

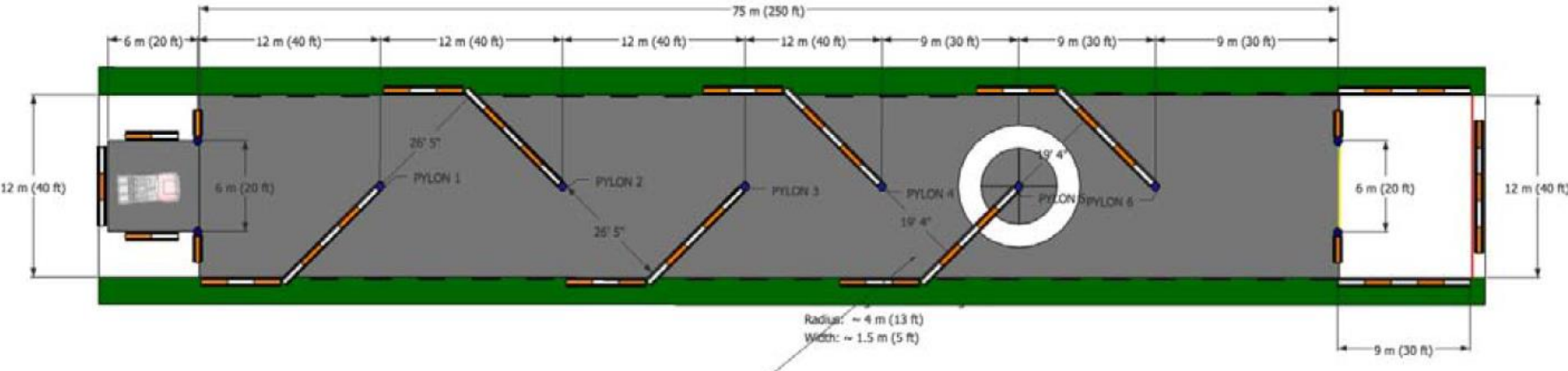
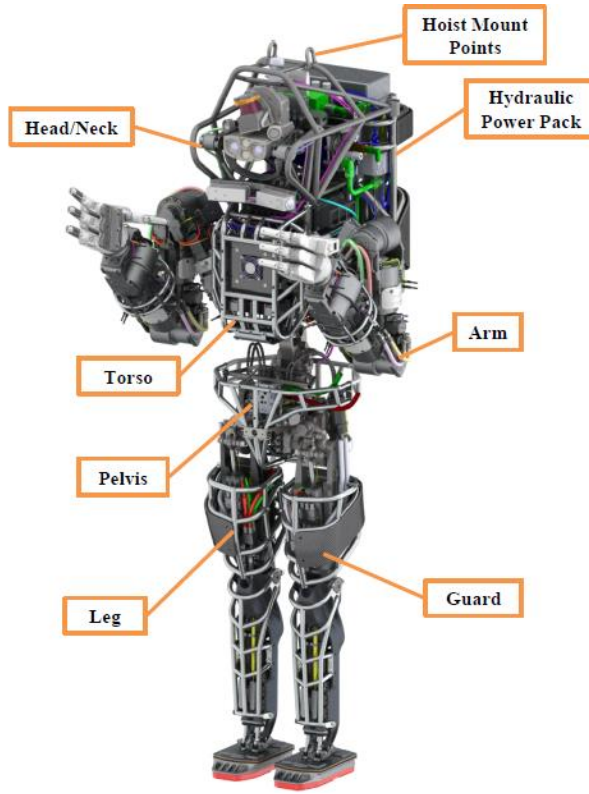
Modeling and Control

12 Feb



DARPA Robotics Challenge

- 4 points task
 - Robot drives the vehicle through the course (1)
 - Robot gets out of the vehicle and travels dismounted out of the end zone (2)
 - Bonus point (1)



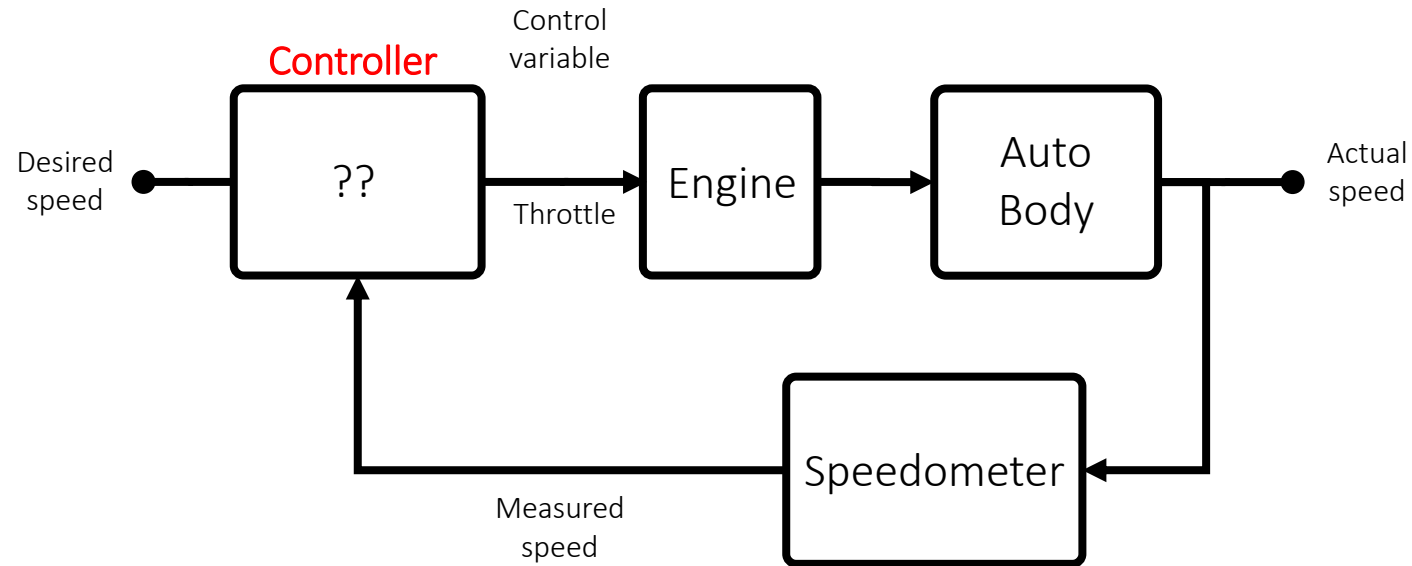


Control?

The process of causing a system variable to conform to some desired value



Example: Cruise Control





What do we want to control?

-> Car position and orientation

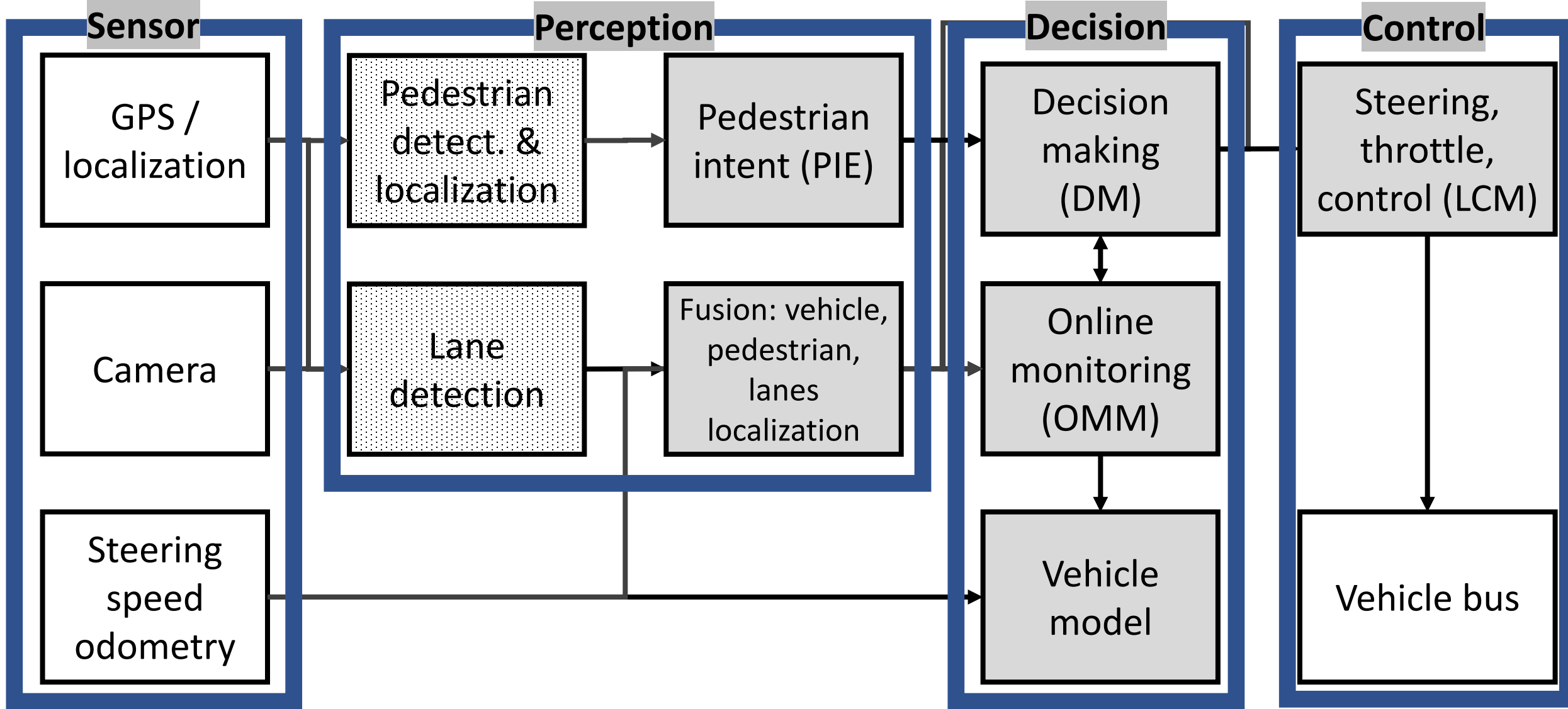
What can we use?

-> Steering wheel and pedal(s)

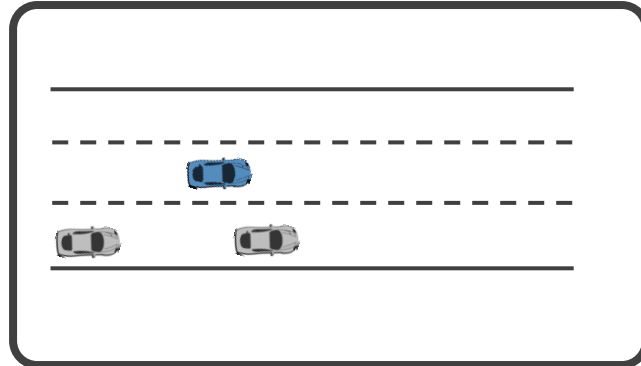
Real Time Factor: 0.47 Sim Time: 00 00:18:35.996 Real Time: 00 00:27:45.996 Iterations: 785399



GEM system



Dynamical system models



Nonlinear dynamics

Generally, nonlinear ODEs do not have closed form solutions!

Dubin's car model

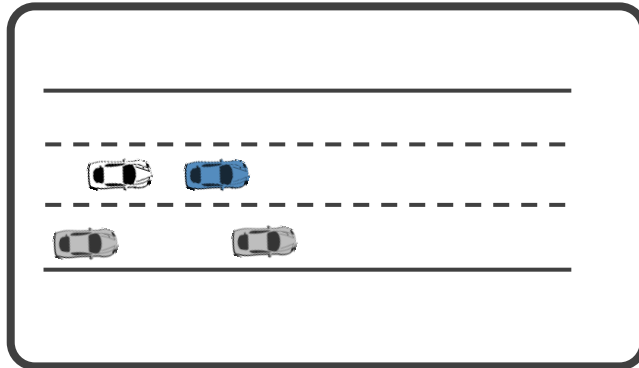
$\dot{v} = a$	Speed
$\frac{ds_x}{dt} = v \cos(\psi)$	Horizontal position
$\frac{ds_y}{dt} = v \sin(\psi)$	Vertical position
$\frac{d\delta}{dt} = v_\delta$	Steering angle
$\frac{d\psi}{dt} = \frac{v}{l} \tan(\delta)$	Heading angle

Physical plant

$\frac{dx}{dt} = f(x, u)$	System dynamics
$x[t + 1] = f(x[t], u[t])$	
$x = [v, s_x, s_y, \delta, \psi]$	State variables
$u = [a, v_\delta]$	Control inputs



Nonlinear hybrid dynamics



Physical plant

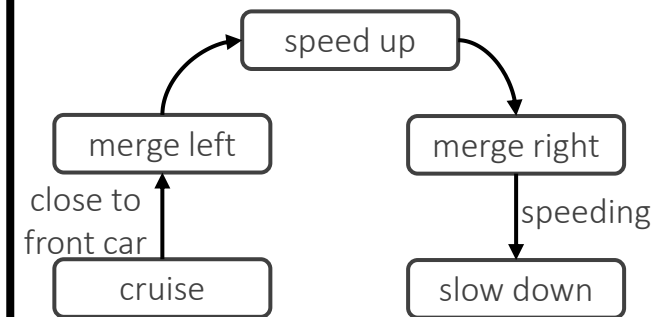
$$\frac{dx}{dt} = f(x, u) \quad \text{System dynamics}$$

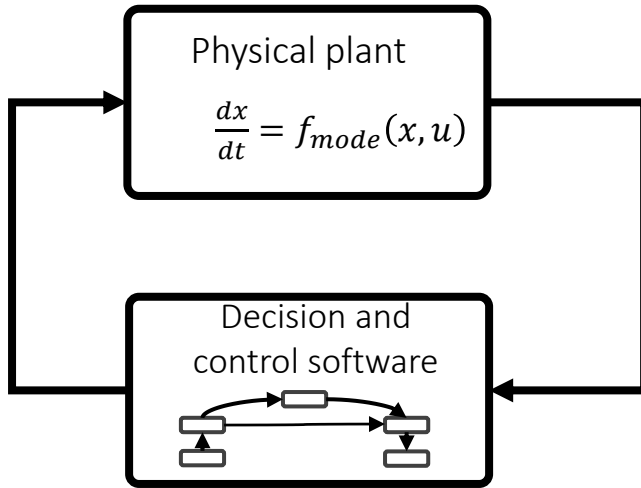
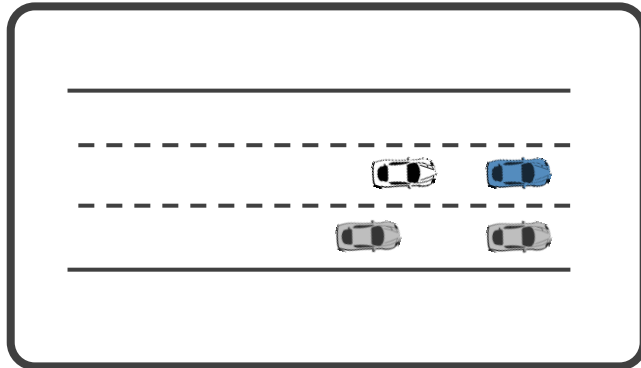
$$x[t + 1] = f(x[t], u[t])$$

$$x = [v, s_x, s_y, \delta, \psi] \quad \text{State variables}$$

$$u = [a, v_\delta] \quad \text{Control inputs}$$

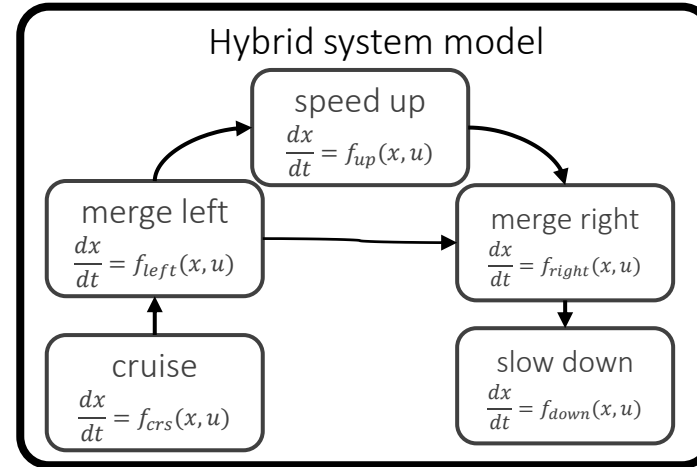
Decision and control software



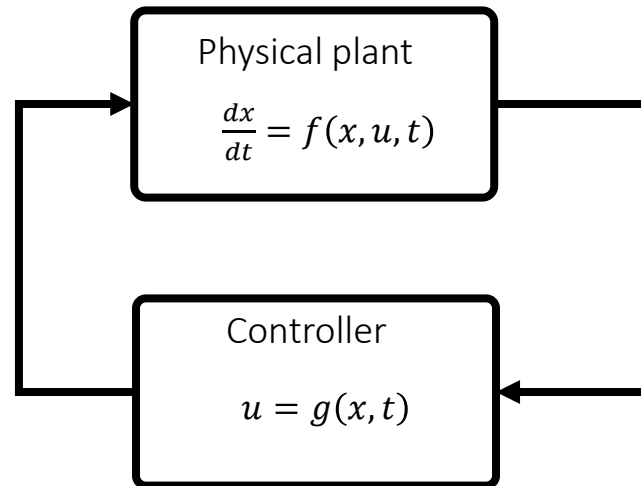


Nonlinear hybrid dynamics

Interaction between computation and physics can lead to unexpected behaviors



Simplified view of a plant and a controller



$$\frac{dx}{dt} = f(x, g(x), t)$$

$$\dot{x} = f(x, g(x), t)$$



Model?

A set of mathematical relationships among the system variables



Dynamical Systems Model

Describe behavior in terms of instantaneous laws

$$\frac{dx(t)}{dt} = f(x(t), u(t), t)$$

$$t \in \mathbb{R}, x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$$

$f: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^n$ dynamic function



Example: Pendulum

Pendulum equation

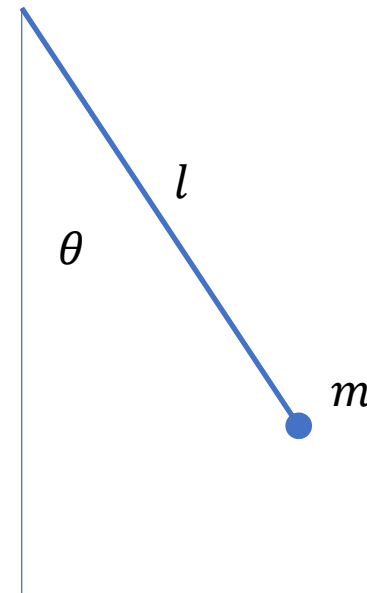
$$x_1 = \theta \quad x_2 = \dot{\theta}$$

$$x_2 = \dot{x}_1$$

$$\dot{x}_2 = -\frac{g}{l} \sin(x_1) - \frac{k}{m} x_2$$

$$\begin{bmatrix} \dot{x}_2 \\ \dot{x}_1 \end{bmatrix} = \begin{bmatrix} -\frac{g}{l} \sin(x_1) - \frac{k}{m} x_2 \\ x_2 \end{bmatrix}$$

k : friction coefficient



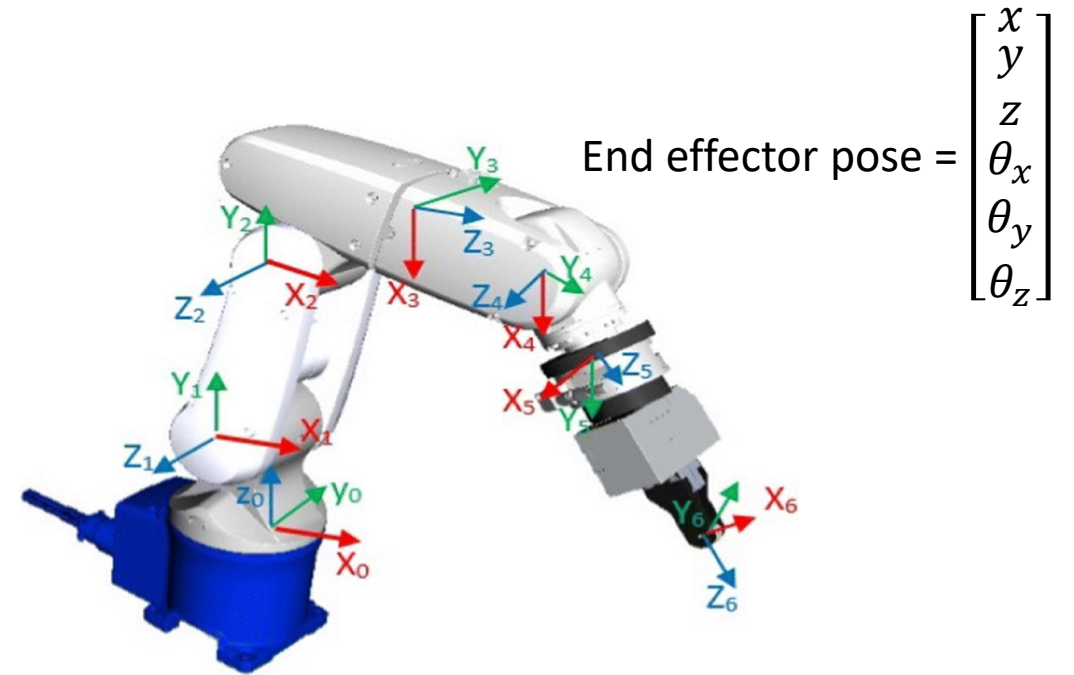
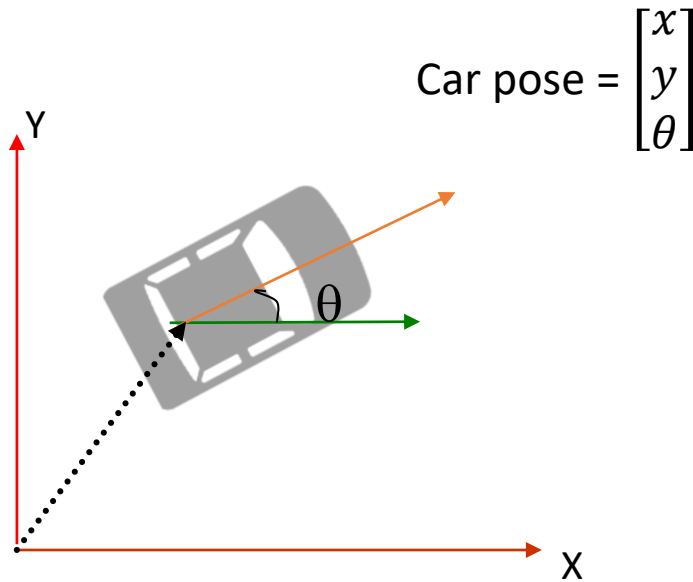
What is described?

-> Center of mass movement relative to the origin

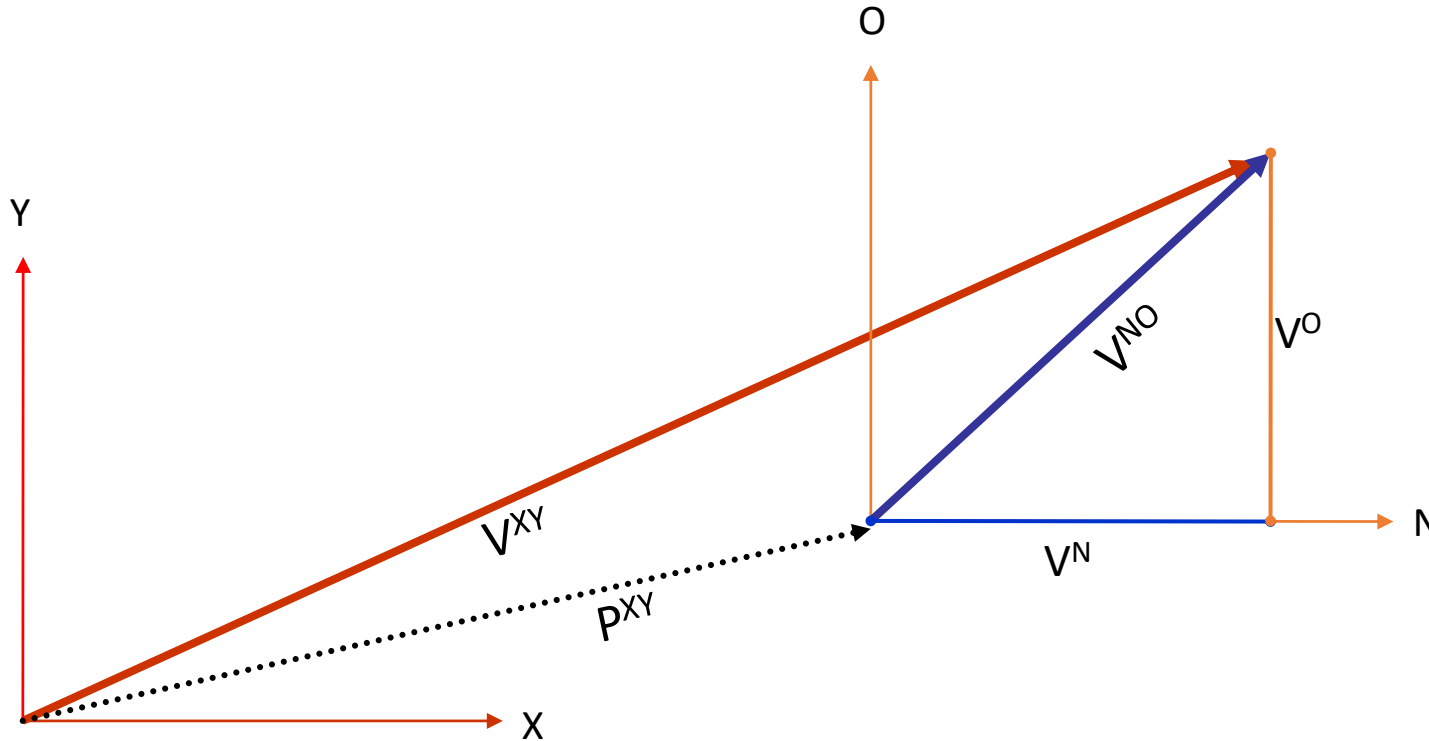


Coordinate system

- Configuration (pose) of robot can be described by position and orientation.



Translation along the X-Axis and Y-Axis



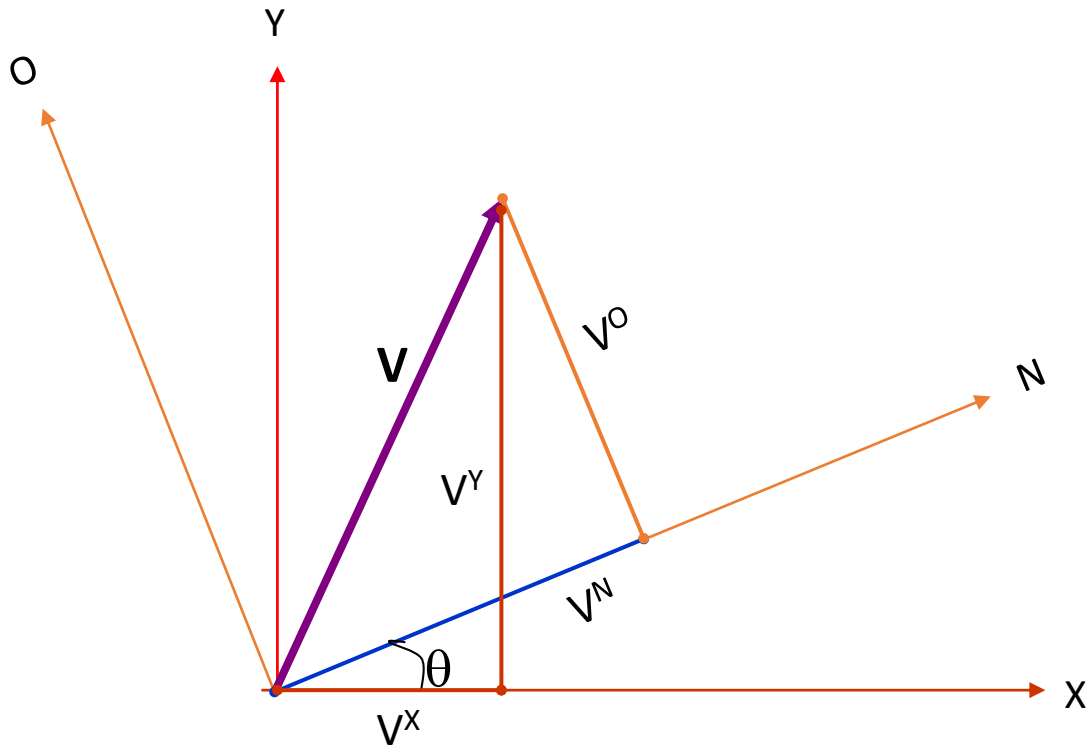
$$\bar{P}^{XY} = \begin{bmatrix} P_x \\ P_y \end{bmatrix}$$

$$\bar{V}^{NO} = \begin{bmatrix} V^N \\ V^O \end{bmatrix}$$

$$\bar{V}^{XY} = \bar{P}^{XY} + \bar{V}^{NO} = \begin{bmatrix} P_x + V^N \\ P_y + V^O \end{bmatrix}$$



Rotation (Z-Axis)



θ = Angle of rotation between the XY and NO coordinate axis

$$\bar{v}^{XY} = \begin{bmatrix} v^X \\ v^Y \end{bmatrix} \quad \bar{v}^{NO} = \begin{bmatrix} v^N \\ v^O \end{bmatrix}$$

$$\bar{v}^{XY} = \begin{bmatrix} v^X \\ v^Y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} v^N \\ v^O \end{bmatrix}$$

- Rotation matrix

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

- An orthogonal (orthonormal) matrix
 - Each column is a unit length vector
 - Each column is orthogonal to all other columns
- The inverse is the same as the transpose

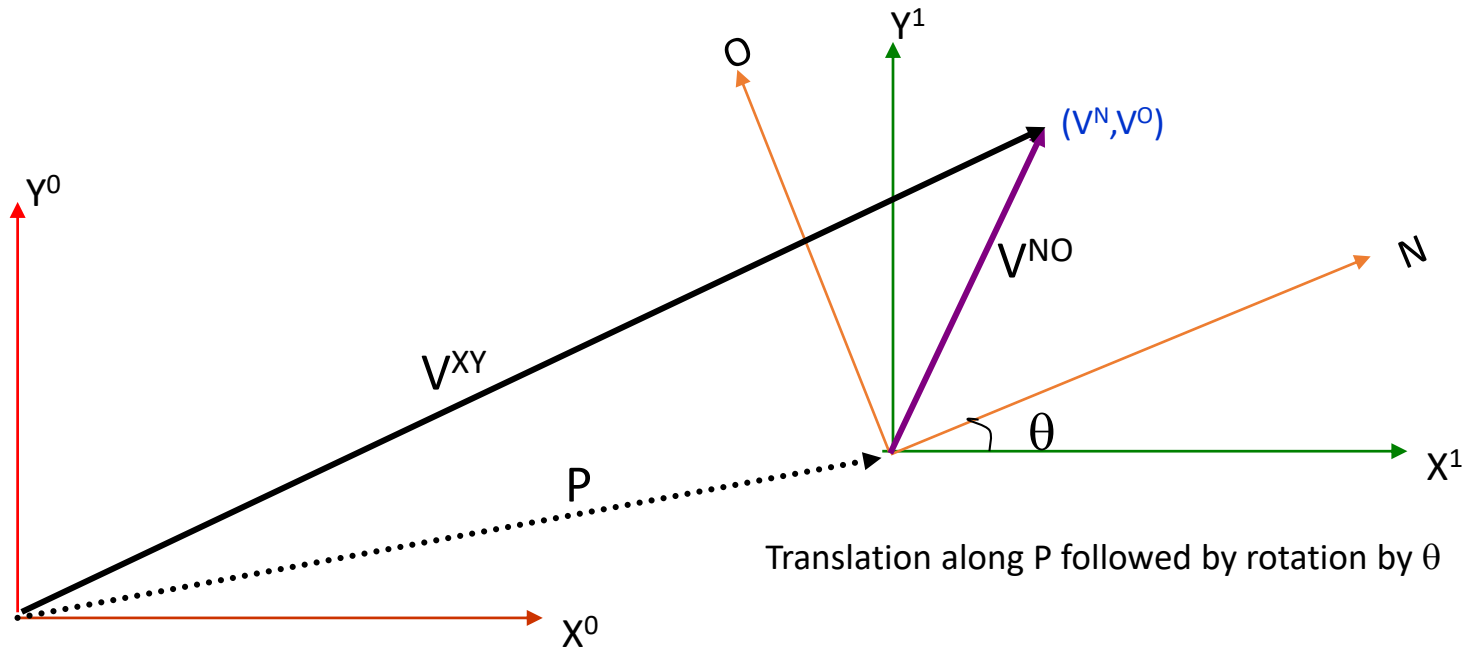
$$R(\theta)^{-1} = R(\theta)^T$$

- The determinant is 1
- Special Orthogonal group of dimension 2

$$R(\theta) \in SO\{2\}$$



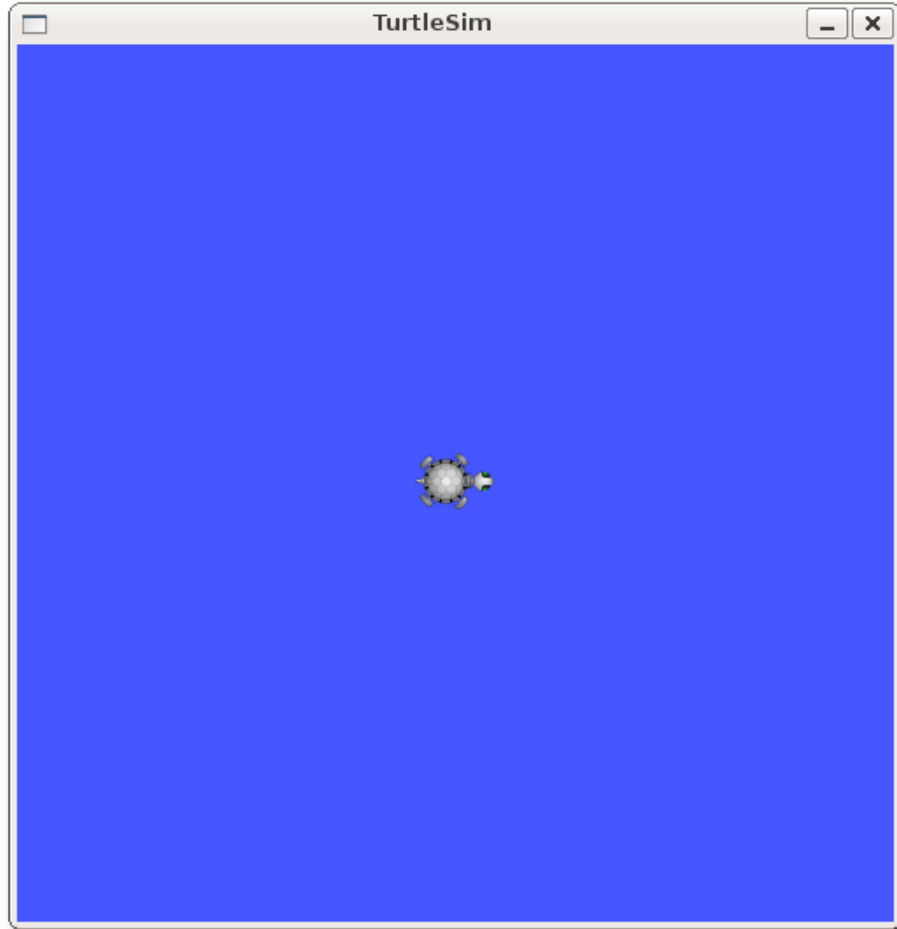
Transformation



$$V^{XY} = \begin{bmatrix} V^X \\ V^Y \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V^N \\ V^O \end{bmatrix}$$



TurtleSim

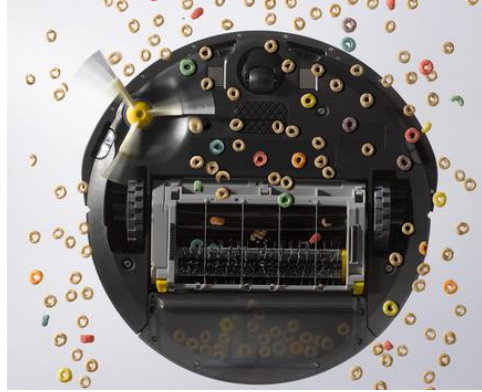


- Turtle simulator
- simple way to learn the basics of ROS
- a ROS package, part of the ROS installation
- Explore the nodes, topics, messages, and services (roscore, rosnode, and rostopic commands)

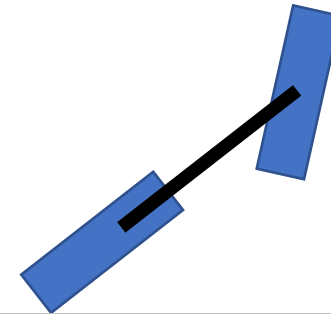


☐ ☆ ↻	Rehaud Dubé	inbox [robotics-worldwide] [jobs] - SevenSense Opportunity - C++ Performance Developer - of mobile robotics. Our mission is to unlock new levels of productivity for entire industries by making robots n...	5:50 AM
☐ ☆ ↻	Tully Tote	inbox [robotics-worldwide] [meetings] ROSCon 2020 will be in New Orleans Nov 14th to 16th - 3A__roscon.ros.org_2020_&d-DwIBaQ&c-clK7kQUTW/LAVEOVlgvi0NU5BOUHhpN0H8p7CSfnc_gf&r-Dw3s...	5:56 AM
☐ ☆ ↻	Dino Accoto (Assoc .	inbox [robotics-worldwide] [jobs] Post-Doc and Research Associate positions at NTU, Singapore - The Robotics Research Centre at Nanyang Technological University (NTU, Singapore; prof. Dino Accoto...	5:33 AM
☐ ☆ ↻	Evan Drumwright	inbox [robotics-worldwide] [jobs] internships for dextrous manipulation - , in robotics? Are you driven to do more in robotics than one-off demo videos? Dextrous Robotics is a startup backed by...	
☐ ☆ ↻	Evan Drumwright	inbox [robotics-worldwide] [jobs] junior/senior software engineer for dextrous manipulation - , in robotics? Are you driven to do more in robotics than one-off demo videos? Dextrous Robotics is a startu...	Feb 9
☐ ☆ ↻	Yue Yufeng	inbox [robotics-worldwide] [jobs] Multiple Research Fellow and Research Associate Positions in Autonomous Robotics at Nanyang Technological University - ranked 6th worldwide in the QS World Univ...	Feb 9
☐ ☆ ↻	Jaime Pulido Fentan.	inbox [robotics-worldwide] [jobs] Computer Vision Engineer @ Saga Robotics - *Saga Robotics are currently looking for a new full-time coworker with experience in computer vision systems and robot	Feb 9
☐ ☆ ↻	Jaime Pulido Fentan.	inbox [robotics-worldwide] [jobs] Software Engineer @ Saga Robotics - Norway Saga Robotics are currently looking for a new full-time coworker with experience in software integration to fulfill	Feb 9
☐ ☆ ↻	Tan U-Xuan	inbox [robotics-worldwide] [jobs] Postdoctoral fellow & research engineer positions - Engineering, Robotics, or related areas - Strong hands on experience Preference will be given to those with prior proj...	Feb 5
☐ ☆ ↻	Ivana Palunko	inbox [robotics-worldwide] [jobs] Postdoc, Phd and engineering positions in a H2020 SeaClear project - systems, robotics and machine learning. The team has access to several aerial and marine robot...	Feb 5
☐ ☆ ↻	YUHONG LIN	inbox [robotics-worldwide] [meetings] Last Call for Registration: Mastering ROS Robot Manipulators Course - : Mastering ROS Robot Manipulators Course FEBRUARY 24 - 28, 2020 IN BARCELONA, SPAIN...	Feb 5
☐ ☆ ↻	Jonathan DeCastro	inbox [robotics-worldwide] [meetings] ICRA 2020 Workshop "Transforming Specifications into Robot Programs: A Survey of Formal Methods Tools for Non-Experts" - applications in robotics. However, ...	Feb 5
☐ ☆ ↻	Aeolus Robotics Inc	inbox [robotics-worldwide] [jobs] Aeolus Robotics:Robotics Software Engineer - Aeolus Robotics Inc. is a start-up robotics company. We are looking for a driven Robotics Software Engineers who are han...	Feb 2
☐ ☆ ↻	giovanna varni	inbox [robotics-worldwide] [jobs] Post Doctoral position on Socially Assistive Robotics in Clinical Geriatrics at Sorbonne University, Paris, France - Socially Assistive Robotics in Clinical Geriatrics ****...	Feb 2
☐ ☆ ↻	khaled.geneidy	inbox [robotics-worldwide] [jobs] Research Associate in Computer Vision and Machine Learning for Robotics at UoL - Learning for Robotics. *Project: * The candidate will join an exciting project on auto...	Feb 2
☐ ☆ ↻	mingxi_zhou_uri	inbox [robotics-worldwide] [jobs] Marine Robotics autonomy, path-planning, and multi-vehicle operation, PhD/Master's student opening, University of Rhode Island (Mingxi Zhou) - experience in roboti...	Feb 2
☐ ☆ ↻	reyhanehpahlevan	inbox [robotics-worldwide] [news] RoboCup 2020 Virtual Robot Competition - Second Call for Participation - AI/robotics research, for example, behavior strategy (eg. multi-agent planning, realtime/anyti...	Jan 26
☐ ☆ ↻	Alberto Rodriguez	inbox [robotics-worldwide] [news] MIT RoboSeminar 2019 - online talks - the MIT Robotics youtube channel: https://urldefense.proofpoint.com/v2/url?u=https-3A__www.youtube.com_channel_LUCK2IKz...	Jan 23
☐ ☆ ↻	reyhanehpahlevan	inbox [robotics-worldwide] [news] RoboCup 2020 Virtual Robot Competition - Second Call for Participation - AI/robotics research, for example, behavior strategy (eg. multi-agent planning, realtime/anyti...	Jan 23
☐ ☆ ↻	YUHONG LIN	inbox [robotics-worldwide] [meetings] Last Call for Early-Bird: Mastering ROS Robot Manipulators Course - : Mastering ROS Robot Manipulators Course* *Date.* February 24 - February 28, 2020 *Locati...	Jan 21
☐ ☆ ↻	Micire, Mark	inbox [robotics-worldwide] [jobs] Robotics, Intelligent, and Autonomous Systems positions in Perth, Australia - Robotics Engineer Intelligent and Autonomous Systems Group, Woodside Energy, Perth, A...	Jan 21
☐ ☆ ↻	Davide Scaramuzza	inbox [robotics-worldwide] [software] Driving Event Camera Datasets (Samsung DVS Gen3) - System (ROS). The format is the one used by the RPG DVS ROS driver. The rosbag files contain events using...	Jan 21

Locomotion of Wheeled System



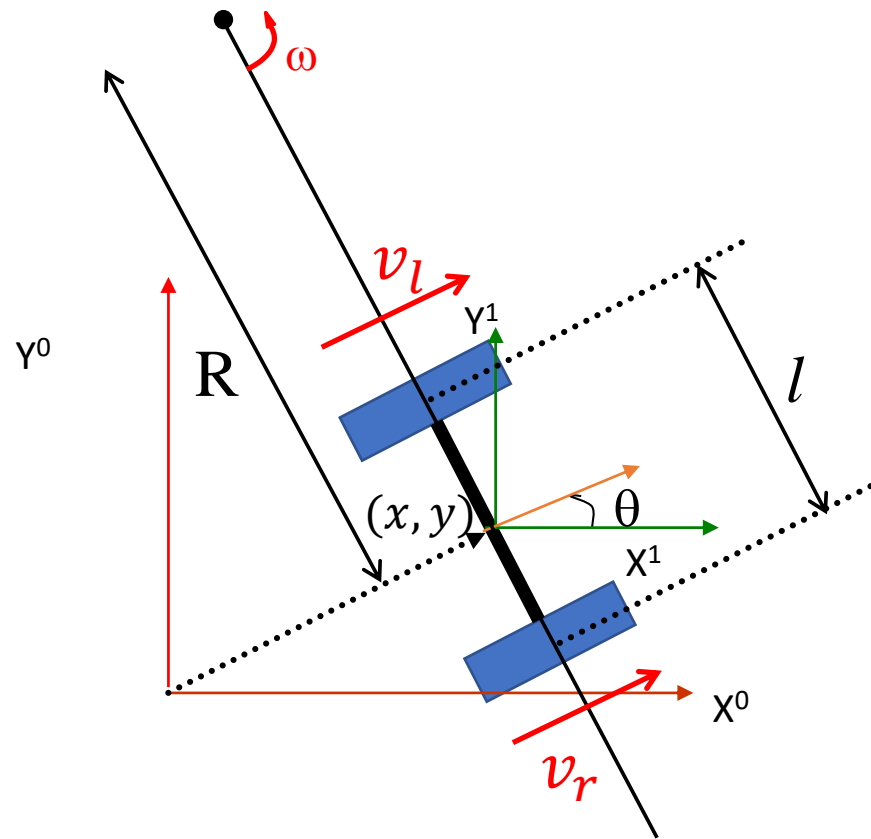
Differential Drive Model



Rear Wheel Model



Differential Drive Model

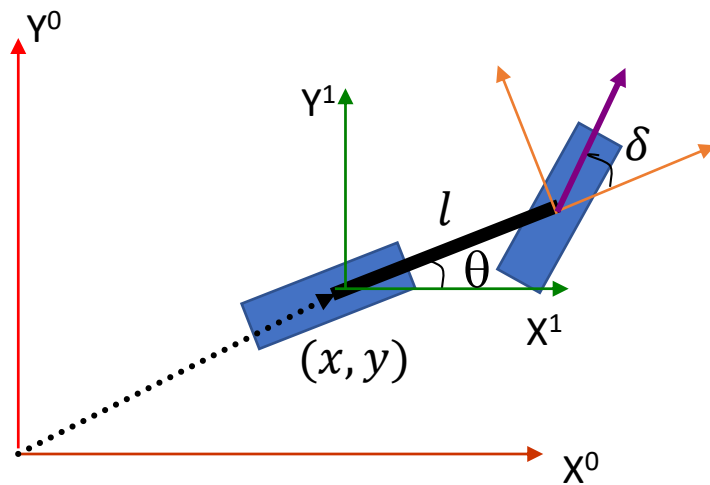


Instantaneous Center of Curvature
 $= [x - R \sin \theta, y + R \cos \theta] = [ICC_x, ICC_y]$

$$\begin{aligned}\omega(R + l/2) &= v_r \\ \omega(R - l/2) &= v_l \\ R &= \frac{l(v_r + v_l)}{2(v_r - v_l)} \\ \omega &= \frac{v_r - v_l}{l}\end{aligned}$$



Rear Wheel Model (Dubin's model)



Car length = l

Car (real wheel) pose = $\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$

Car speed = v

Car (front wheel) steering angle = δ

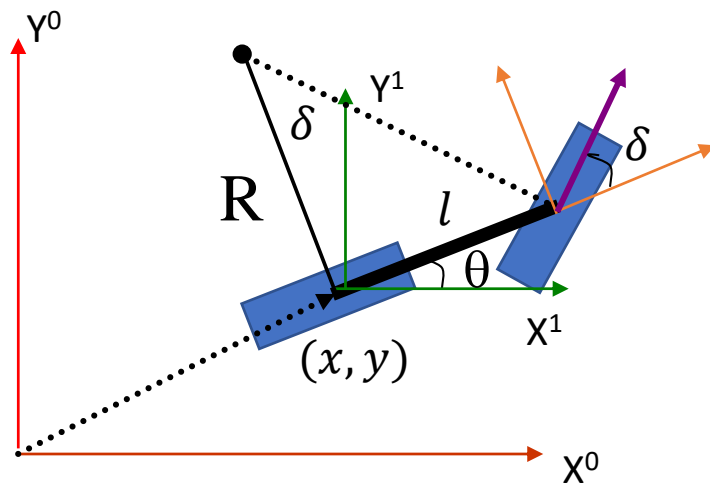
$$\dot{x} = v \cos\theta$$

$$\dot{y} = v \sin\theta$$

$$\dot{\theta} = \frac{v}{l} \tan\delta$$



Rear Wheel Model (Dubin's model)



Car length = l

Car (real wheel) pose = $\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$

Car speed = v

Car (front wheel) steering angle = δ

$$\dot{x} = v \cos\theta$$

$$\dot{y} = v \sin\theta$$

$$\dot{\theta} = \frac{v}{l} \tan\delta$$

$$R d\theta = dp_r$$

$$R \frac{d\theta}{dt} = \frac{dp_r}{dt} = v$$

$$\tan\delta = \frac{l}{R} \rightarrow R = \frac{l}{\tan\delta}$$

