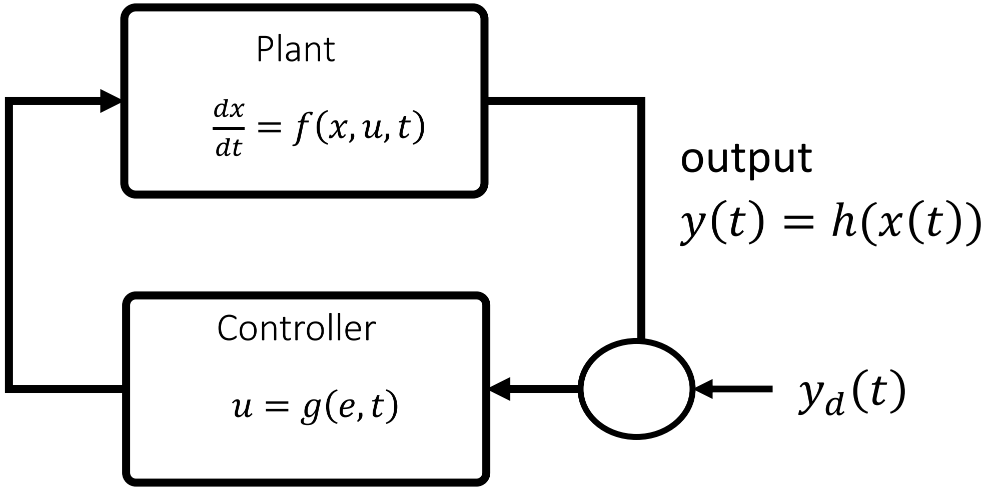
Modeling and Control

17 Feb

Simplified view of a plant and a controller



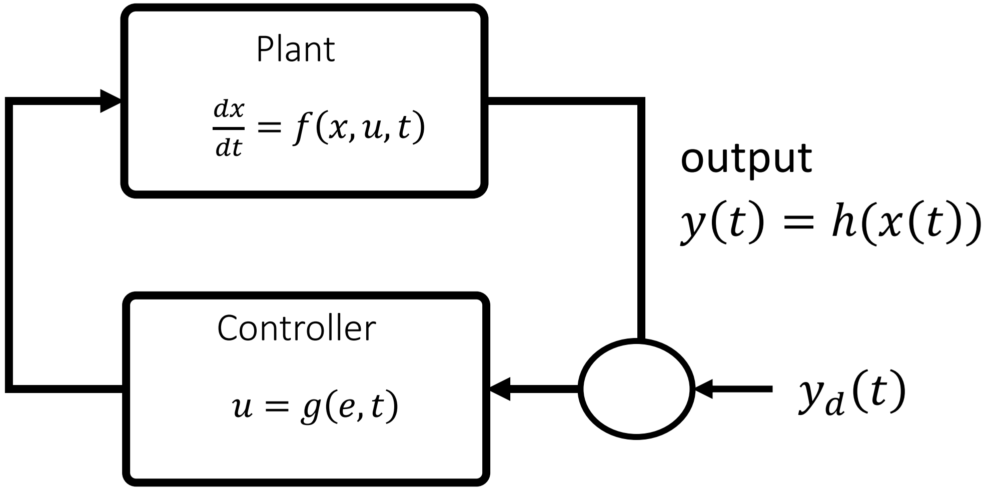
$$\frac{dx}{dt} = f(x, u(t), t); \quad y(t) = h\big(x(t)\big);$$

$$e(t) = y(t) - y_d(t)$$

$$u(t) = g(e(t), t)$$

Control?

The process of causing a system variable to conform to some desired value



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$$e(t) = y(t) - y_d(t)$$

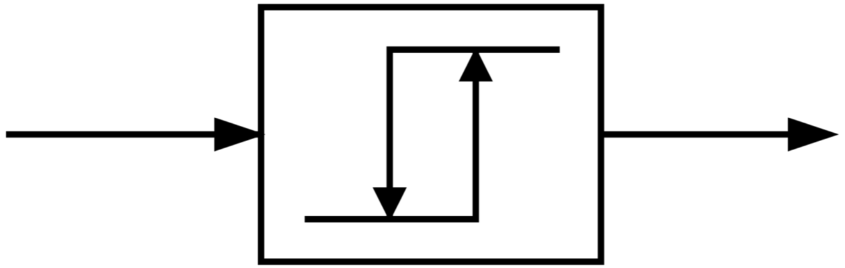
$$u(t) = g(e(t), t)$$

Use action u to make error $e = 0$

Bang-Bang Controller

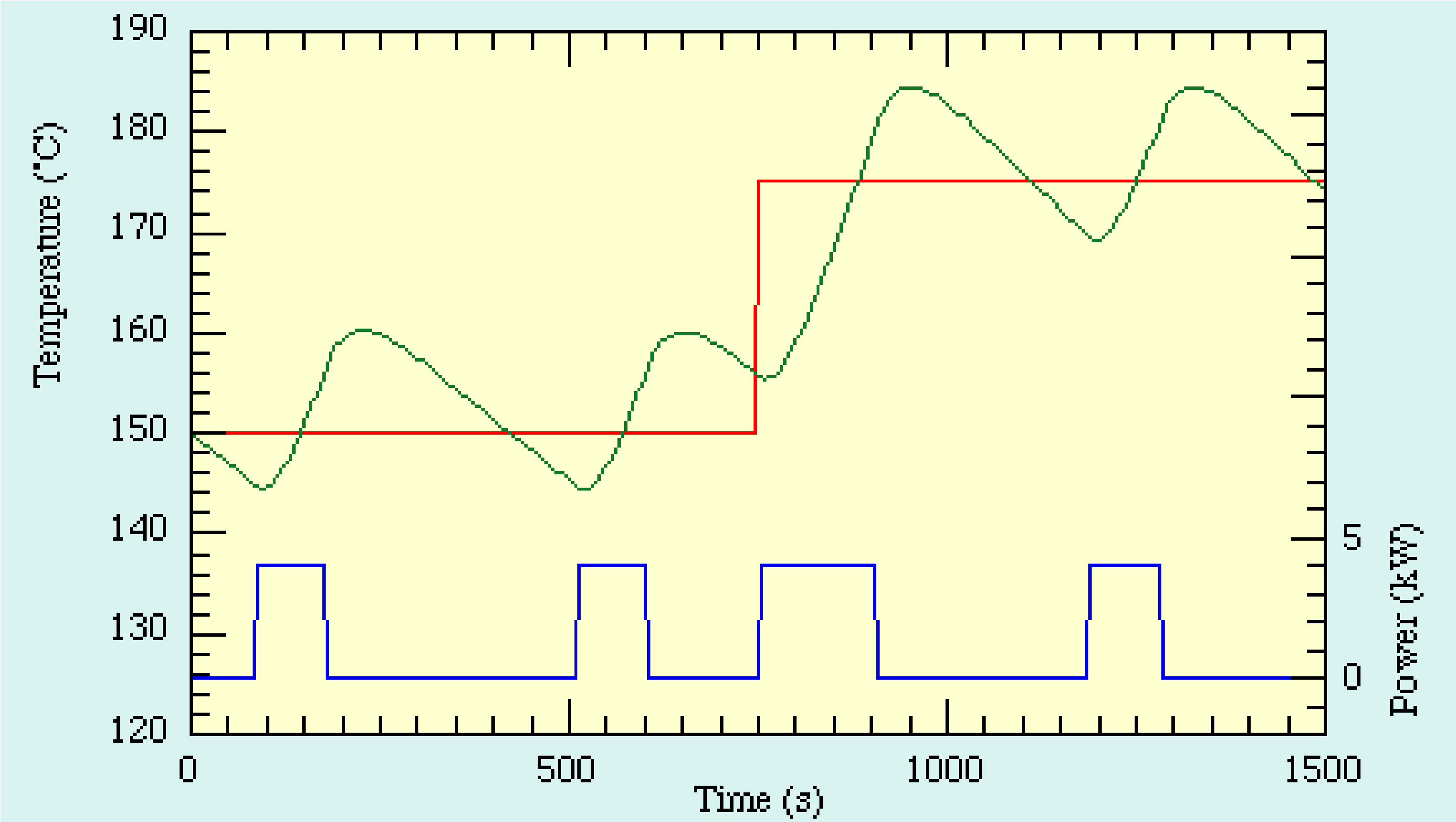
- Controller
  - $e(t) < 0 \Rightarrow u(t) = constant$
  - $e(t) > 0 \Rightarrow u(t) = -constant$

- To prevent chattering around $e(t) = 0$
  - $e(t) < -a \Rightarrow u(t) = constant$
  - $e(t) > a \Rightarrow u(t) = -constant$



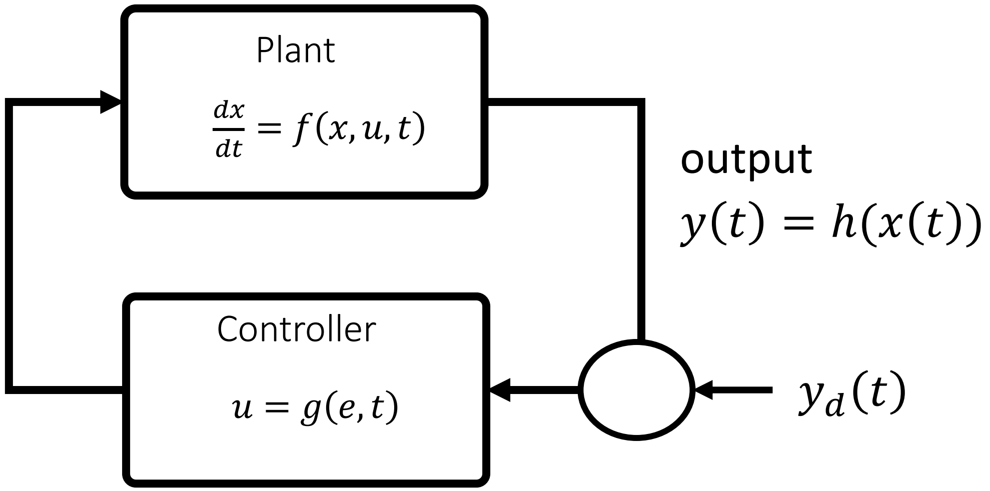
- Most common residential thermostats are bang–bang controllers.

Bang-Bang controller



- Good for reaching the setpoint
- It cannot track the setpoint accurately.

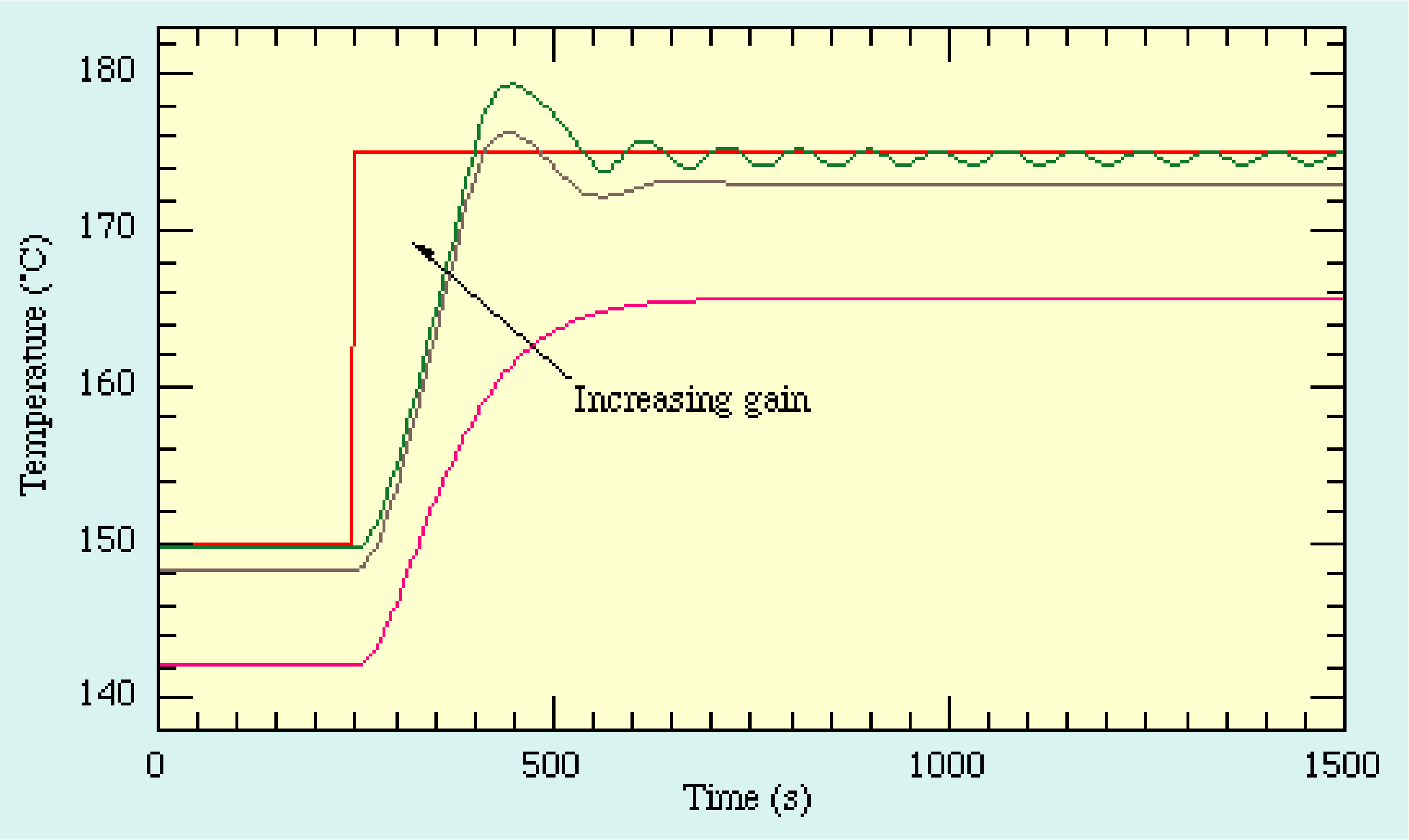
Proportional Control



- Use action u to make error $e = 0$
  - $u(t) = k_p e(t)$

- The controller gain k_p determines how quickly the system responds to error.
  - If k_p is small, error $e(t)$ can be large.
  - If k_p is too large, the system may oscillate or become unstable.

Proportional Control



- Increasing gain approaches setpoint faster
- Can lead to overshoot, and even instability
- Steady-state offset

Proportional-Integral (PI) Control

- We can add an integral term to the Proportional controller.

$$u(t) = k_p e(t) + k_I \int_0^t e(\tau)\, d\tau$$

- Advantages of PI control
  - Eliminating steady-state offset

Proportional-Derivative (PD) control

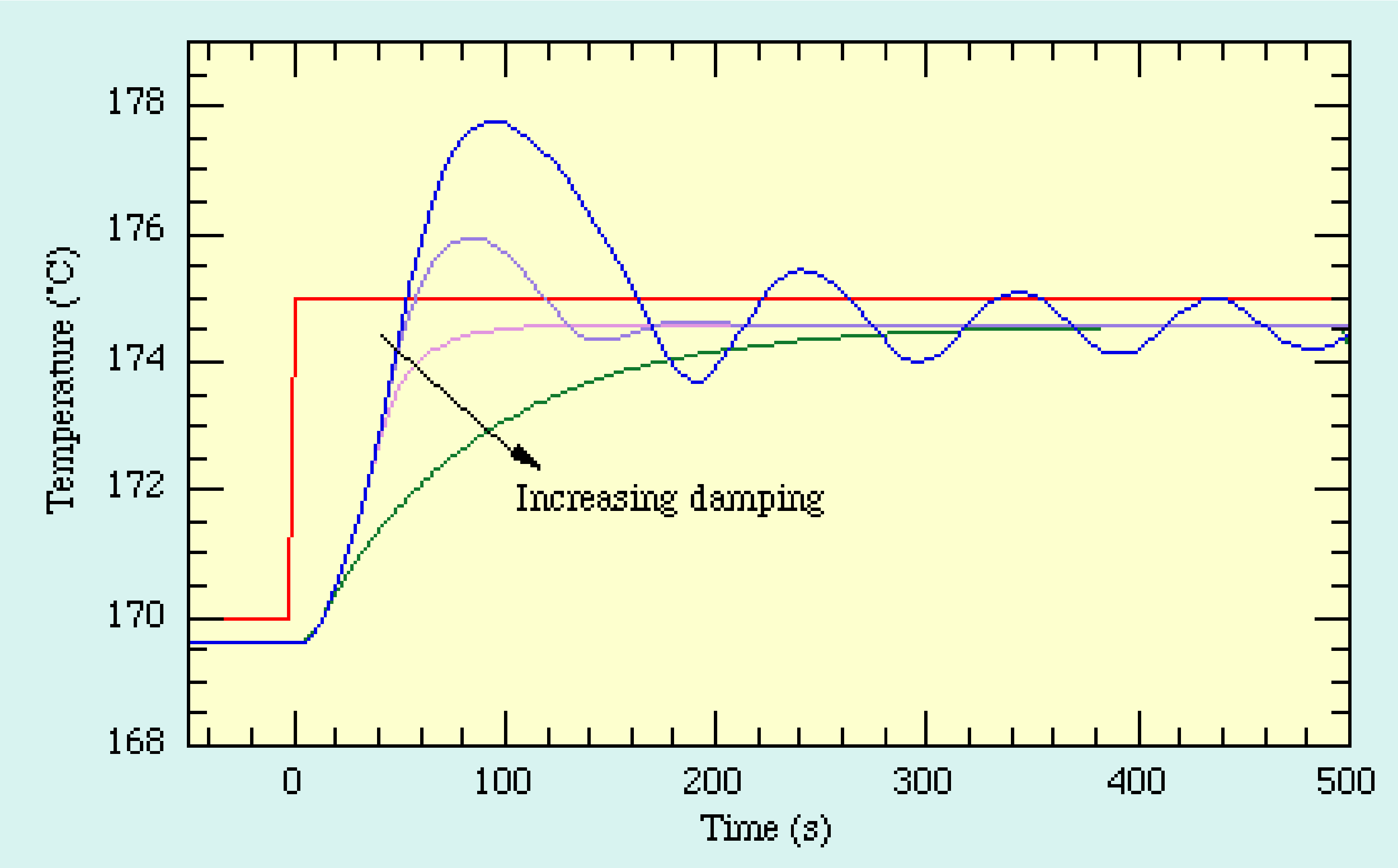
- Damping friction is a force opposing motion, proportional to velocity
- We can add the rate of change of the error.

$$u(t) = k_p e(t) + k_d \frac{de}{dt}$$

- Advantages of PD control
  - Reducing the overshoot of a P controller response
  - Improving the system's tolerance to external disturbances

- Estimating a derivative from measurements is not easy, and it can amplify noise.

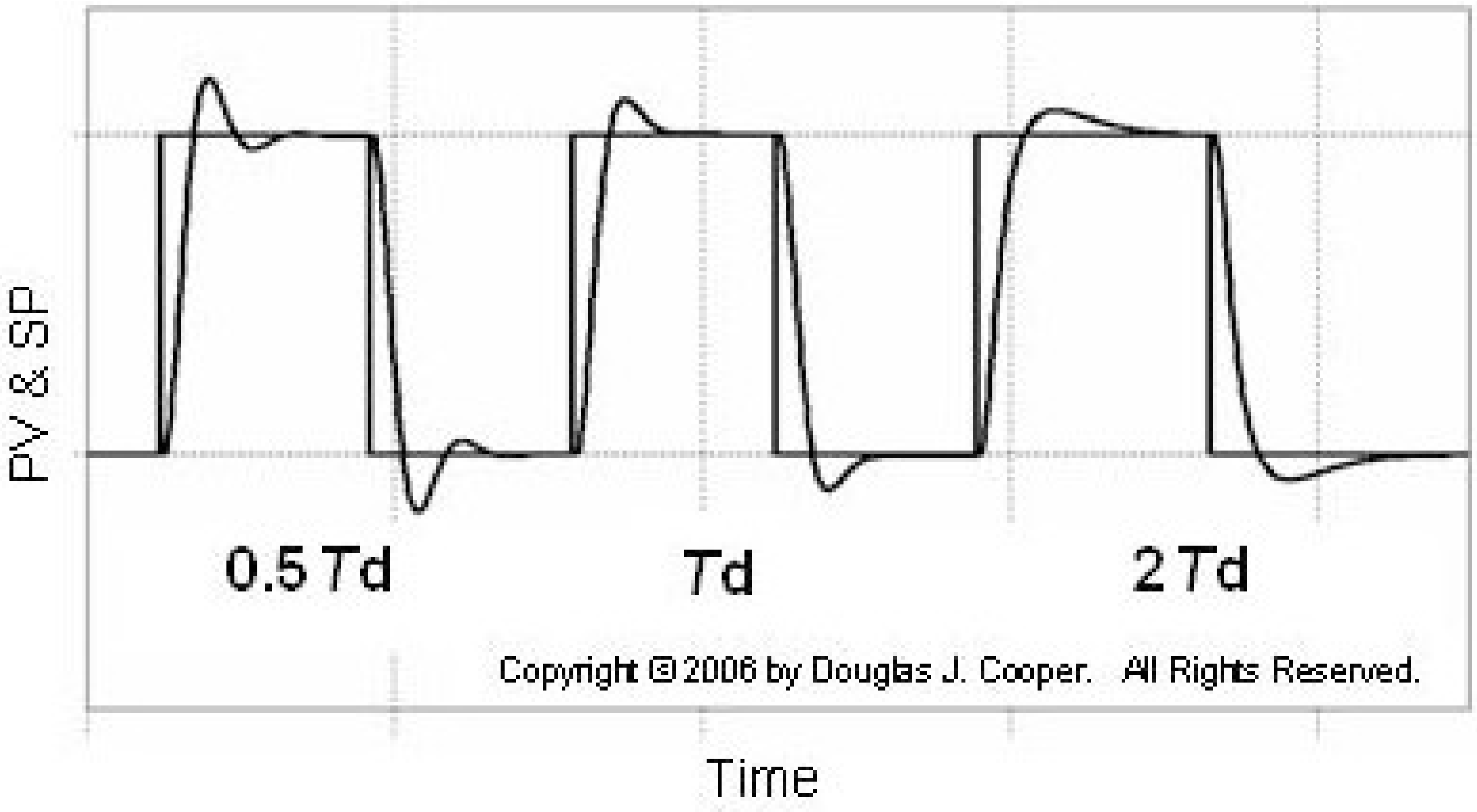
Derivative Control



- Reducing overshoot and oscillation

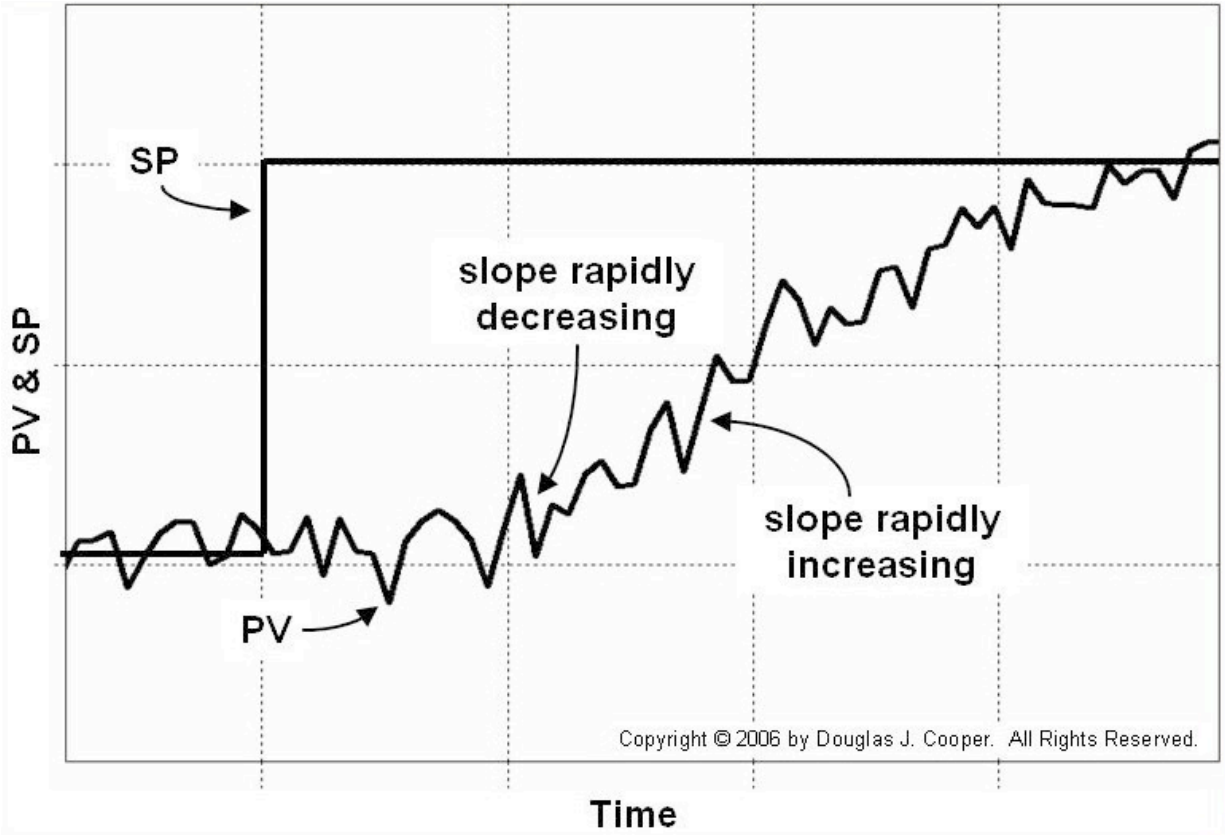
Effect of Derivative Control

- Different amount of damping



Derivatives Amplify Noise

- A problem if control output depends on slope with a high gain



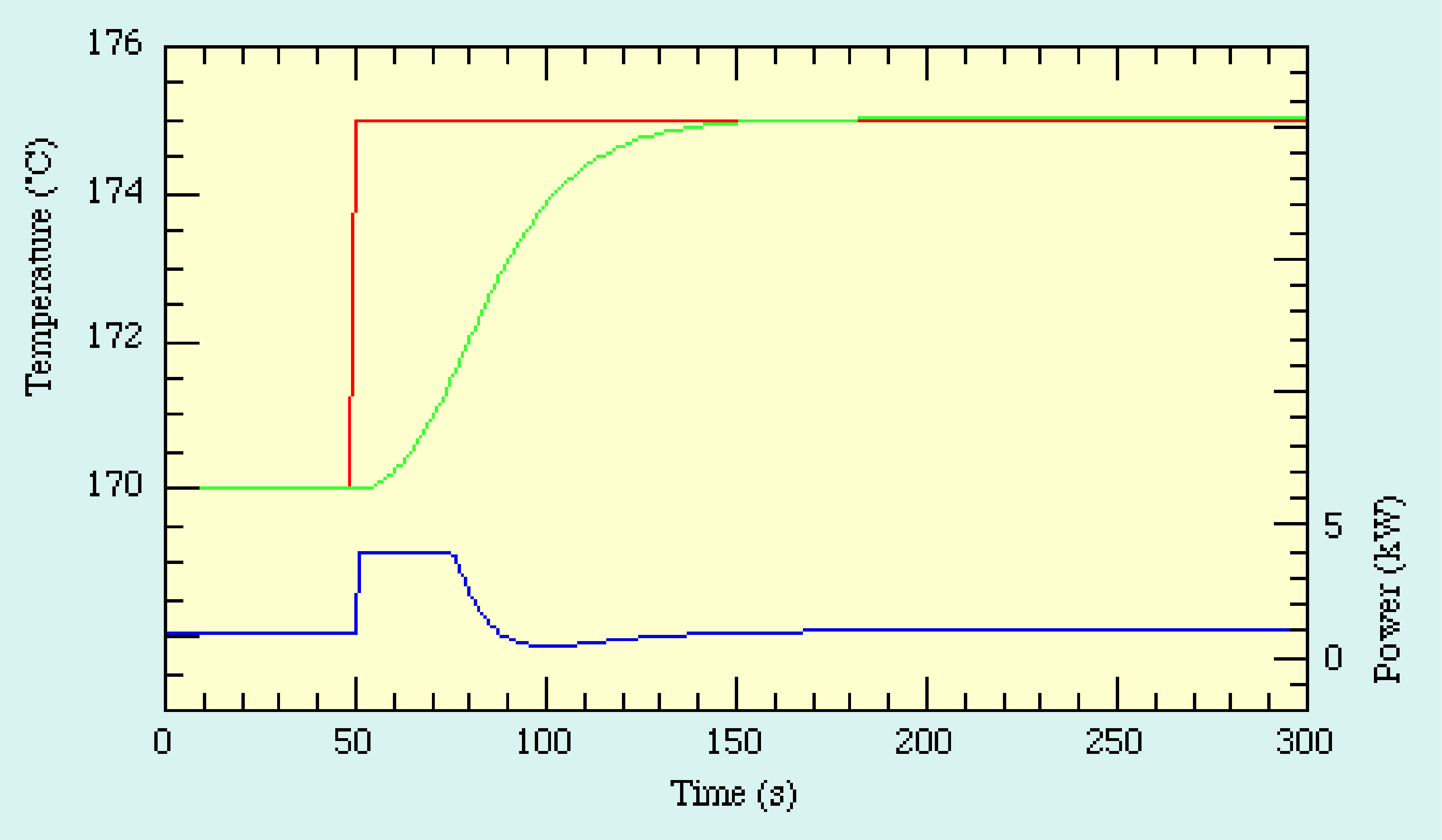
PID control

- 90% (or more) of control loops in industry are PID
- Simple control design model → simple controller
- The standard form of a PID controller:

$$u(t) = k_p e(t) + k_I \int_0^t e(\tau)\, d\tau + k_d \frac{de}{dt}$$

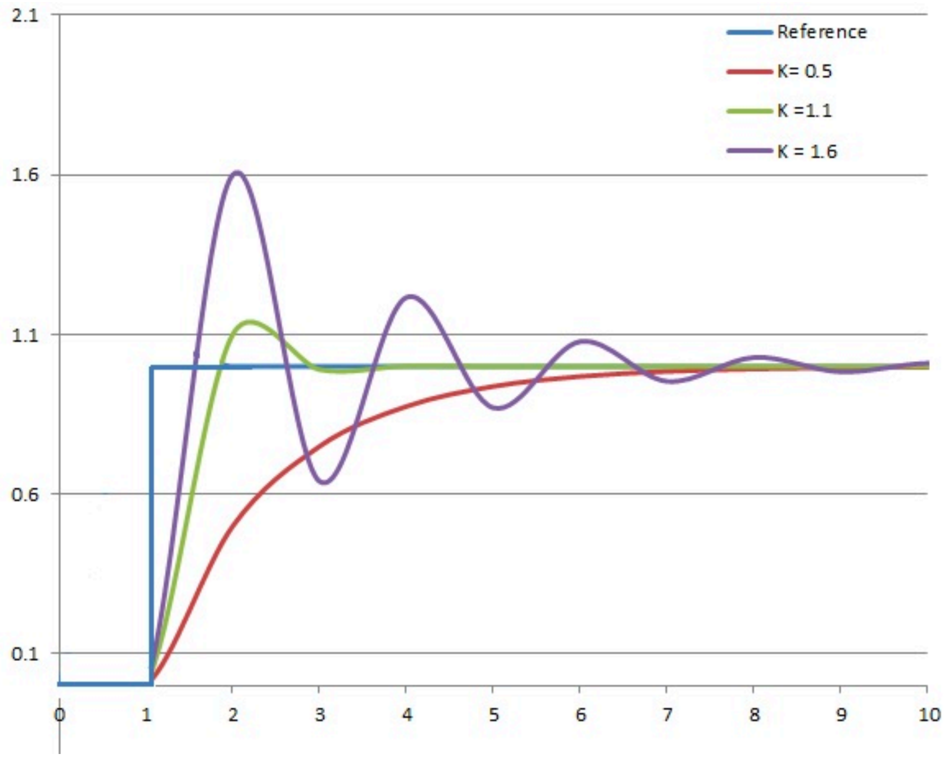
- where the error term $e(t) = y(t) - y_d(t)$
- $y_d(t)$: desired output or setpoint value
- k_p, k_I, k_d: constant gains
- Advantages
  - Removal of steady-state error
  - Reducing overshoots
  - Transient response time

PID Control



…when it is tuned well

Choosing proportional gain k_p in PID



Response of y(t) to step change of $y_d(t)$ vs time, for three values of K_p (K_i and K_d held constant) Fig. from wikipedia

Ziegler-Nichols method of tuning PID

1. Set k_d and k_i to zero.
2. Increase k_p until the system start to oscillate at a frequency.
3. Set the oscillation period as T_u and $k_u = k_p$.
4. $u(t) = k_p e(t) + k_I \int_0^t e(\tau)\, d\tau + k_d \frac{de}{dt}$

	k_p	k_I	k_d
P	$0.5k_u$		
PI	$0.5k_u$	$1.2k_u/T_u$	
PD	$0.8k_u$		$k_uT_u/8$
PID	$0.5k_u$	$2k_u/T_u$	$k_uT_u/8$