Particle filters
- Bayes filter
- Histogram filter
- Alternative Nonparametric filter
- $\text{bel}(x_t)$ represented by finite set of parameters

Overview: $\text{bel}(x_t)$ represented by a set of random state samples

Example

Nonparametric representation

Nonlinear transformation on the representation is going to be easier

Parties
- Samples of the distribution $\text{bel}(x_t)$ are called particles
- $M$ particles denoted as $x_t^{[0]}, x_t^{[2]}, \ldots, x_t^{[M]}$, together $X_t$
Each \( x_t^{[m]} \), \( 1 \leq m \leq M \), is state vector for the system.

E.g. Rear wheel vehicle model

\[
x_t^{[m]} = \langle x_t^m, y_t^m, \theta_t^m \rangle
\]

\( [x_t, y_t, \theta_t] \)

\( M \approx 1000 \)

\( x_t^{[m]} \) should be included in \( X_t \) with probability

\[
\text{bel}(x_t) = P(x_t | z_{1:t}, u_{1:t})
\]

Relationship only hold asymptotically \( M \to \infty \)

Basic PF Algorithm

Recursive Algorithm

\[
X_t = x_t^{[1]} \ldots x_t^{[m]}
\]
\[ X_t = x_t^{[1]}, x_t^{[2]}, ..., x_t^{[M]} \] particles

Algorithm Particle_filter(\(X_{t-1}, u_t, z_t\)):

\[
\bar{X}_{t-1} = X_t = \emptyset
\]

for all \(m\) in \([M]\) do:

sample \(x_t^{[m]} \sim p(x_t | u_t, x_{t-1}^{[m]})\)

\[
w_t^{[m]} = p(z_t | x_t^{[m]})
\]

\[
\bar{X}_t = \bar{X}_t + \{x_t^{[m]}, w_t^{[m]}\}
\]

end for

for all \(m\) in \([M]\) do:

draw \(i\) with probability \(\propto w_t^{[i]}\)

add \(x_t^{[i]}\) to \(X_t\)

end for

return \(X_t\) \(\rightarrow\) New particles prediction + correction

**Resampling in Particle filtering**

"Survival of fittest"

We want to sample from \(\text{Bel}(X_t)\)

But we only get to sample from \(\text{Bel}(X_t)\)

E.g. compute prob \(x \in A\) acc density function \(f\) but we only have samples from \(g\).

\[
\mathbb{E}_f [I(x \in A)] = \int f(x) I(x \in A) \, dx
\]
Provided
\[ f(x) > 0 \Rightarrow g(x) > 0 \]

\[ \int \frac{f(x)}{g(x)} g(x) I(x \in A) \, dx = w(x) \]

\[ = \int w(x) g(x) I(x \in A) \, dx \]

\[ = \mathbb{E} \left[ w(x) I(x \in A) \right] \]

bel \((x+)\)

Samples \(f\) obtained by weighting \(f(x)/g(x)\) to each sample of \(g\)