Spectral Theory of Partial Differential Equations

Professor Richard Laugesen <Laugesen@illinois.edu>

Math 595 section STP (CRN 58570), Fall 2011
17 October to 7 December (2nd half of semester)

When and where: MWF 4-5pm, room 345 AH. **Text:** none.

Course Material: Spectral methods permeate the theory of PDEs. One solves linear PDEs by separation of variables, getting eigenvalues when the spectrum is discrete, and continuous spectrum when it is not. Linearized stability of a steady state or traveling wave of a nonlinear PDEs depends on the sign of the first eigenvalue, or on the location of the continuous spectrum in the complex plane. We aim at highlights of spectral theory:

Part I - Linear PDEs: wave, diffusion, Schrödinger, beam/plate (4th order)
Discrete spectrum and variational methods
Computable special cases (interval, rectangle, disk, ball, harmonic oscillator)
Boundary conditions: imposed and natural
Geometric bounds for low eigenvalues, Weyl asymptotic for high eigenvalues
Can you hear the shape of a drum? (inverse spectral problem)
Continuous spectrum
Computable special cases
Schrödinger with potential

Part II - Nonlinear PDE: thin fluid film, KdV, nonlinear Schrödinger
Stability of steady states, stationary waves, traveling waves, similarity solutions.

What will this course be like?
Research is different from standard coursework. Research often starts with questions motivated by analogy, or by trying to generalize special cases. Normally we find answers in a nonlinear fashion, slowly developing a coherent theory by linking up and extending our scraps of known information. We cannot predict what we will need to know in order to succeed, and we certainly do not have enough time to study all relevant background material.
To succeed in research, we must develop a rough mental map of the surrounding mathematical landscape, so that we know the key concepts and canonical examples (without necessarily knowing the proofs). Then when we need to learn more about a topic, we know where to begin.
This course aims to develop your mental map of spectral theory, as it relates to partial differential equations.

Is this course right for you? [Prerequisites]
If you have taken a graduate course in partial differential equations (e.g. Math 553), then give this course a try! Graduate students from other departments are welcome.

Assessment: based on a 20-30 minute presentation on a topic relevant to the course.