Particle filters

→ Bayes filter

Histogram filter

→ Alternative Nonparametric filter

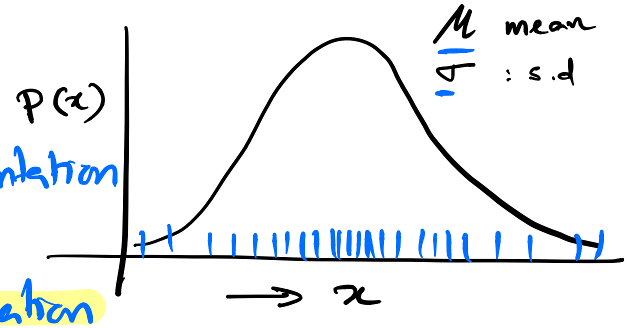
→ bel(x_t) represented by finite set of parameters

Overview : bel(x_t) represented by a set of random state samples

Example

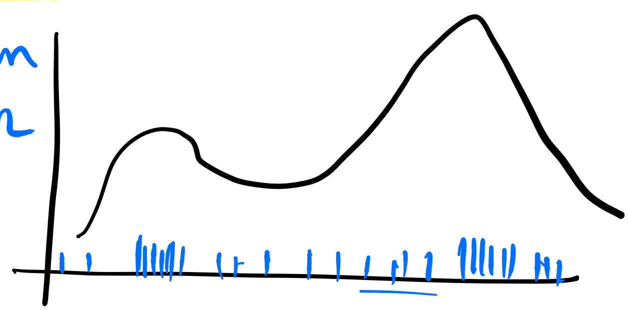
μ mean   Parameters

σ : s.d



nonparametric representation

Nonlinear transformation on the representation is going to be easier



Particles

Samples of the distribution bel(x_t) are called particles

M particles denoted as

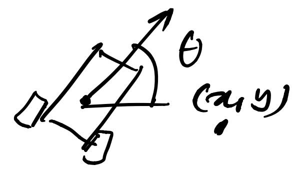
$x_t^{[1]}, x_t^{[2]}, \ldots, x_t^{[M]}$ together X_t

each $x_t^{[m]}$ $1 \le m \le M$

$\in$ state vector for the system

E.g. Rear wheel vehicle model

$\underline{x}_t^{[m]} = \langle \underline{x}_t^m, y_t^m, \theta_t^m \rangle$



$[\underline{x}, y, \theta]$

$M \approx 1000$

$x_t^{[m]}$ should be included in $\underline{X}_t$

with probability $= \underline{bel(x_t)} = P(x_t \mid z_{1:t}, u_{1:t})$

Relationship only hold asymptotically

$M \to \infty$

Basic PF Algorithm

Recursive Algorithm

$X_t = x_t^{[1]} \quad \ldots \quad x_t^{[m]}$

$X_t = x_t^{[1]}, x_t^{[2]}, \ldots x_t^{[M]}$ particles

Previous belief

Recent control int u_t

measurement z_t

$\overline{bel}(x_{t-1})$

Algorithm Particle_filter(X_{t-1}, u_t, z_t):

$\bar{X}_{t-1} = X_t = \emptyset$

for all m in [M] do:

sample $x_t^{[m]} \sim p(x_t | u_t, x_{t-1}^{[m]})$ ST — State transition model

$w_t^{[m]} = p\left(z_t \middle| x_t^{[m]}\right)$ → measurement weights — measurement model

$\bar{X}_t = \bar{X}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$ — intermediate particles

end for

for all m in [M] do:

draw i *with probability* $\propto w_t^{[i]}$

add $x_t^{[i]}$ *to* X_t

end for

Resampling step — importance sampling "trick"

return X_t → New particles prediction + correction

Resampling in Particle filtering

"Survival of fittest"

We want to sample from $bel(x_t)$ x_t f

But we only get to sample from $\overline{bel}(x_t)$ g

E.g compute prob $x \in A$ acc density function f but we only have samples from g.

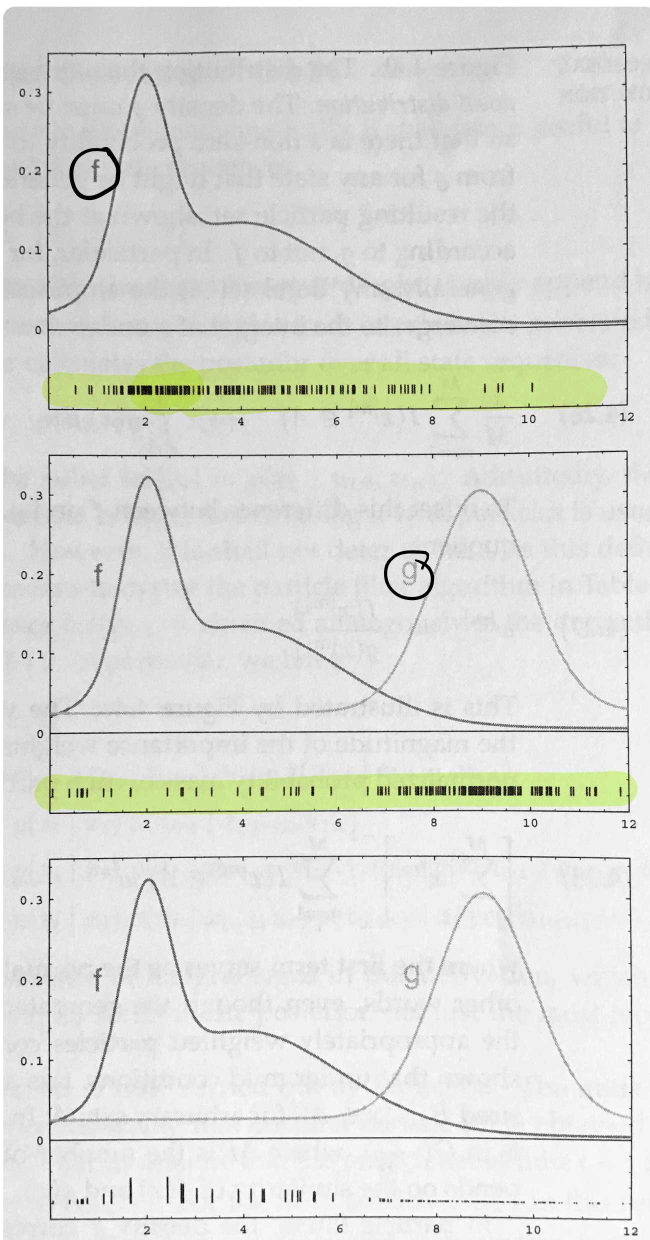
$$E_f[I(x \in A)] = \int f(x)\, I(x \in A)\, dx$$

Provided
$f(x) > 0 \Rightarrow g(x) > 0$

$$= \int \underbrace{\frac{f(x)}{g(x)}}_{= w(x)} g(x)\, I(x \in A)\, dx$$

$$= \int w(x)\, g(x)\, I(x \in A)\, dx$$

$$= E_g\left[w(x)\, I(x \in A)\right]$$



bel (x_t)

Samples from g
bel (x_t)

Samples of f obtained by weighting $\frac{f(x)}{g(x)}$ to each sample of g