Principles of Safe Autonomy: Probabilistic Sensor Models

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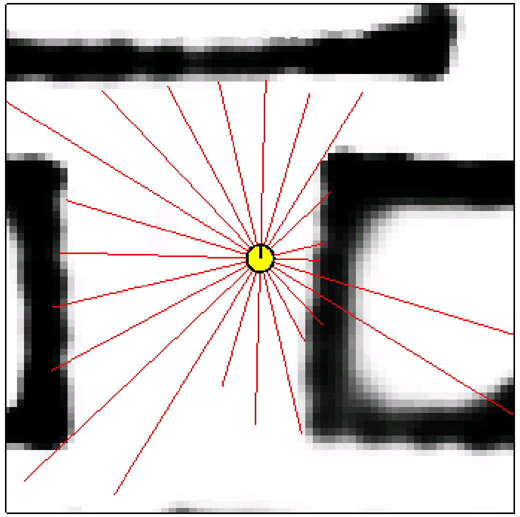
Reference: Probabilistic Robotics by Sebastian Thrun, Wolfram Burgard, and Dieter Fox

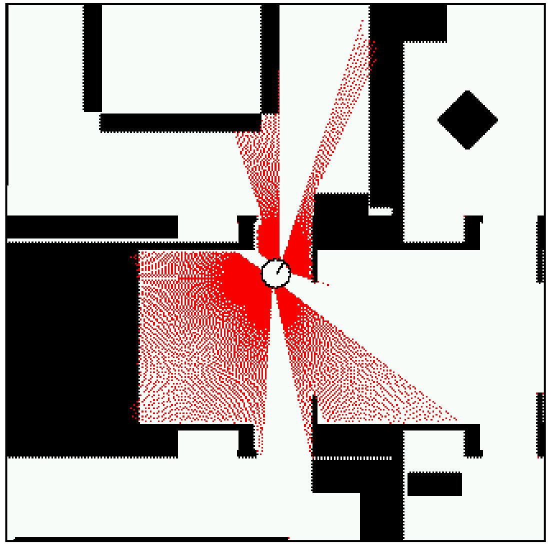
Slides: From the book's website

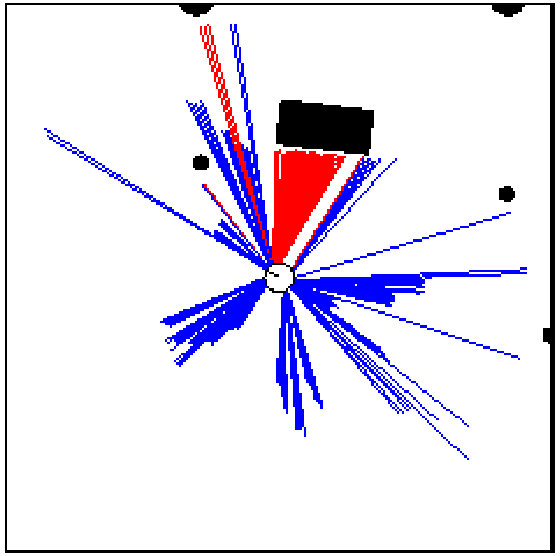
Sensors for Mobile Robots

- Contact sensors: Bumpers
- Internal sensors
  - Accelerometers (spring-mounted masses)
  - Gyroscopes (spinning mass, laser light)
  - Compasses, inclinometers (earth magnetic field, gravity)
- Proximity sensors
  - Sonar (time of flight)
  - Radar (phase and frequency)
  - Laser range-finders (triangulation, tof, phase)
  - Infrared (intensity)
- Visual sensors: Cameras
- Satellite-based sensors: GPS

Proximity Sensors







- The central task is to determine *P(z|x)*, i.e., the probability of a measurement *z* given that the robot is at position *x*.
- **Question**: Where do the probabilities come from?
- **Approach**: Let's try to explain a measurement.

Beam-based Sensor Model

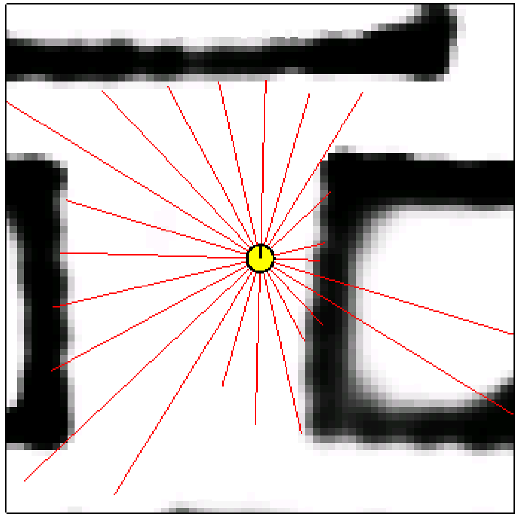
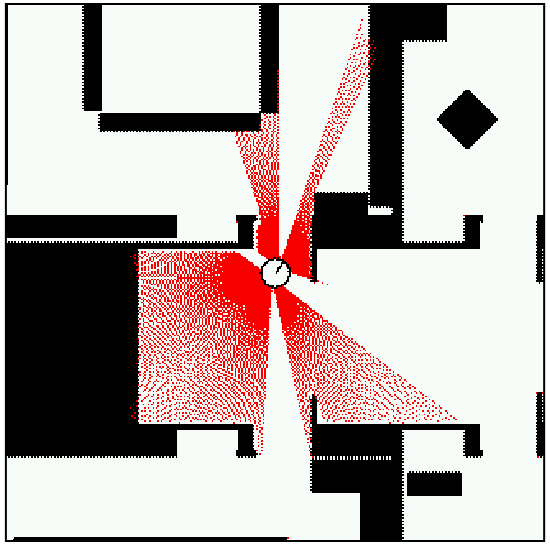
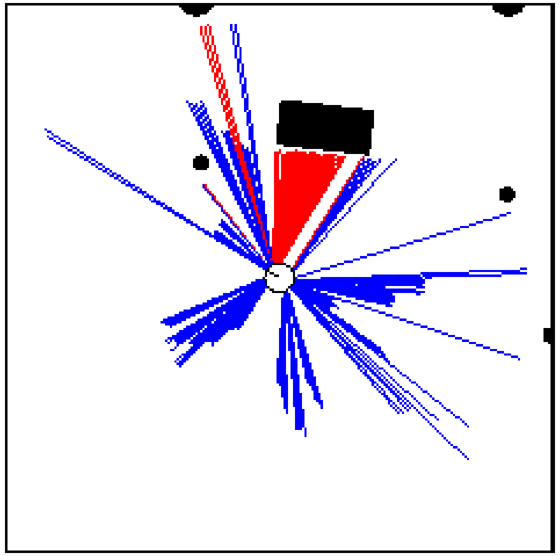
- Scan *z* consists of *K* measurements.

$$z = \{z_1, z_2, \ldots, z_K\}$$

- Individual measurements are independent given the robot position.

$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

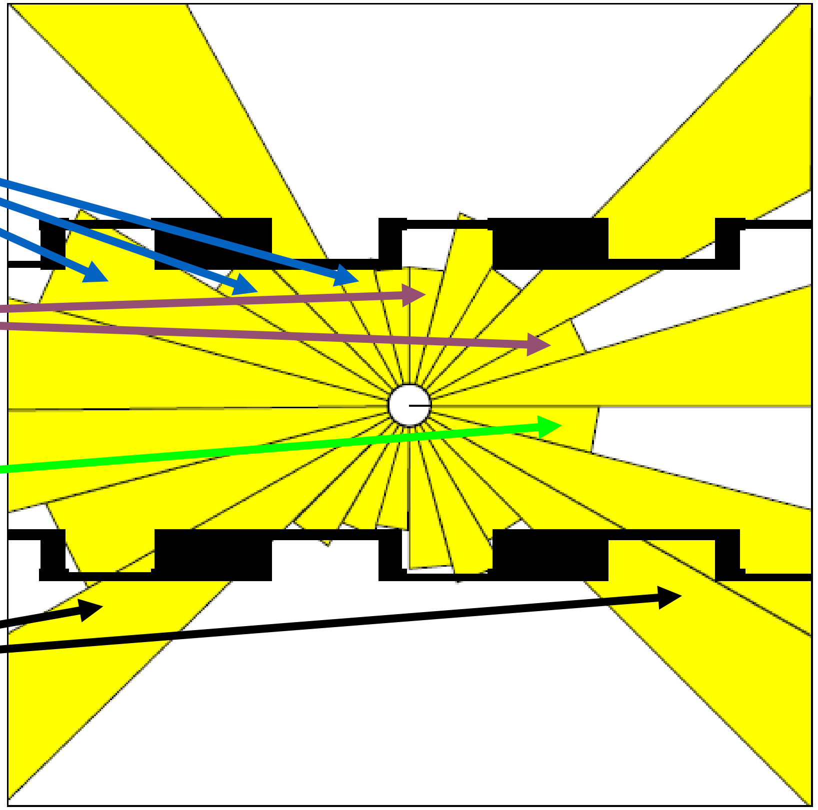
Beam-based Sensor Model

$$P(z \mid x, m) = \prod_{k=1}^{K} P(z_k \mid x, m)$$

Typical Range Measurement Errors

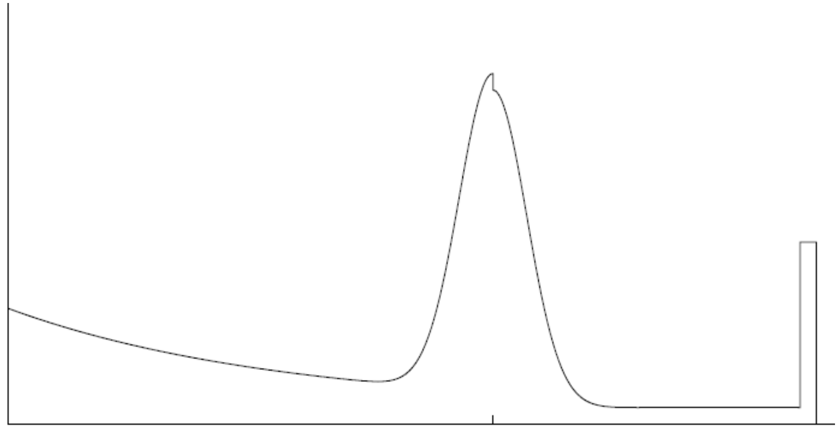
1. Beams reflected by obstacles
2. Beams reflected by persons / caused by crosstalk
3. Random measurements
4. Maximum range measurements



Proximity Measurement

- Measurement can be caused by …
  - a known obstacle.
  - cross-talk.
  - an unexpected obstacle (people, furniture, …).
  - missing all obstacles (total reflection, glass, …).
- Noise is due to uncertainty …
  - in measuring distance to known obstacle.
  - in position of known obstacles.
  - in position of additional obstacles.
  - whether obstacle is missed.

Resulting Mixture Density

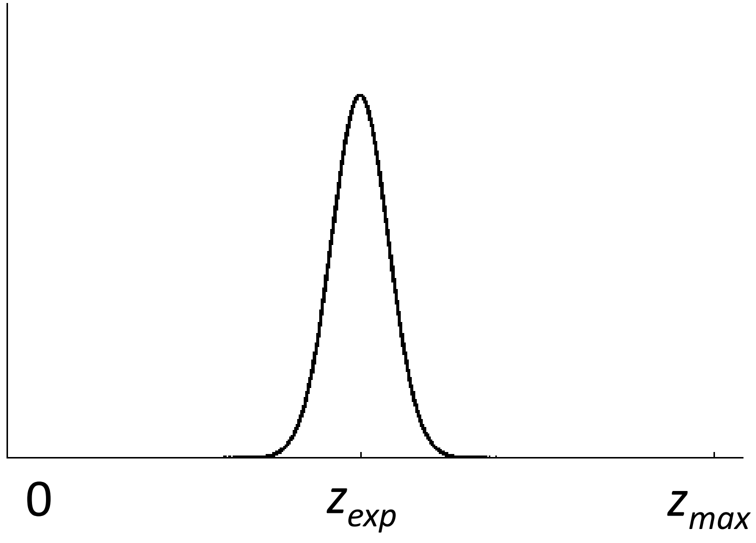


- Correct range with local measurement noise
- Unexpected objects
- Failures
- Random measurements

$$p(z_t^k|x_t,m) = \begin{pmatrix} z_{hit} \\ z_{short} \\ z_{max} \\ z_{rand} \end{pmatrix}^T \cdot \begin{pmatrix} p_{hit}(z_t^k|x_t,m) \\ p_{short}(z_t^k|x_t,m) \\ p_{max}(z_t^k|x_t,m) \\ p_{rand}(z_t^k|x_t,m) \end{pmatrix}$$

Beam-based Proximity Model

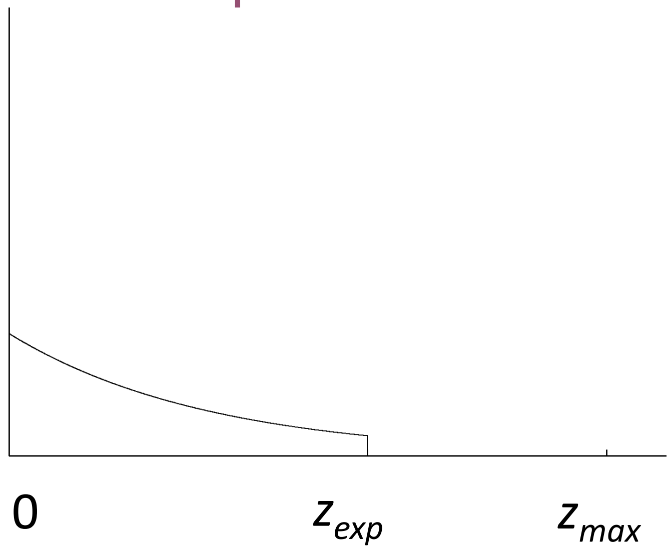
Measurement noise



Gaussian distribution ($b = \sigma_{hit}{}^2$)

$$P_{hit}(z \mid x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2}\frac{(z - z_{\text{exp}})^2}{b}}$$

Unexpected obstacles

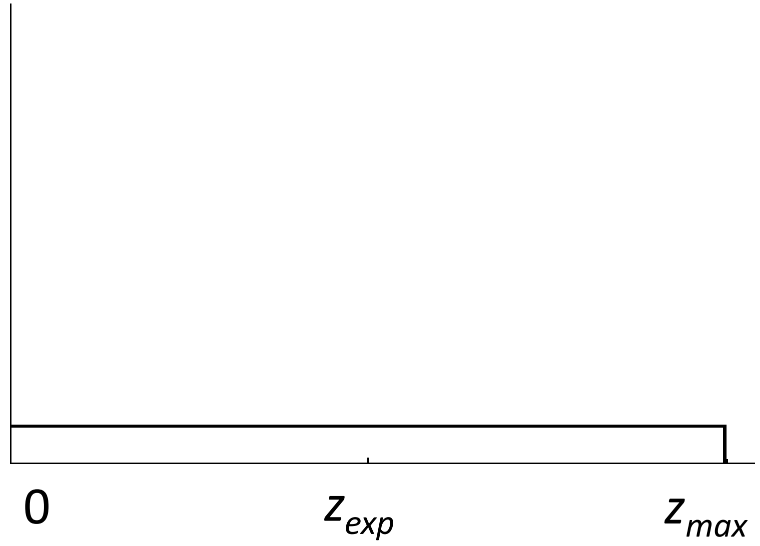


Exponential distribution

$$P_{\text{unexp}}(z \mid x, m) = \begin{cases} \eta \, \lambda \, \mathrm{e}^{-\lambda z} & z < z_{\text{exp}} \\ 0 & otherwise \end{cases}$$

Beam-based Proximity Model

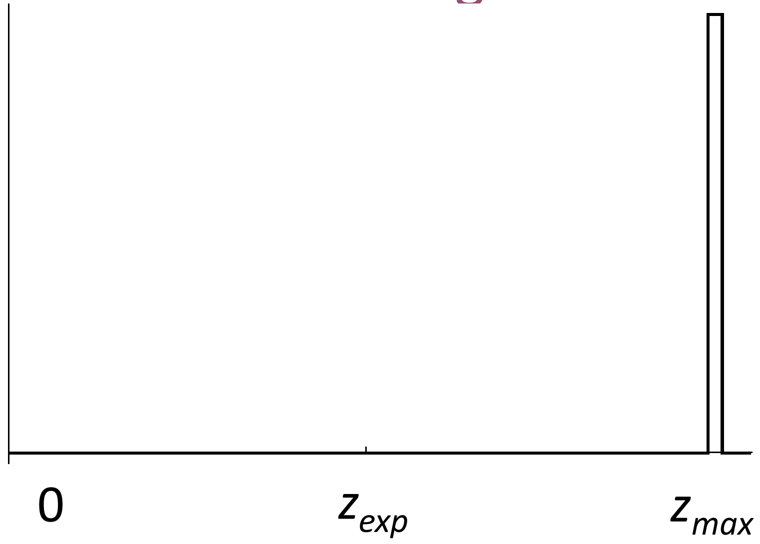
Random measurement



Uniform distribution

$$P_{rand}(z \mid x, m) = \eta \frac{1}{z_{\max}}$$

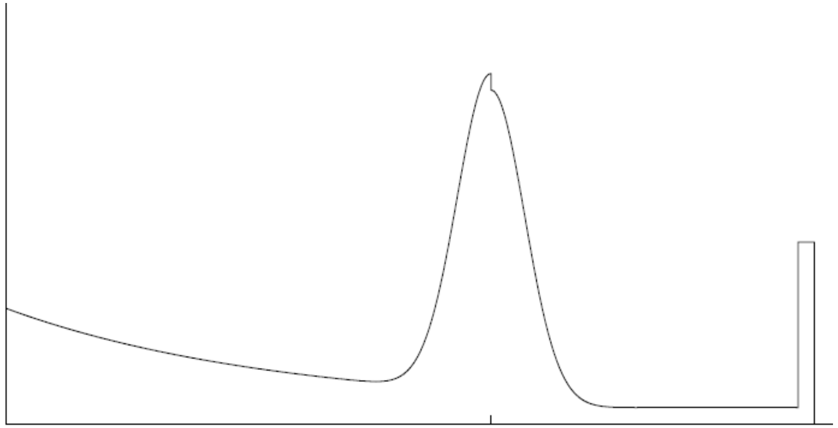
Max range



Uniform distribution

$$P_{\max}(z \mid x, m) = \eta \frac{1}{z_{small}}$$

Resulting Mixture Density



$$p(z_t^k|x_t,m) = \begin{pmatrix} z_{hit} \\ z_{short} \\ z_{max} \\ z_{rand} \end{pmatrix}^T \cdot \begin{pmatrix} p_{hit}(z_t^k|x_t,m) \\ p_{short}(z_t^k|x_t,m) \\ p_{max}(z_t^k|x_t,m) \\ p_{rand}(z_t^k|x_t,m) \end{pmatrix}$$

$$z_{hit} + z_{short} + z_{max} + z_{rand} = 1$$

Computing the likelihood

Algorithm beam_range_finder_model(z_t, x_t, m)**:**

$q = 1$

for $k = 1$ to K do

compute z_t^{k*} for the measurement z_t^k using ray casting

$p = z_{\text{hit}} \cdot p_{\text{hit}}(z_t^k \mid x_t, m) + z_{\text{short}} \cdot p_{\text{short}}(z_t^k \mid x_t, m)$

$+ z_{\text{max}} \cdot p_{\text{max}}(z_t^k \mid x_t, m) + z_{\text{rand}} \cdot p_{\text{rand}}(z_t^k \mid x_t, m)$

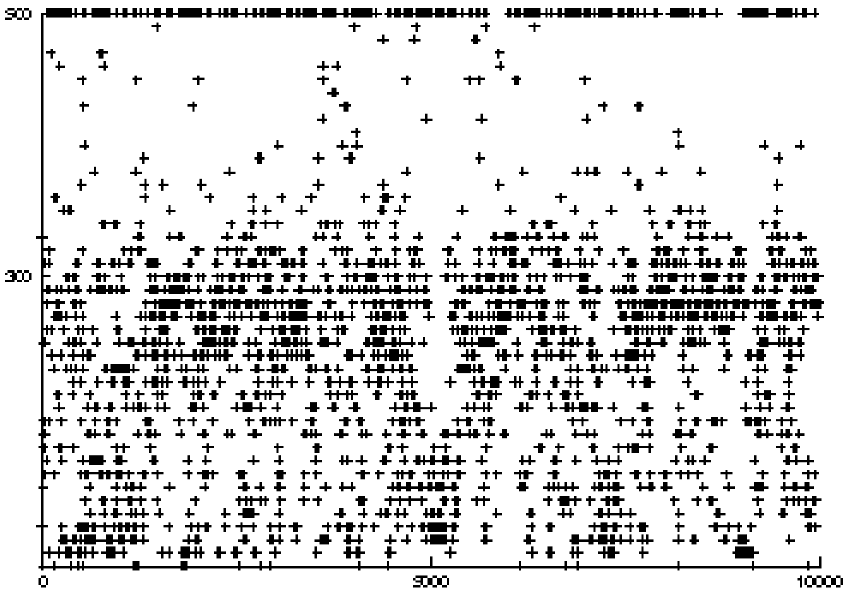
$q = q \cdot p$

return q

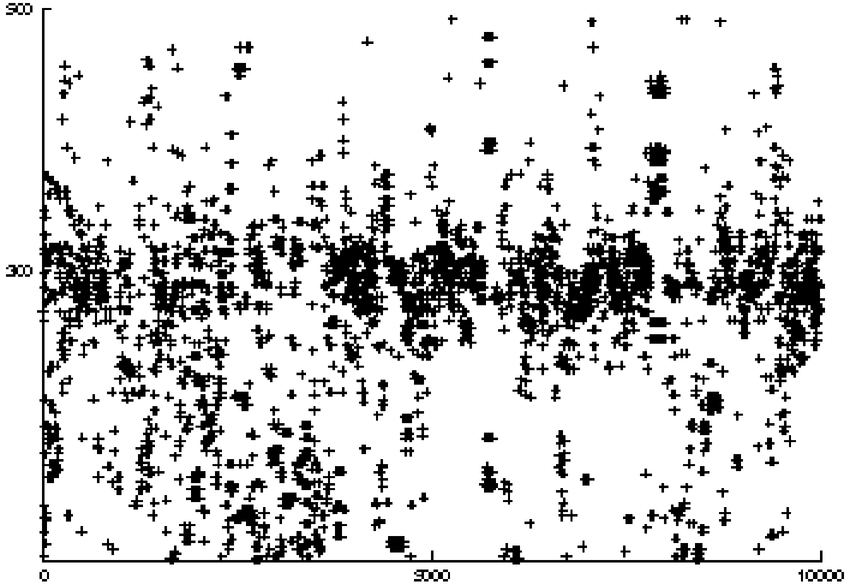
How can we determine the model parameters?

Raw Sensor Data

Measured distances for expected distance of 300 cm.



Sonar



Laser

Estimation

- Maximize log likelihood of the data

$$p(Z \mid X, m, \Theta)$$

- Θ: all intrinsic parameters
  - $z_{hit}, z_{short}, z_{max}, z_{rand}$, and $\sigma_{hit}, \lambda_{max}$

- Line 3-9 : estimate auxiliary variables
- Line 10-15: estimate the intrinsic parameters

1: **Algorithm learn_intrinsic_parameters**(Z, X, m)**:**

2: repeat until convergence criterion satisfied

3: for all z_i in Z do

4: $\eta = [\, p_{\text{hit}}(z_i \mid x_i, m) + p_{\text{short}}(z_i \mid x_i, m) + p_{\text{max}}(z_i \mid x_i, m) + p_{\text{rand}}(z_i \mid x_i, m)\,]^{-1}$

5: calculate z_i^*

6: $e_{i,\text{hit}} = \eta\; p_{\text{hit}}(z_i \mid x_i, m)$

7: $e_{i,\text{short}} = \eta\; p_{\text{short}}(z_i \mid x_i, m)$

8: $e_{i,\text{max}} = \eta\; p_{\text{max}}(z_i \mid x_i, m)$

9: $e_{i,\text{rand}} = \eta\; p_{\text{rand}}(z_i \mid x_i, m)$

10: $z_{\text{hit}} = |Z|^{-1} \sum_i e_{i,\text{hit}}$

11: $z_{\text{short}} = |Z|^{-1} \sum_i e_{i,\text{short}}$

12: $z_{\text{max}} = |Z|^{-1} \sum_i e_{i,\text{max}}$

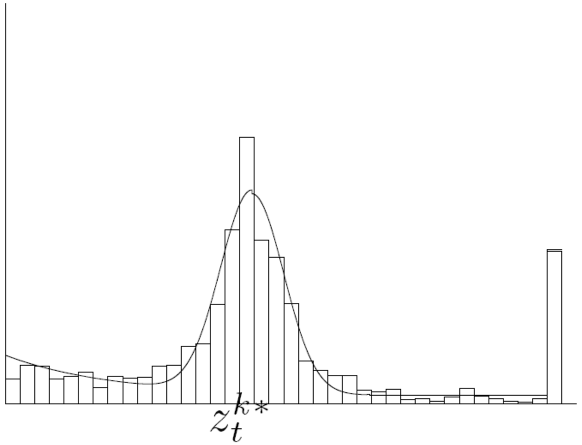
13: $z_{\text{rand}} = |Z|^{-1} \sum_i e_{i,\text{rand}}$

14: $\sigma_{\text{hit}} = \sqrt{\frac{1}{\sum_i e_{i,\text{hit}}} \sum_i e_{i,\text{hit}} (z_i - z_i^*)^2}$

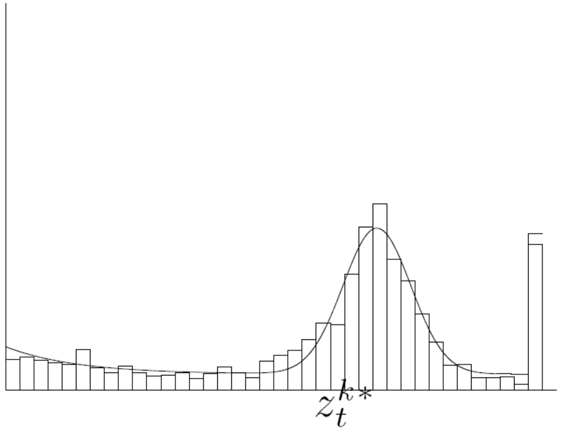
15: $\lambda_{\text{short}} = \frac{\sum_i e_{i,\text{short}}}{\sum_i e_{i,\text{short}}\, z_i}$

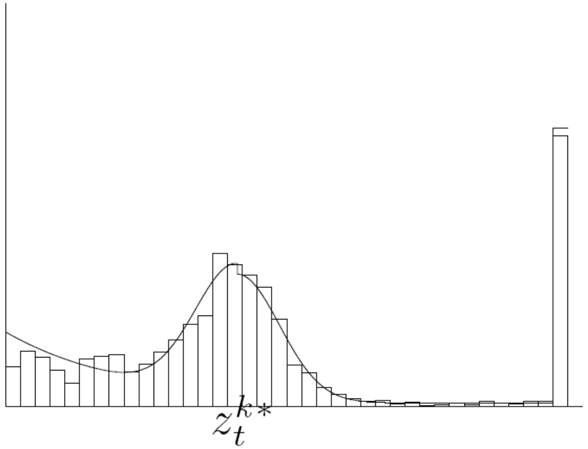
16: return $\Theta = \{z_{\text{hit}}, z_{\text{short}}, z_{\text{max}}, z_{\text{rand}}, \sigma_{\text{hit}}, \lambda_{\text{short}}\}$

Estimation Results

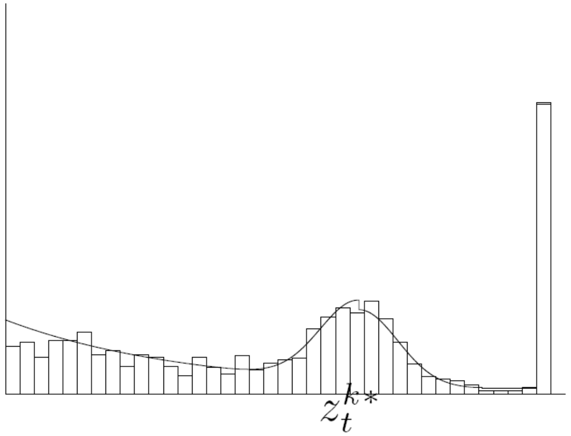


Laser





Sonar

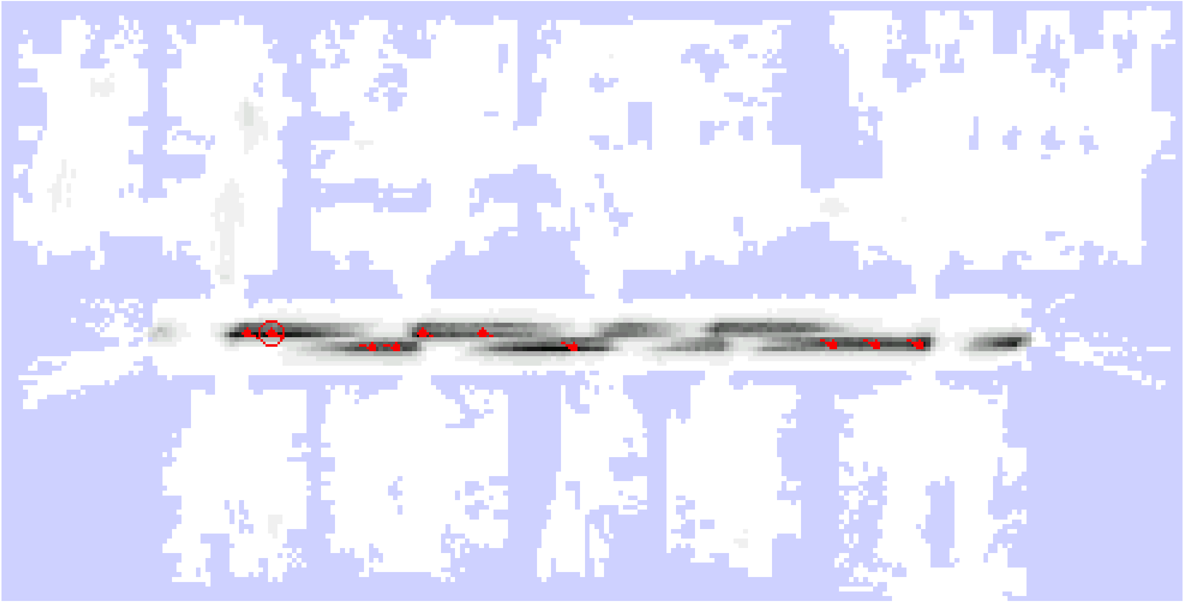


300cm

400cm

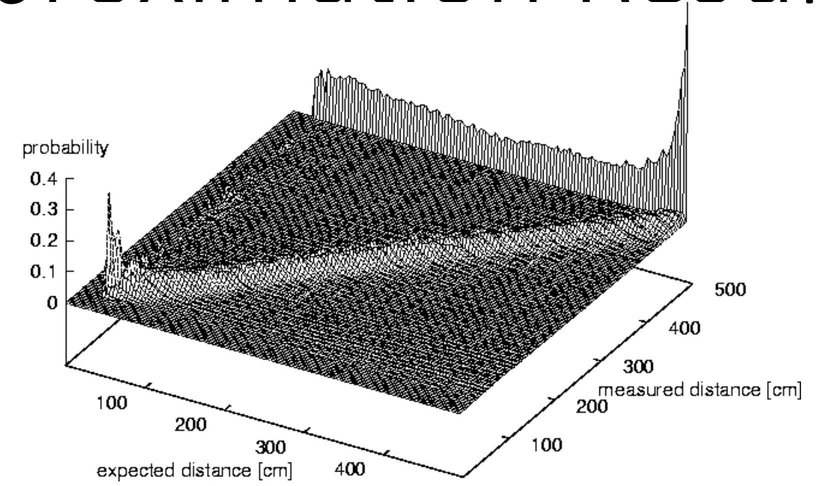
Example

z

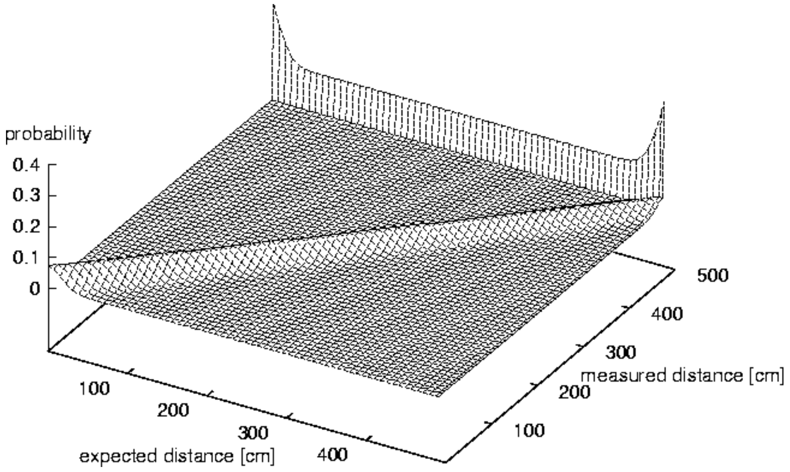


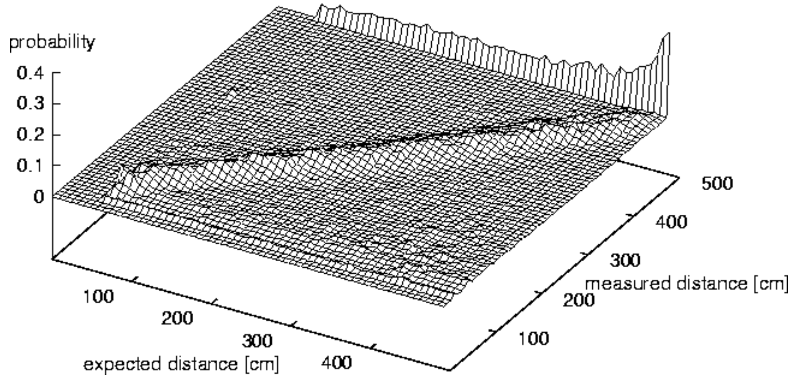
$P(z|x,m)$

Approximation Results

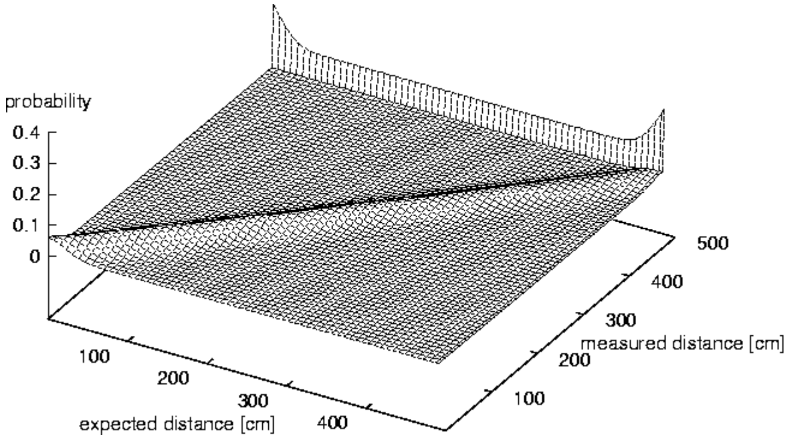


Laser





Sonar



Scan-based Model

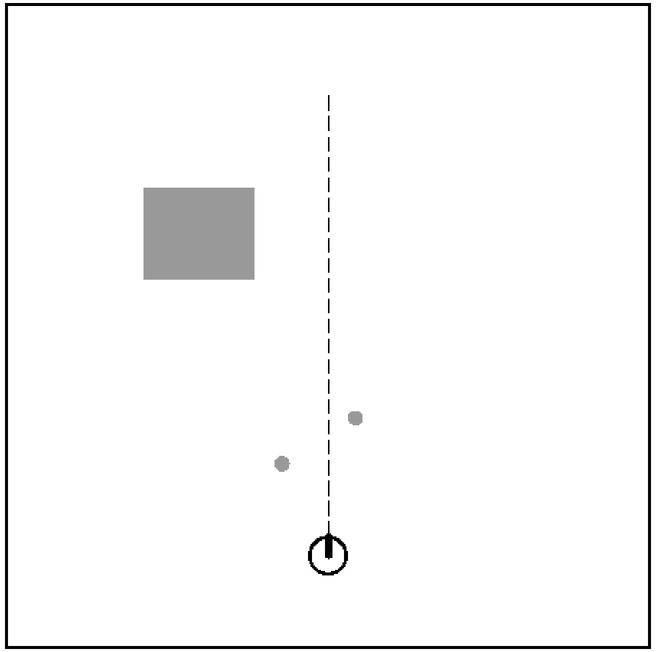
- Beam-based model is …
  - not smooth for small obstacles and at edges.
  - not very efficient.

- Idea: Instead of following along the beam, just check the end point.

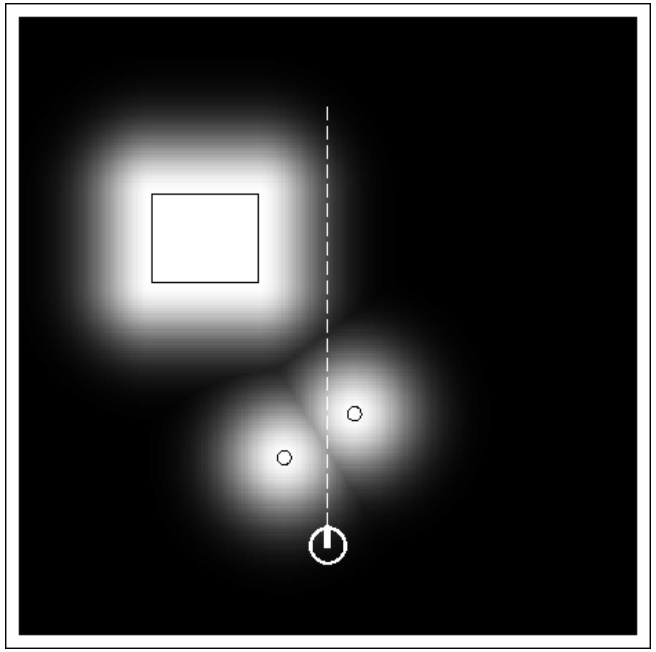
Scan-based Model

- Probability is a mixture of …
  - a Gaussian distribution with mean at distance to closest obstacle,
  - a uniform distribution for random measurements, and
  - a small uniform distribution for max range measurements.
- Again, independence between different components is assumed.

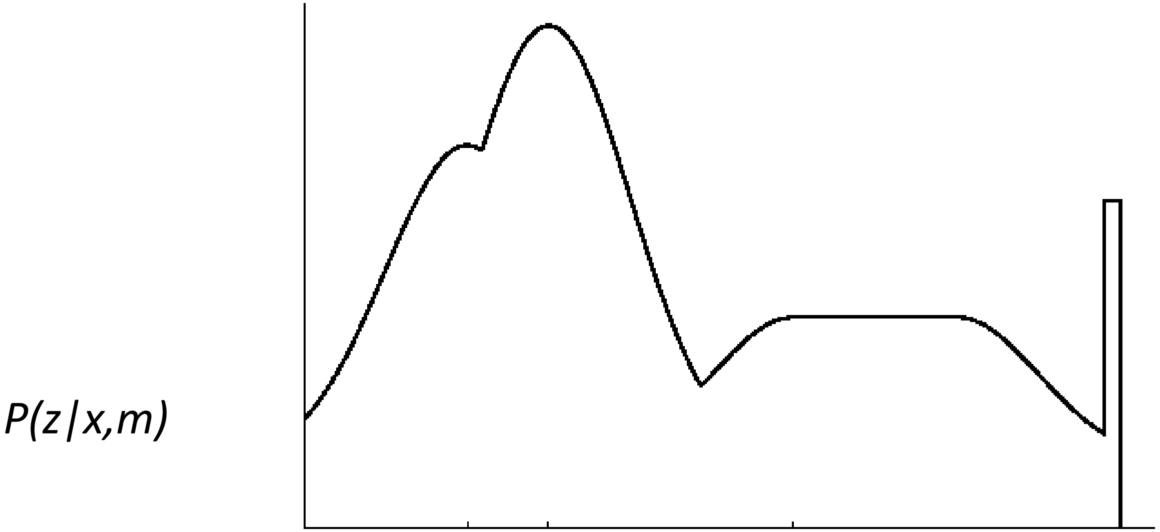
Example



Map *m*



Likelihood field



How to compute the Likelihood

Algorithm likelihood_field_range_finder_model(z_t, x_t, m):

$q = 1$

for all k do

if $z_t^k \neq z_{\max}$

$x_{z_t^k} = x + x_{k,\text{sens}} \cos\theta - y_{k,\text{sens}} \sin\theta + z_t^k \cos(\theta + \theta_{k,\text{sens}})$

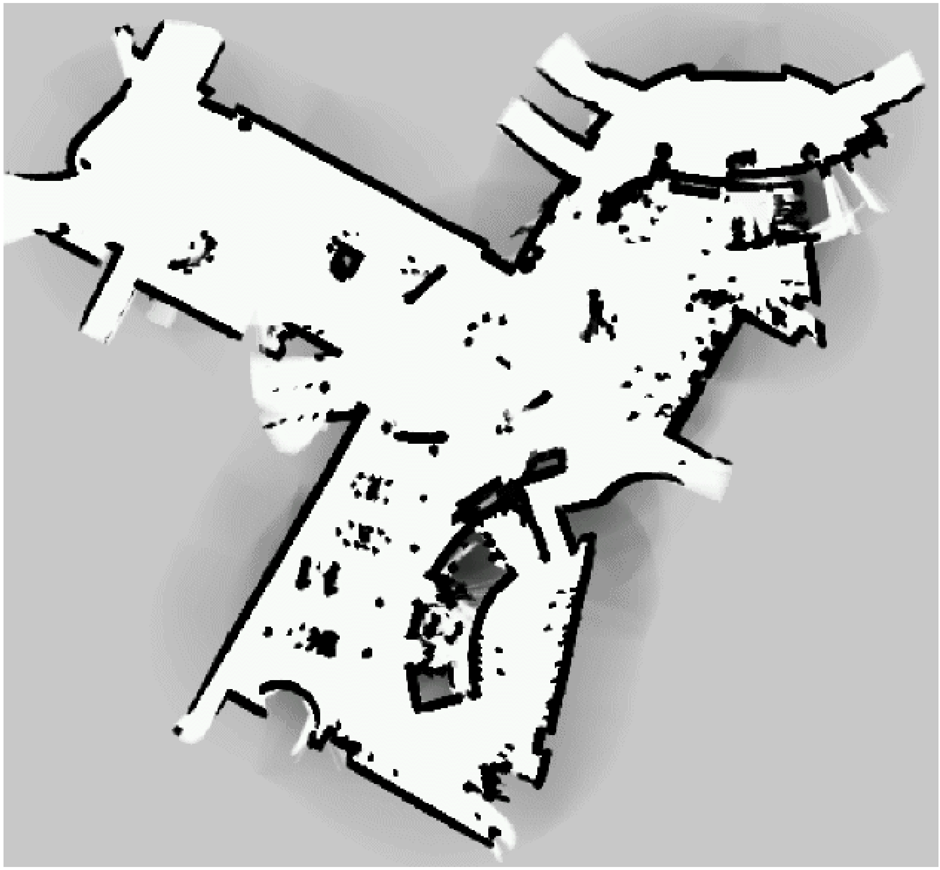
$y_{z_t^k} = y + y_{k,\text{sens}} \cos\theta + x_{k,\text{sens}} \sin\theta + z_t^k \sin(\theta + \theta_{k,\text{sens}})$

$dist^2 = \min\limits_{x',y'} \left\{ (x_{z_t^k} - x')^2 + (y_{z_t^k} - y')^2 \;\middle|\; \langle x', y' \rangle \text{ occupied in } m \right\}$

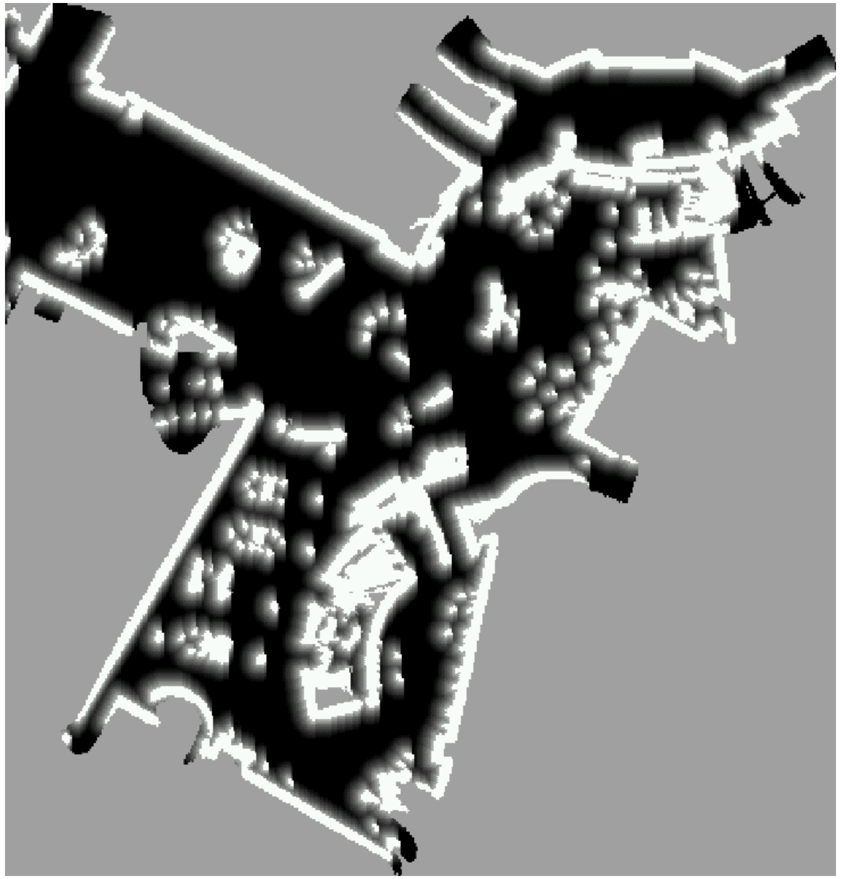
$q = q \cdot \left(z_{\text{hit}} \cdot \textbf{prob}(dist\ , \sigma_{\text{hit}}) + \frac{z_{\text{random}}}{z_{\max}} \right)$

return q

San Jose Tech Museum



Occupancy grid map



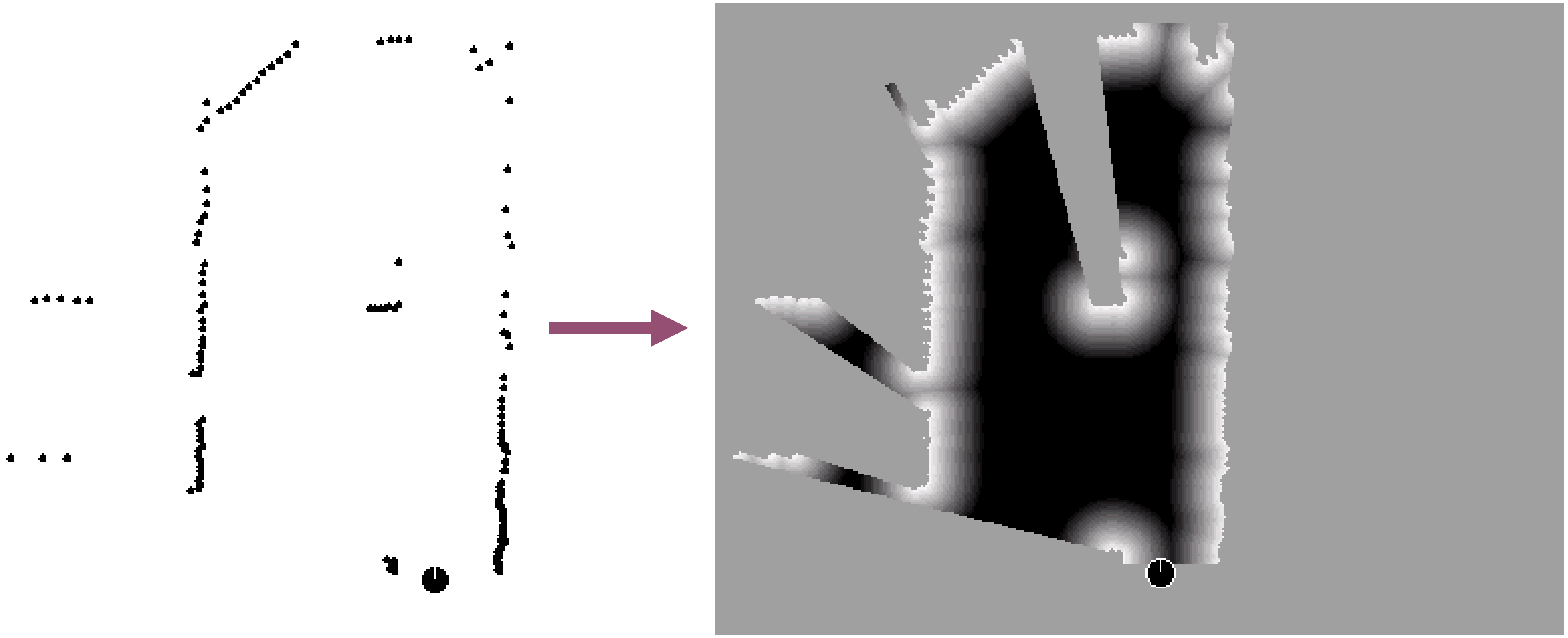
Likelihood field

Summary Beam-based Model

- Assumes independence between beams.
    - Justification?
    - Overconfident!
- Models physical causes for measurements.
    - Mixture of densities for these causes.
    - Assumes independence between causes. Problem?
- Implementation
    - Learn parameters based on real data.
    - Different models should be learned for different angles at which the sensor beam hits the obstacle.
    - Determine expected distances by ray-tracing.
    - Expected distances can be pre-processed.

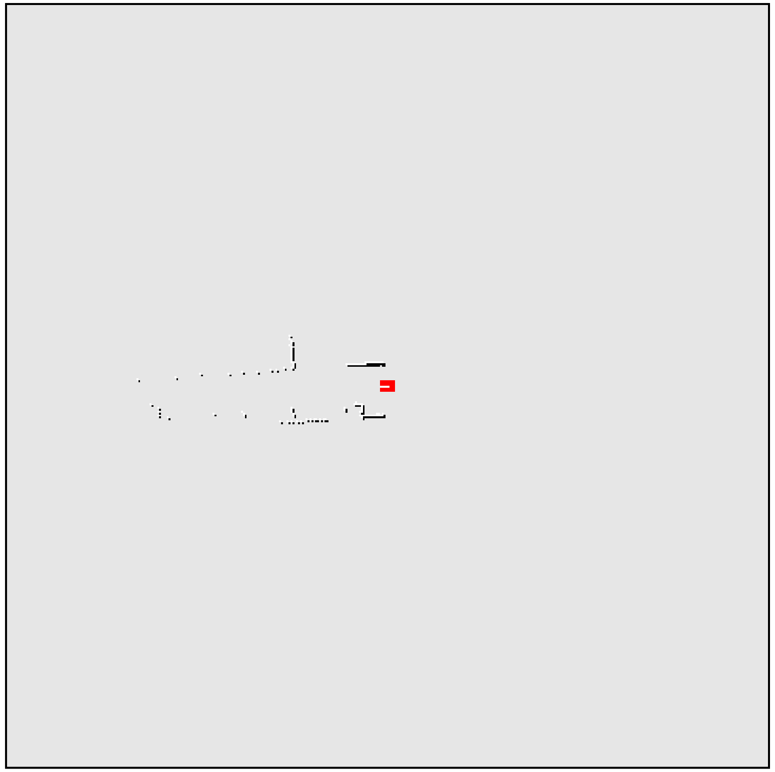
Scan Matching

- Extract likelihood field from scan and use it to match different scan.



Scan Matching

- Extract likelihood field from first scan and use it to match second scan.



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Properties of Scan-based Model

- Highly efficient, uses 2D tables only.
- Smooth w.r.t. to small changes in robot position.
- Allows gradient descent, scan matching.
- Ignores physical properties of beams.
- Will it work for ultrasound sensors?

Additional Models of Proximity Sensors

- Map matching (sonar,laser): generate small, local maps from sensor data and match local maps against global model.
- Scan matching (laser): map is represented by scan endpoints, match scan into this map.
- Features (sonar, laser, vision): Extract features such as doors, hallways from sensor data.

Landmarks

- Active beacons (*e.g.*, radio, GPS)
- Passive (*e.g.*, visual, retro-reflective)
- Standard approach is triangulation

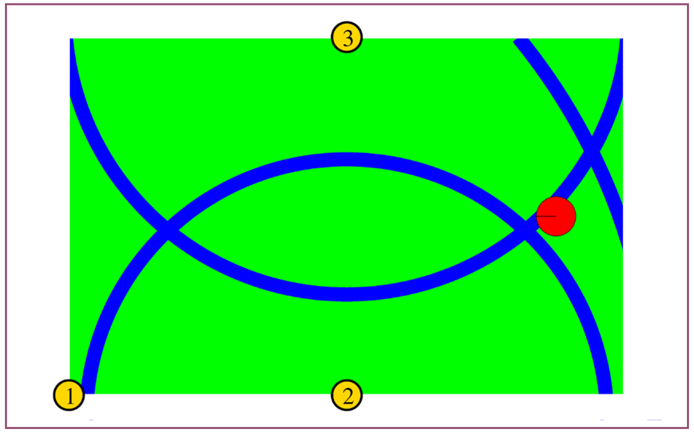
- Sensor provides
  - distance, or
  - bearing, or
  - distance and bearing.

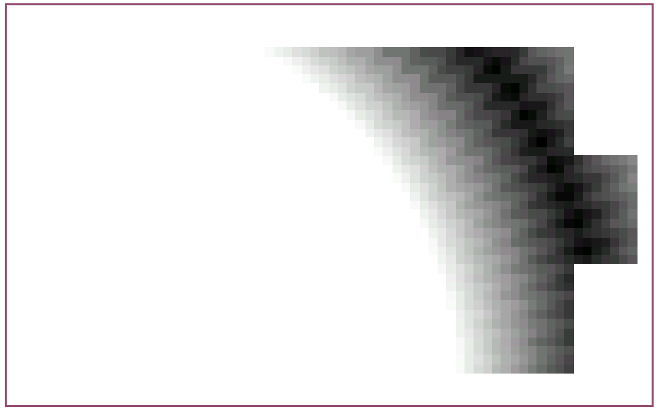
Distance and Bearing

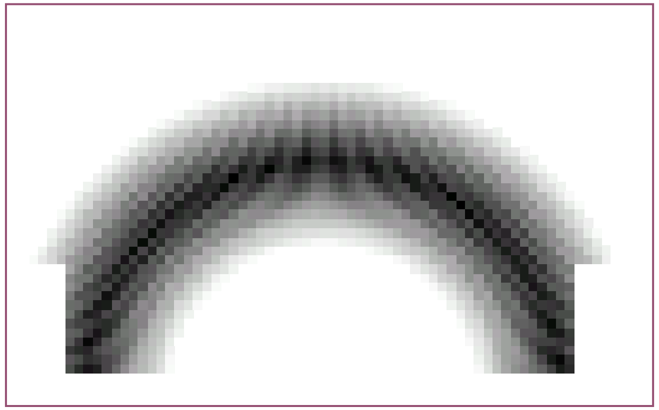
Probabilistic Model

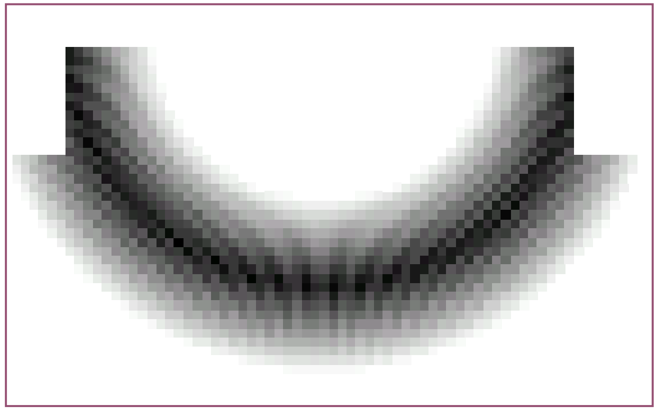
1. Algorithm **landmark_detection_model**(z,x,m):
   $z = \langle i, d, \alpha \rangle, x = \langle x, y, \theta \rangle$
2. $\hat{d} = \sqrt{(m_x(i) - x)^2 + (m_y(i) - y)^2}$
3. $\hat{a} = \text{atan2}(m_y(i) - y, m_x(i) - x) - \theta$
4. $p_{\text{det}} = \text{prob}(\hat{d} - d, \varepsilon_d) \cdot \text{prob}(\hat{\alpha} - \alpha, \varepsilon_\alpha)$
5. Return $z_{\text{det}} p_{\text{det}} + z_{\text{fp}} P_{\text{uniform}}(z \mid x, m)$

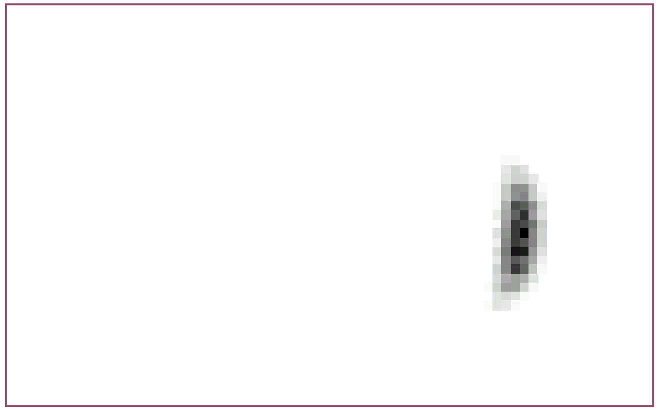
Distributions





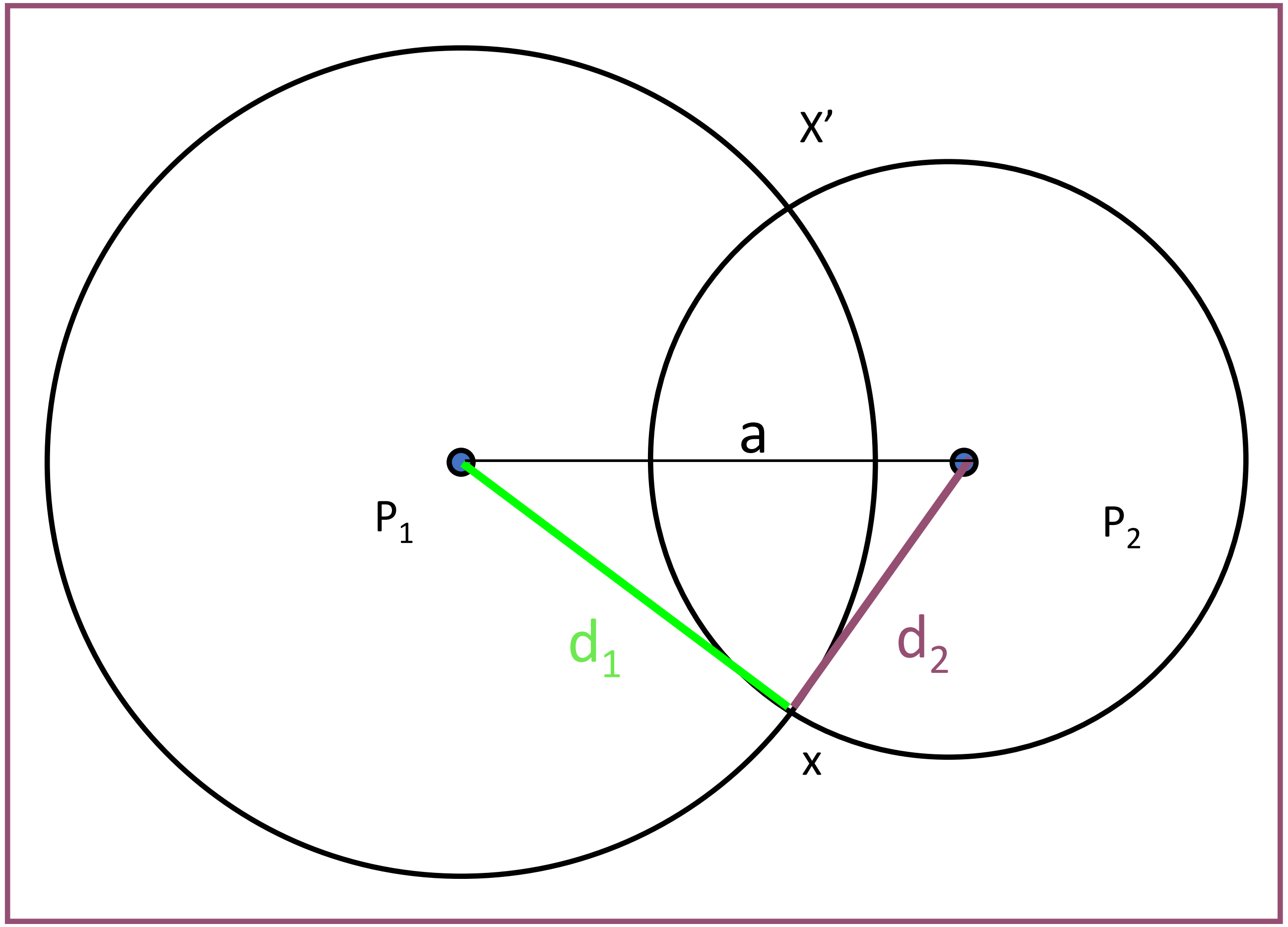






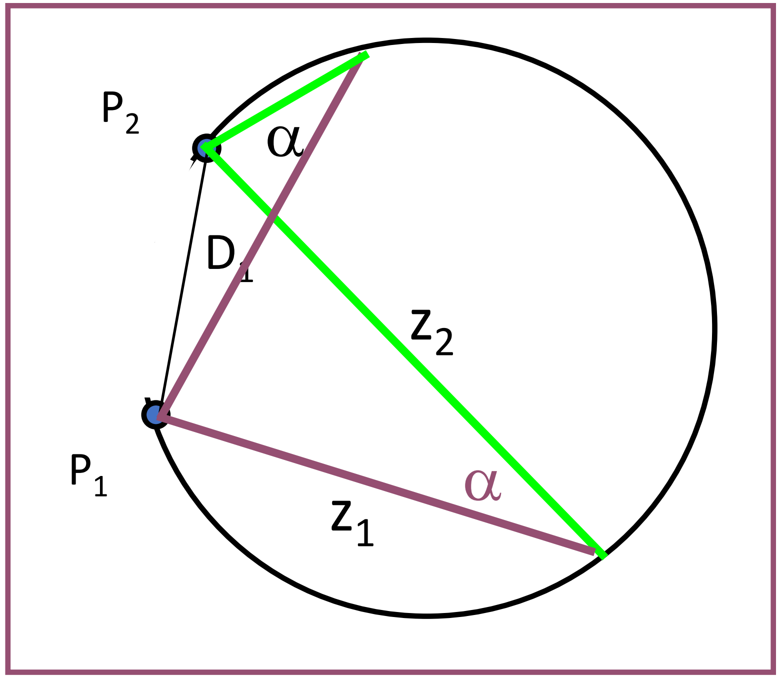
Distances Only
No Uncertainty

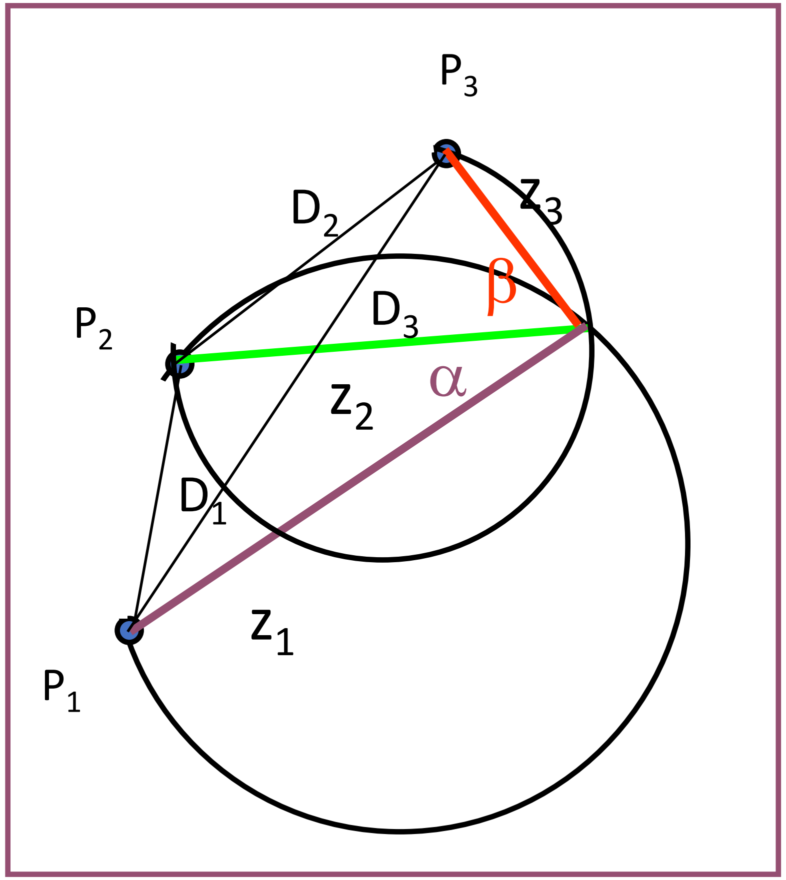
$$x = (a^2 + d_1^2 - d_2^2) / 2a$$
$$y = \pm\sqrt{(d_1^2 - x^2)}$$



$P_1=(0,0)$
$P_2=(a,0)$

Bearings Only
No Uncertainty





Law of cosine

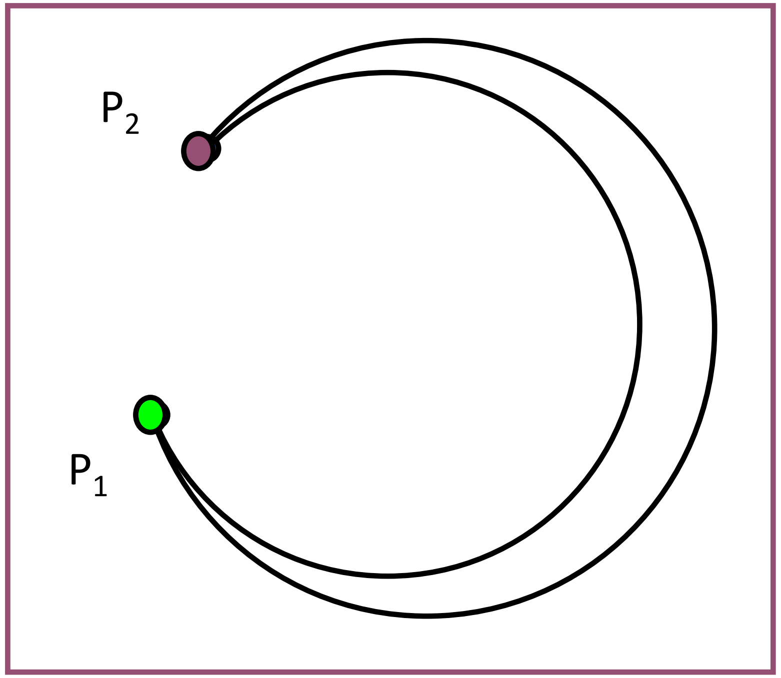
$$D_1^2 = z_1^2 + z_2^2 - 2\, z_1 z_2 \cos\alpha$$

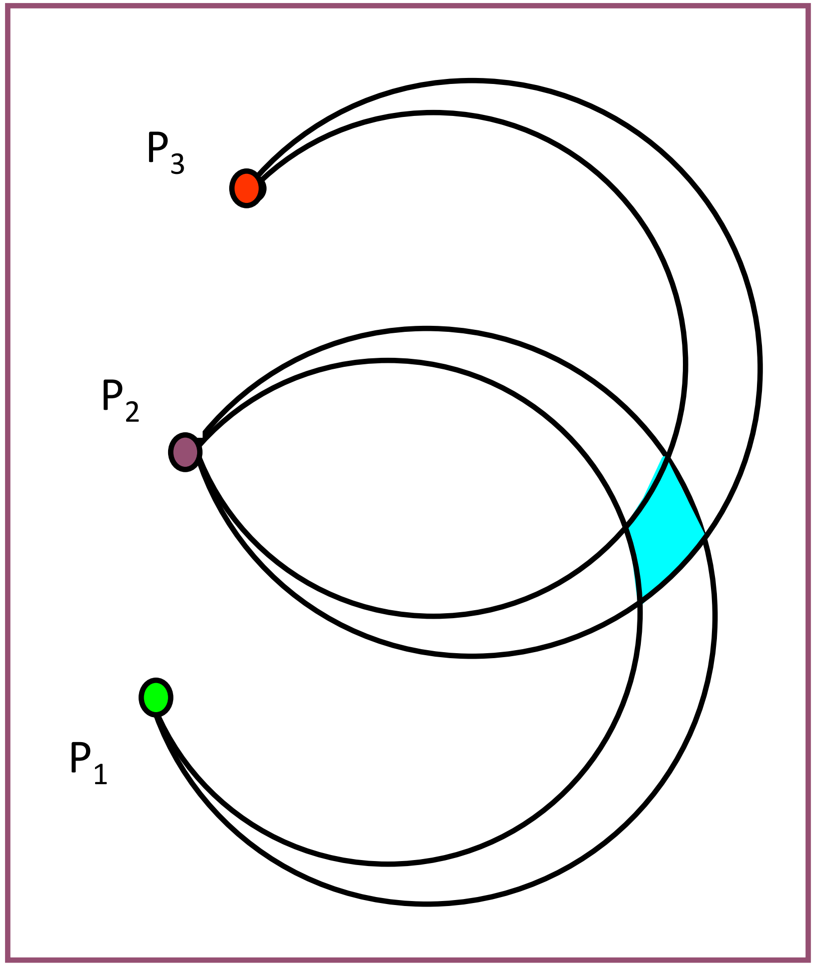
$$D_1^2 = z_1^2 + z_2^2 - 2\, z_1 z_2 \cos(\alpha)$$

$$D_2^2 = z_2^2 + z_3^2 - 2\, z_1 z_2 \cos(\beta)$$

$$D_3^2 = z_1^2 + z_3^2 - 2\, z_1 z_2 \cos(\alpha + \beta)$$

Bearings Only With Uncertainty





Most approaches attempt to find estimation mean.

Summary of Sensor Models

- Explicitly modeling uncertainty in sensing is key to robustness.
- In many cases, good models can be found by the following approach:
  1. Determine parametric model of noise free measurement.
  2. Analyze sources of noise.
  3. Add adequate noise to parameters (eventually mix in densities for noise).
  4. Learn (and verify) parameters by fitting model to data.
  5. Likelihood of measurement is given by “probabilistically comparing” the actual with the expected measurement.
- This holds for motion models as well.
- It is extremely important to be aware of the underlying assumptions!