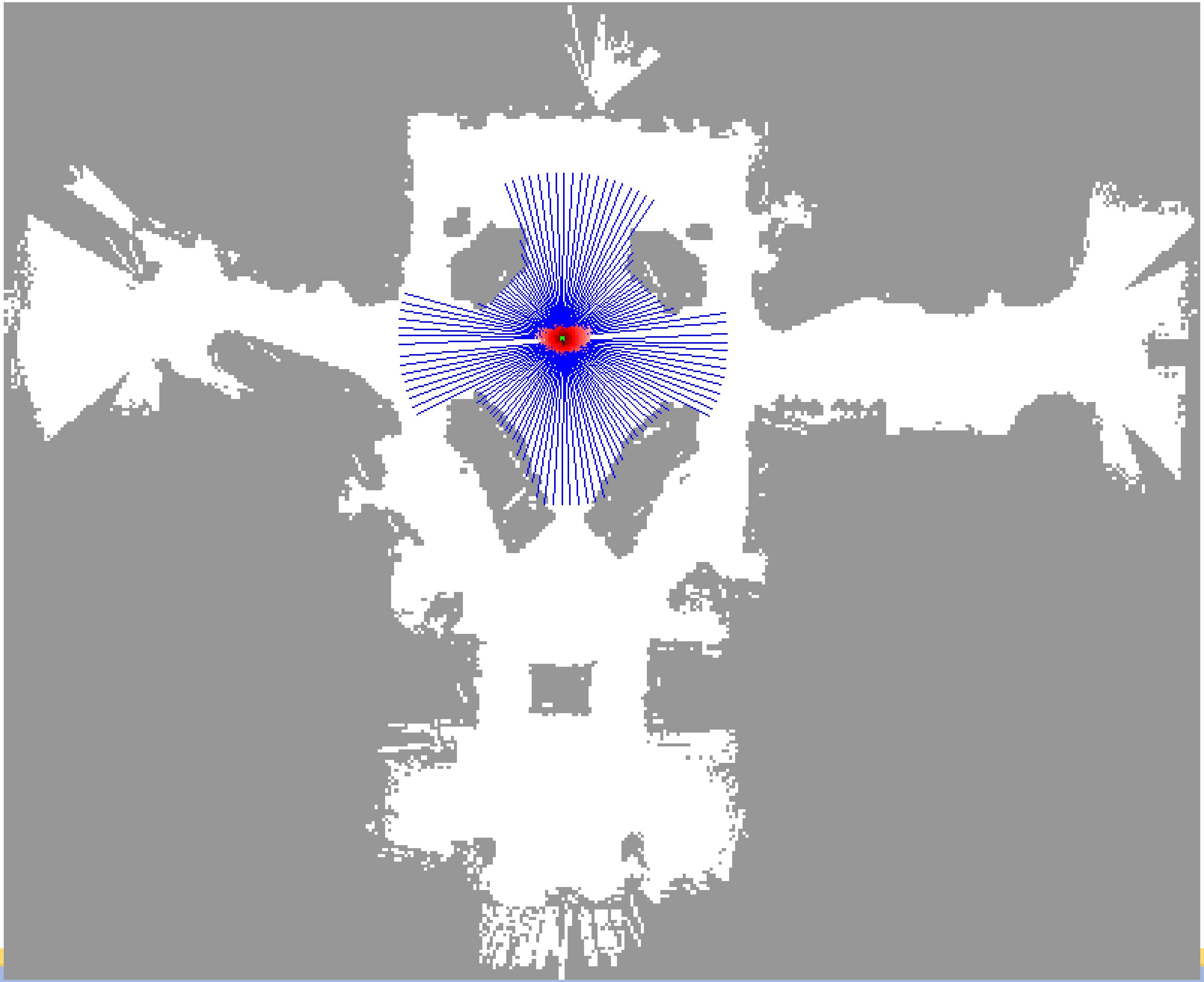


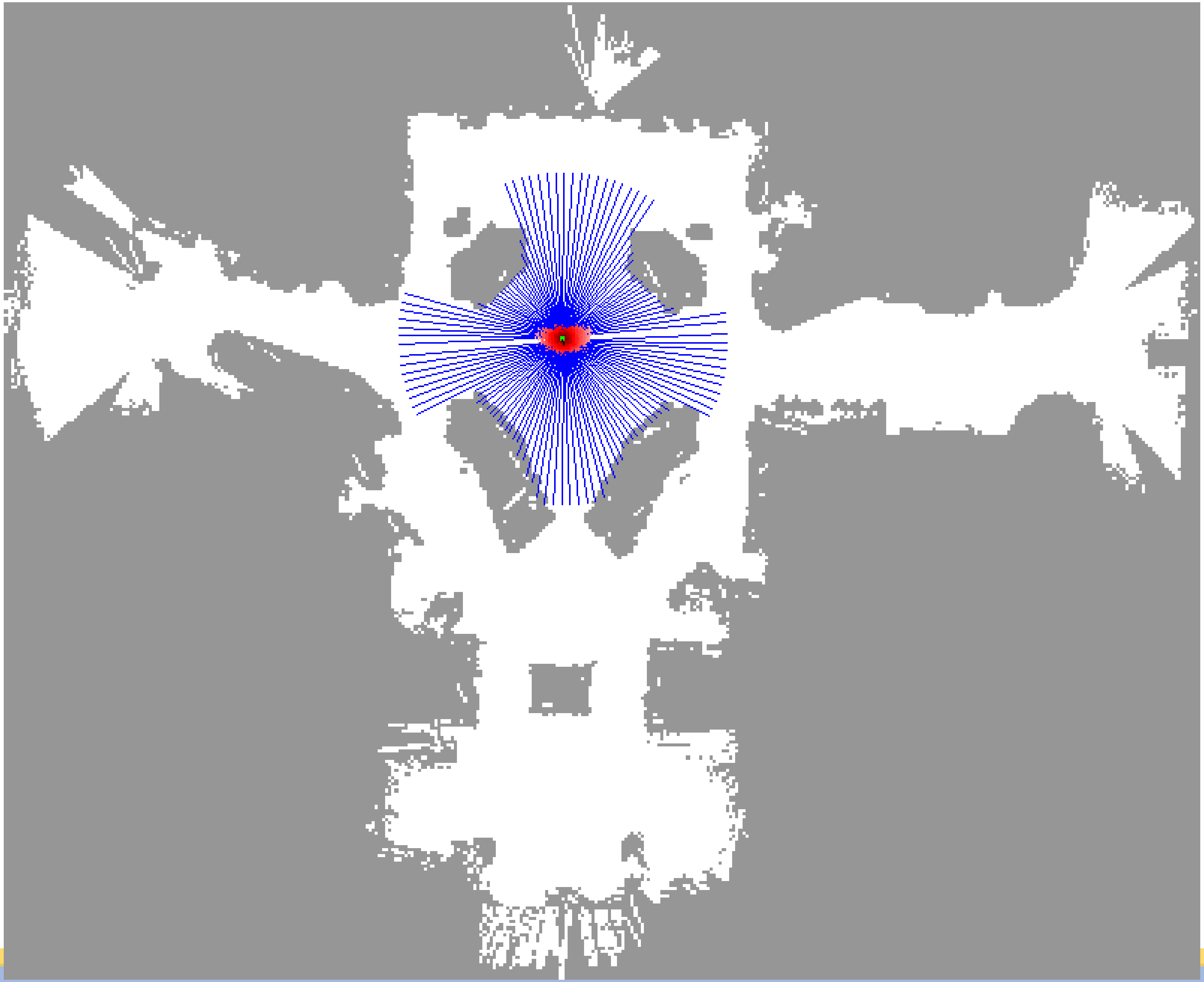




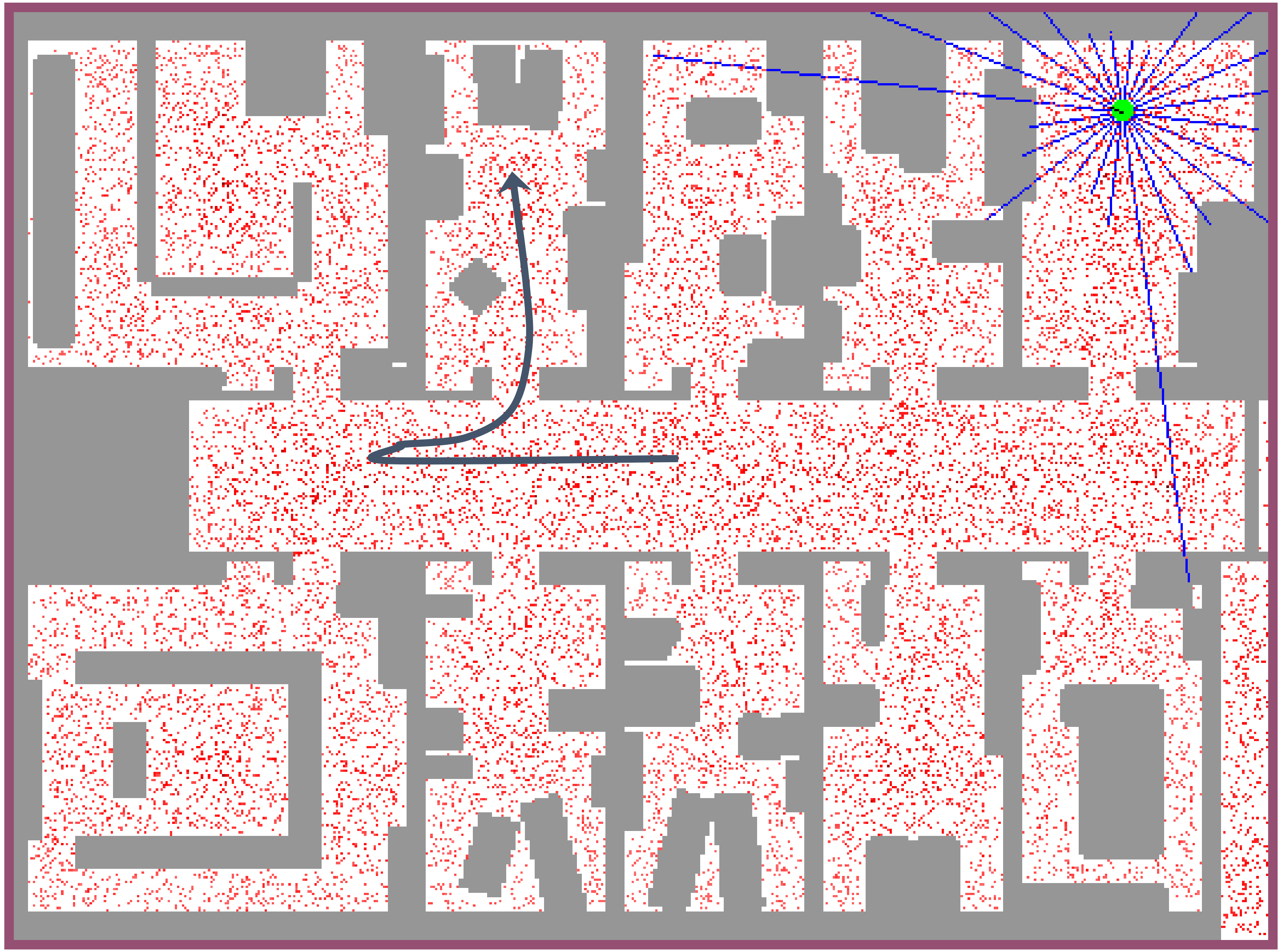




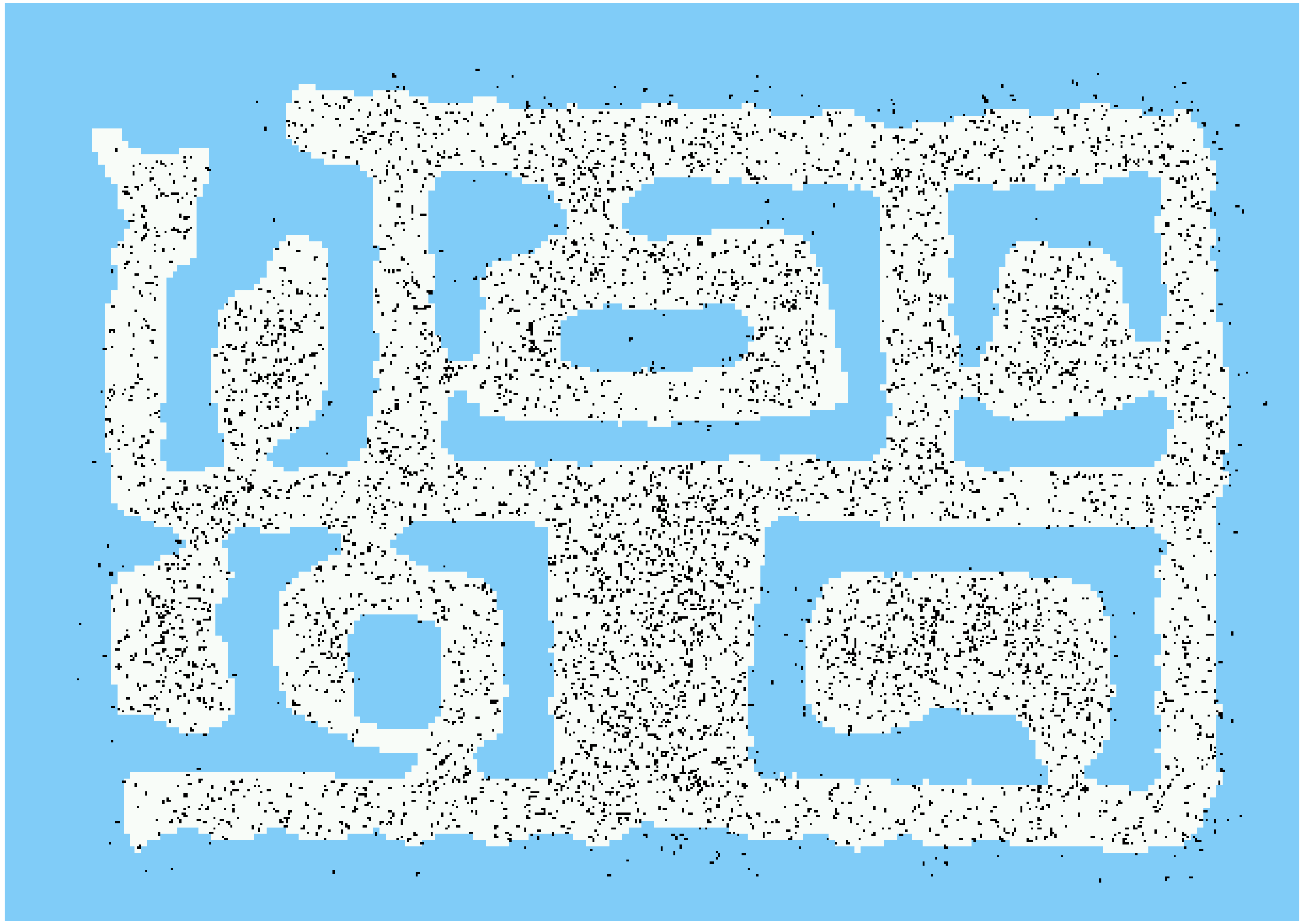




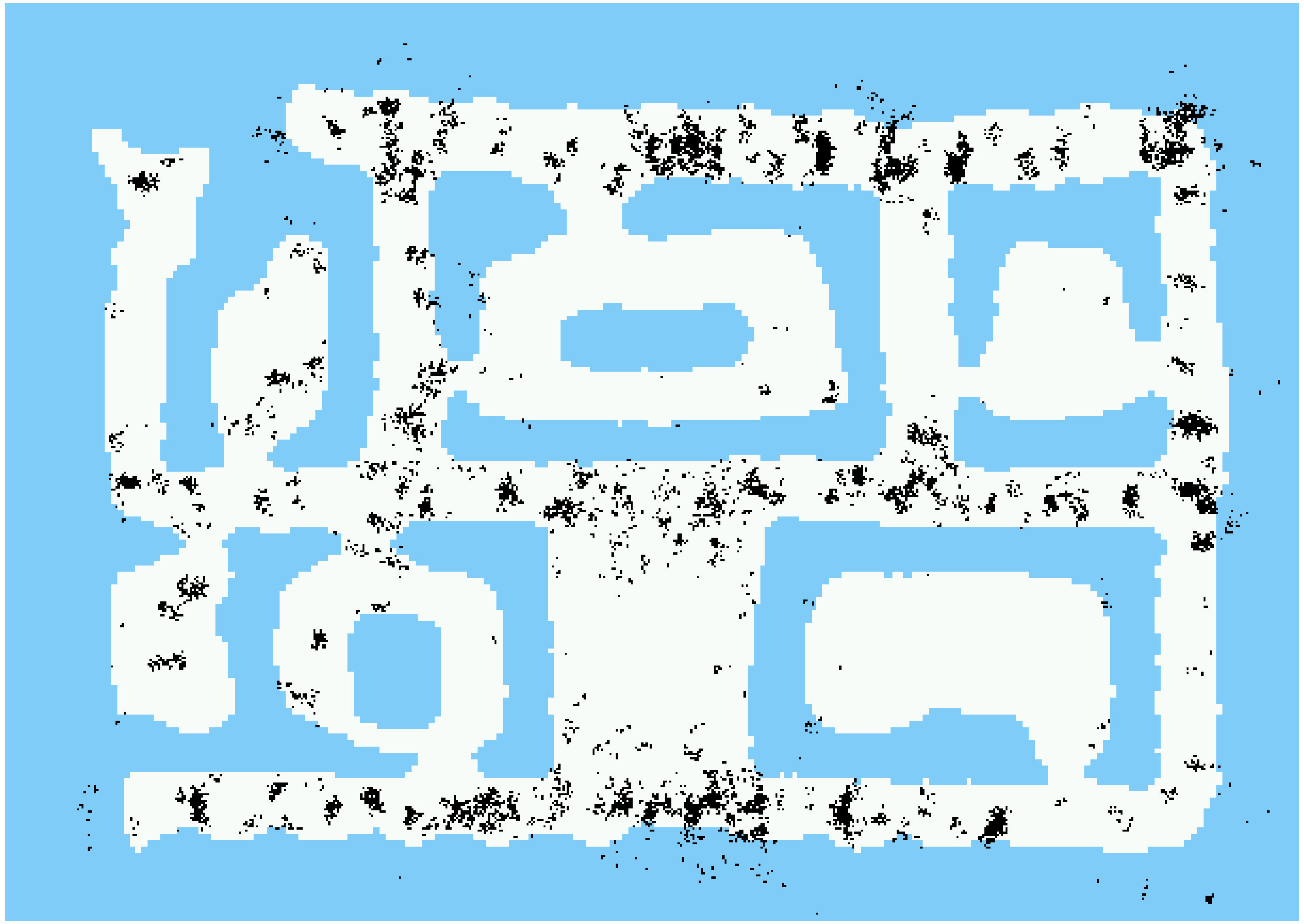
Sample-based Localization (sonar)



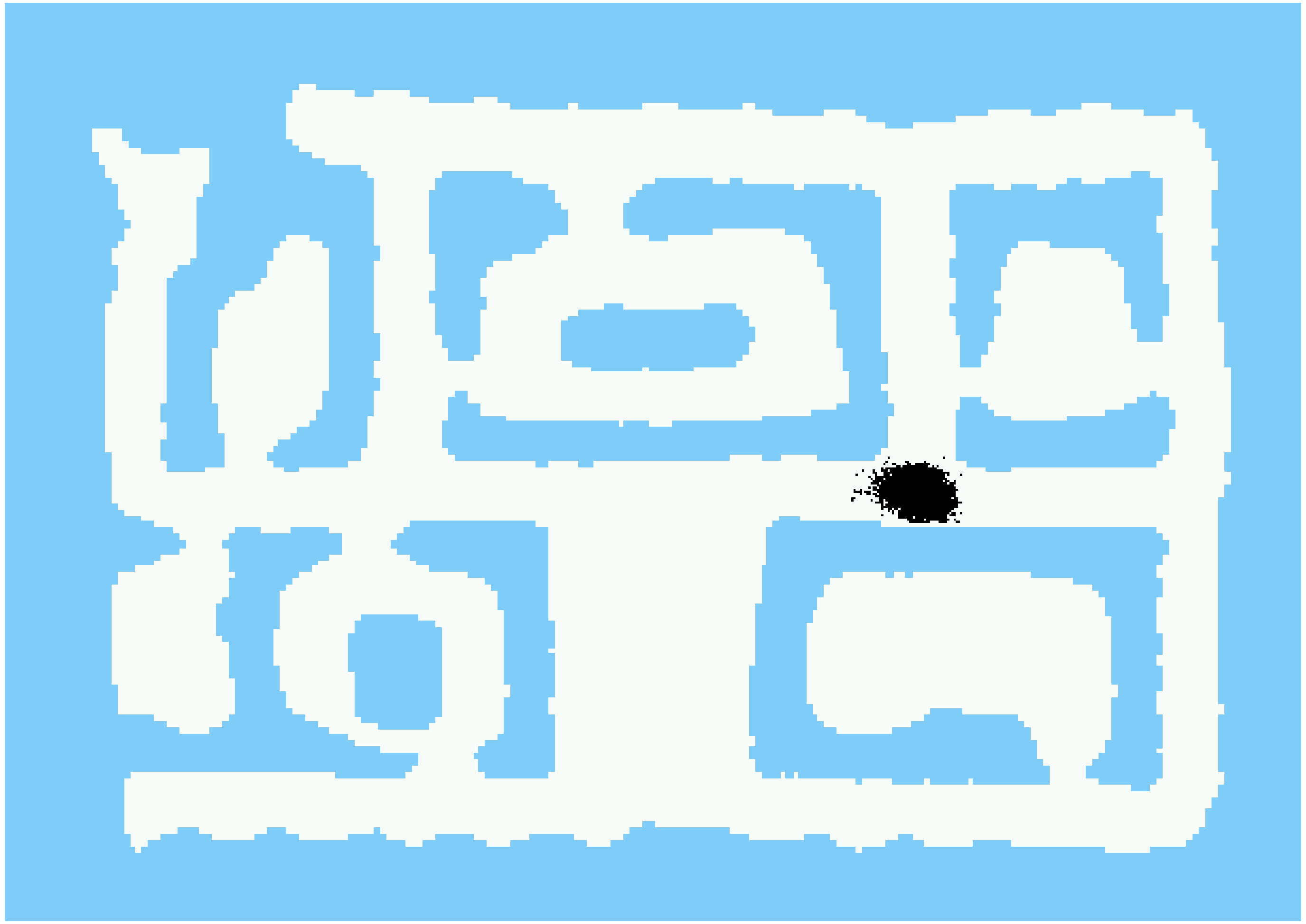
Initial Distribution



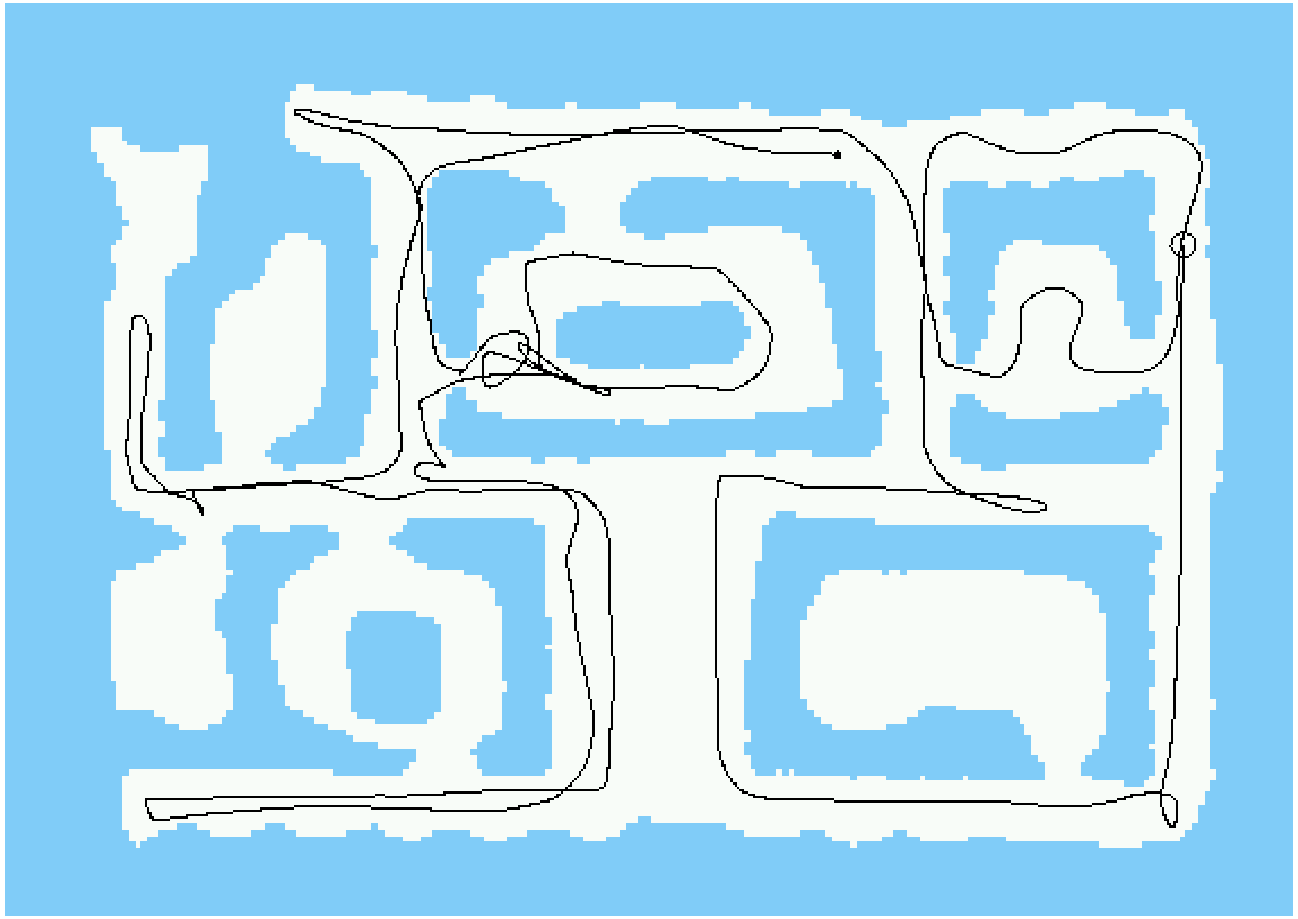
After Incorporating Ten Ultrasound Scans



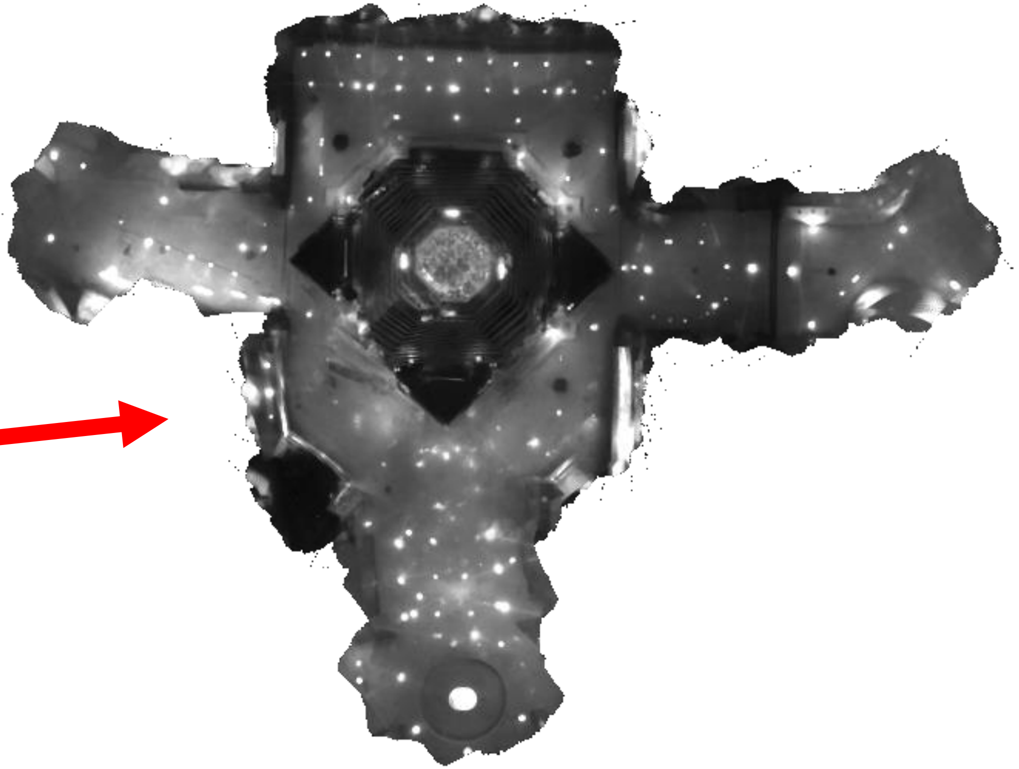
After Incorporating 65 Ultrasound Scans



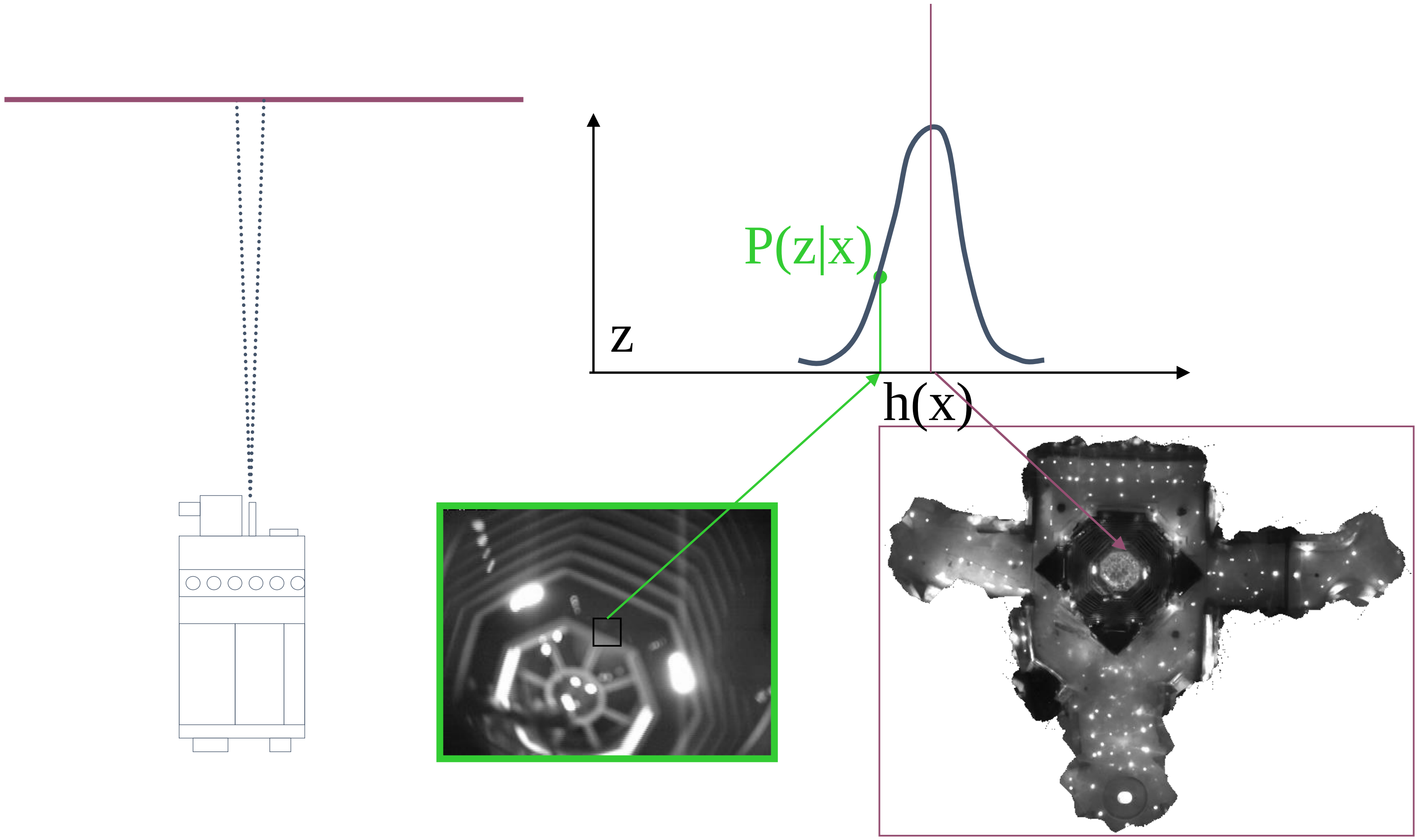
Estimated Path



Using Ceiling Maps for Localization

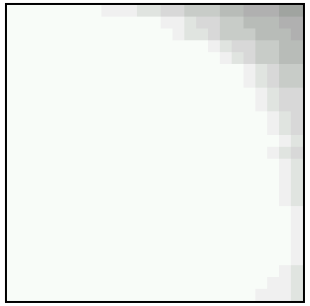


Vision-based Localization

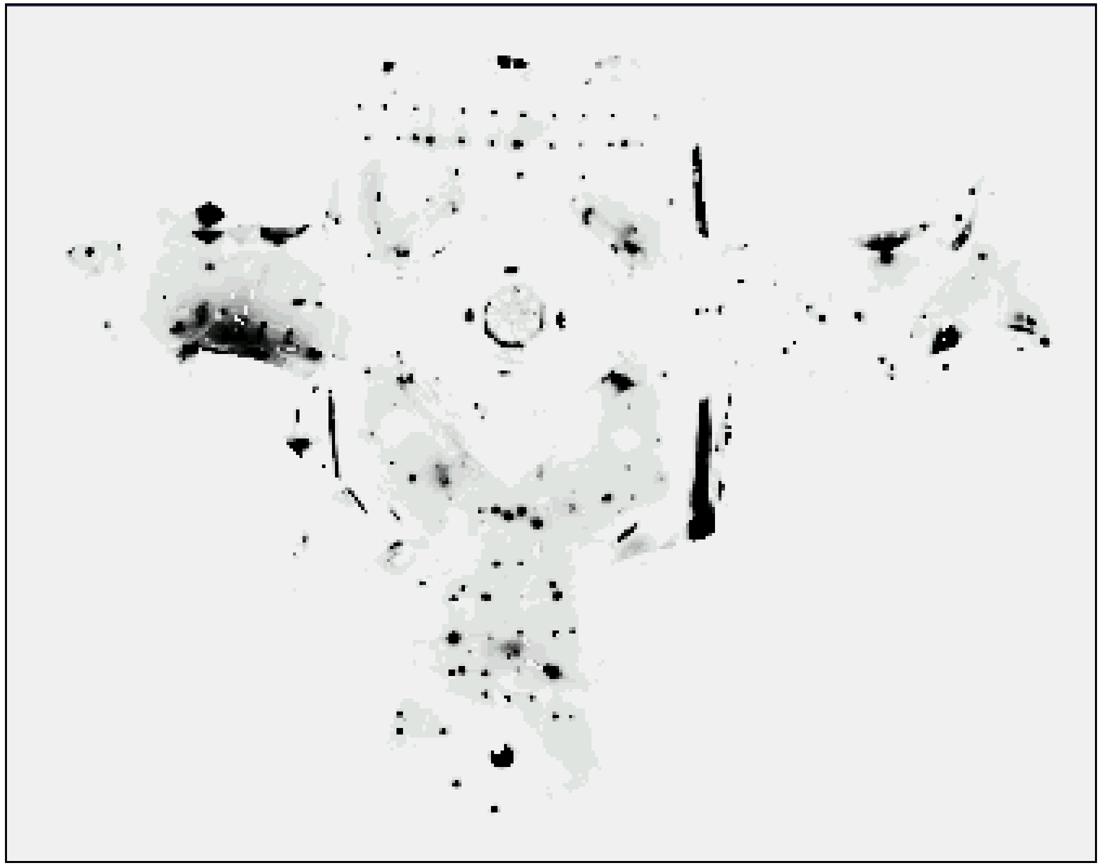


Under a Light

Measurement z:

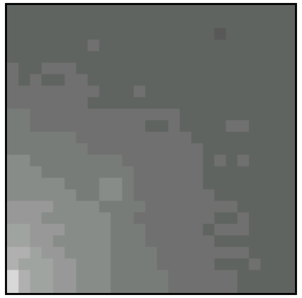


***P(z|x)*:**

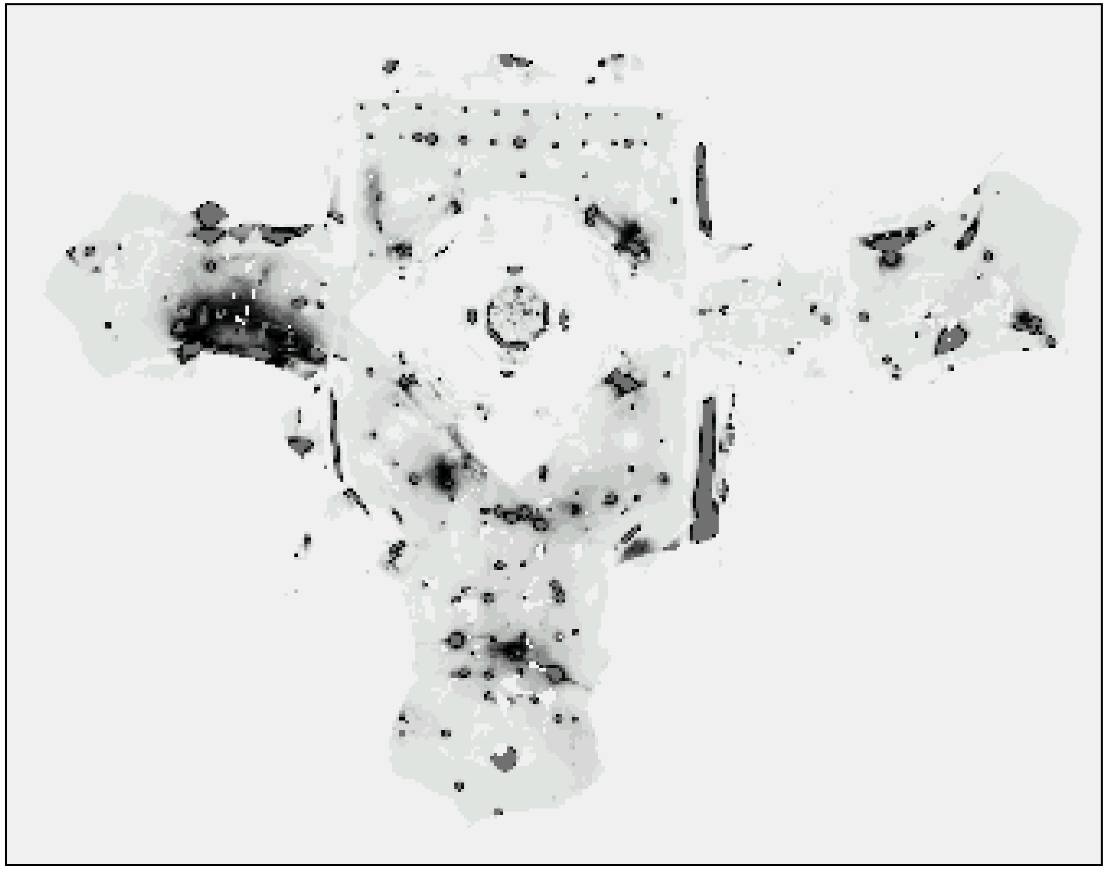


Next to a Light

Measurement z:



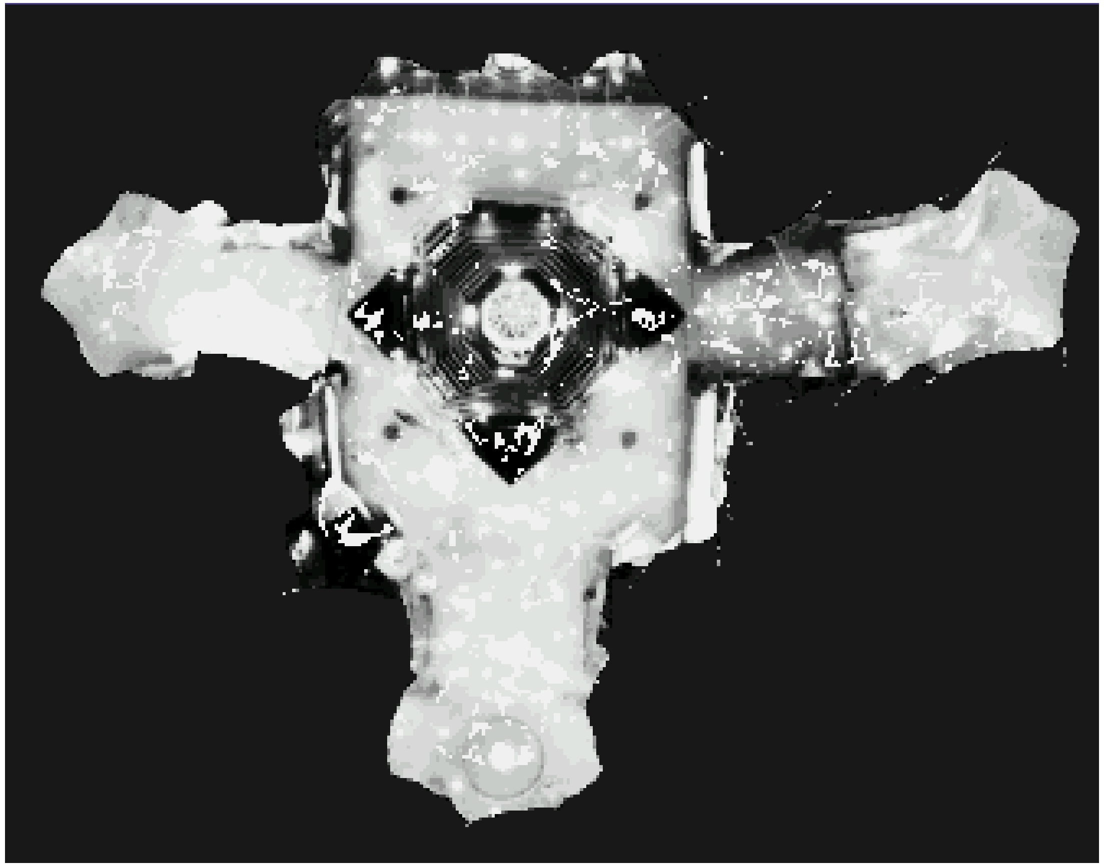
***P(z|x)*:**



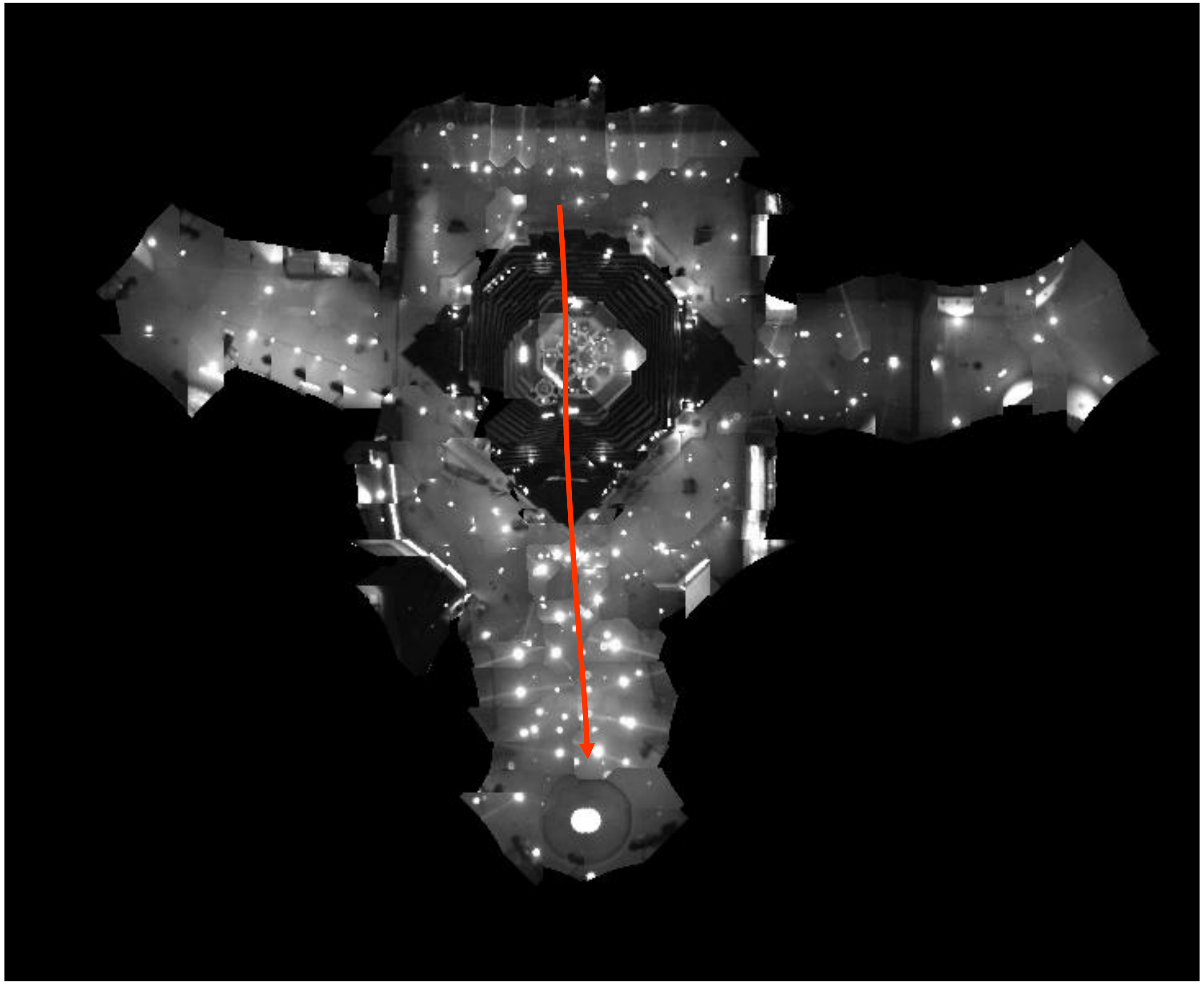
Elsewhere

Measurement z:

***P(z|x)*:**



Global Localization Using Vision



Limitations

- The approach described so far is able to
  - track the pose of a mobile robot and to
  - globally localize the robot.

- Can we deal with localization errors (i.e., the kidnapped robot problem)?
- How to handle localization errors/failures?
  - Particularly serious when the number of particles is small

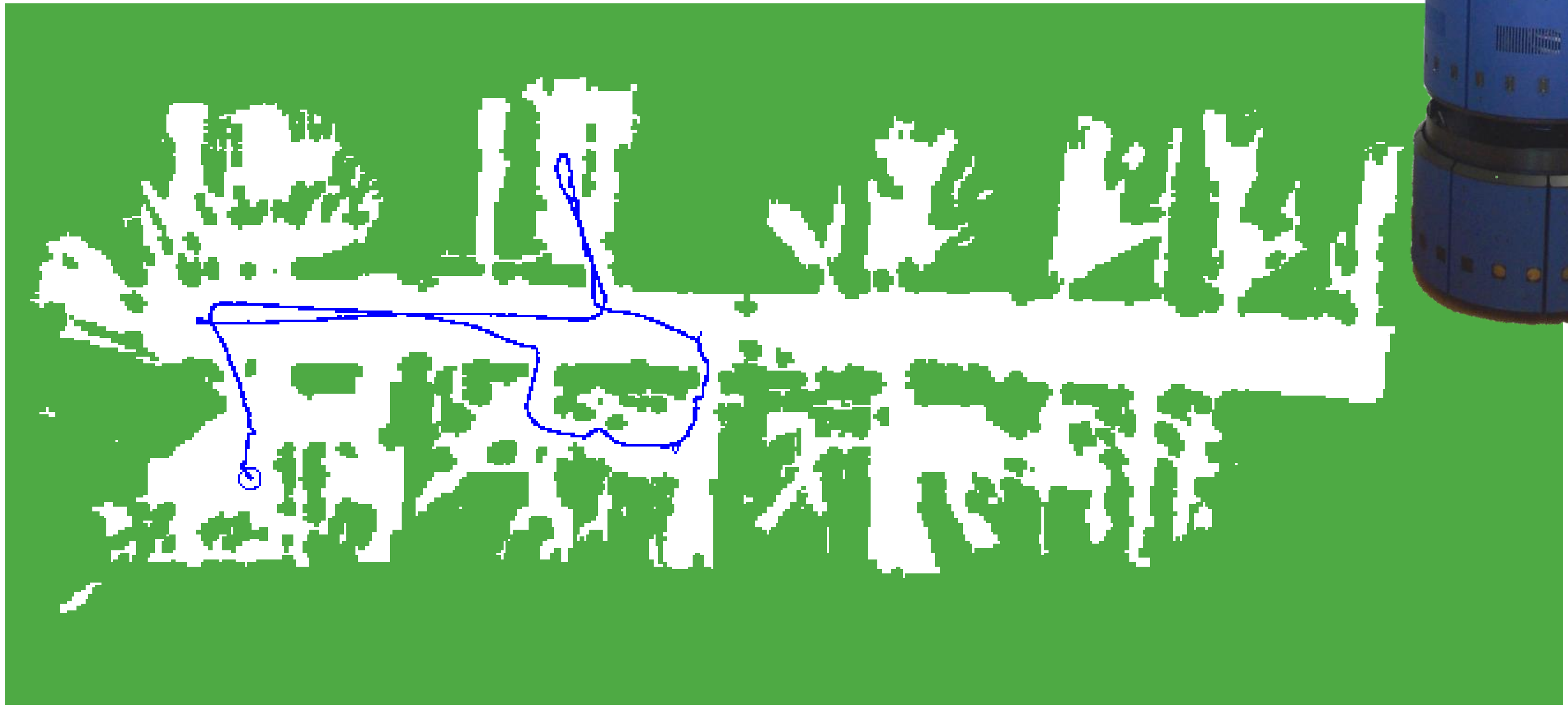
Approaches

- Randomly insert samples
  - Why?
  - The robot can be teleported at any point in time
- How many particles to add? With what distribution?
  - Add particles according to localization performance
  - Monitor the probability of sensor measurements $p(z_t | z_{1:t-1}, u_{1:t}, m)$
  - For particle filters: $p(z_t | z_{1:t-1}, u_{1:t}, m) \approx \frac{1}{M} \sum w_t^{[m]}$
- Insert random samples proportional to the average likelihood of the particles (the robot has been teleported with higher probability when the likelihood of its observations drops).

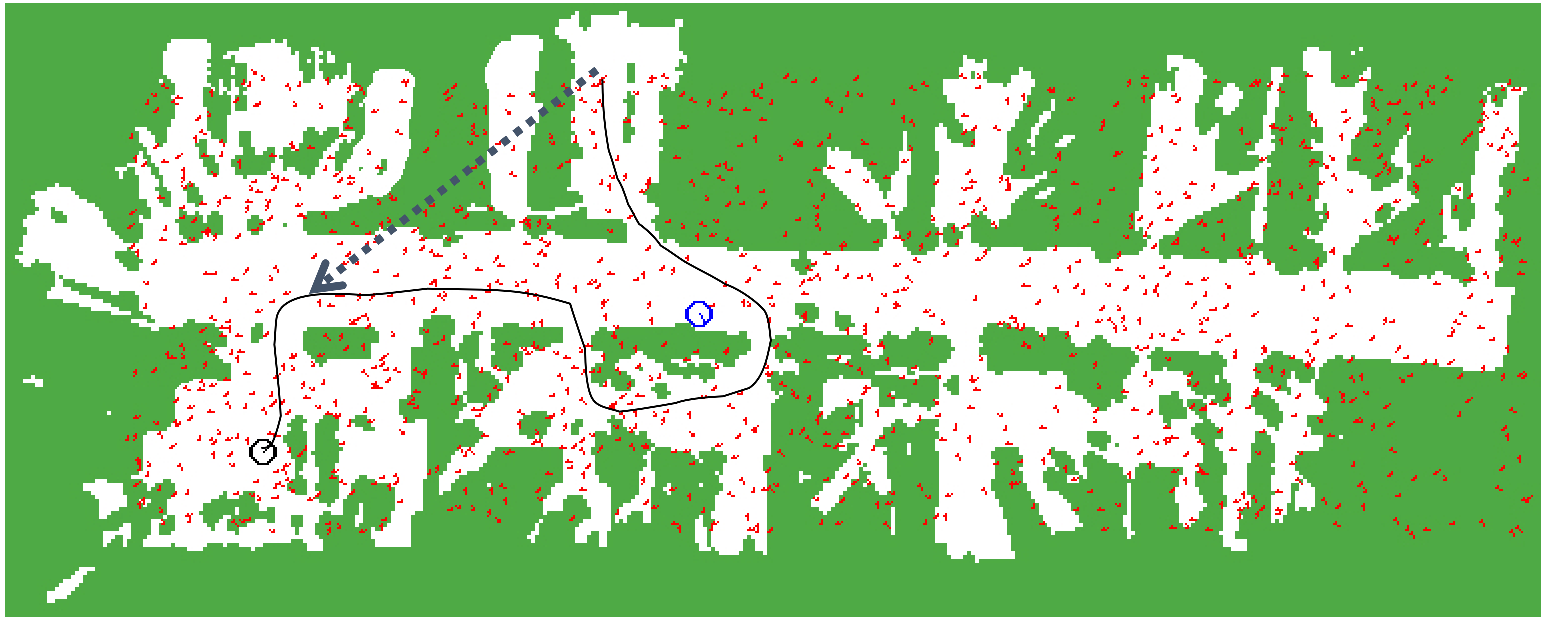
Random Samples
Vision-Based Localization

936 Images, 4MB, .6secs/image

Trajectory of the robot:



Kidnapping the Robot



Summary

- Particle filters are an implementation of recursive Bayesian filtering
- They represent the posterior by a set of weighted samples.
- In the context of localization, the particles are propagated according to the motion model.
- They are then weighted according to the likelihood of the observations.
- In a re-sampling step, new particles are drawn with a probability proportional to the likelihood of the observation.