Dependable Demand Response Management in the Smart Grid: A Stackelberg Game Approach

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Abstract—Demand Response Management (DRM) is a key component in the smart grid to effectively reduce power generation costs and user bills. However, it has been an open issue to address the DRM problem in a network of multiple utility companies and consumers where every entity is concerned about maximizing its own benefit. In this paper, we propose a Stackelberg game between utility companies and end-users to maximize the revenue of each utility company and the payoff of each user. We derive analytical results for the Stackelberg equilibrium of the game and prove that a unique solution exists. We develop a distributed algorithm which converges to the equilibrium with only local information available for both utility companies and end-users. Though DRM helps to facilitate the reliability of power supply, the smart grid can be susceptible to privacy and security issues because of communication links between the utility companies and the consumers. We study the impact of an attacker who can manipulate the price information from the utility companies. We also propose a scheme based on the concept of shared reserve power to improve the grid reliability and ensure its dependability.

Index Terms—Demand response management, dependability, reliability, reserve power, security, smart grid, Stackelberg game.

I. INTRODUCTION

The demand of electricity consumers has been growing due to increased use of machines and the new types of appliances such as plug-in hybrid electric vehicles. The concern towards the impact on environment and on the reliability of power supply, has also been rising. However, traditional power grids are not able to meet these demands and requirements because of their inflexible designs and lack of prompt communications between the supply and the demand sides. Recent blackouts [1] have indicated the inefficiency and serious reliability issues of the traditional grid. Therefore, it is essential to transform the traditional power grid into a more responsive, efficient and reliable system. Smart grid [2] is a future power grid system that incorporates a smart metering infrastructure capable of sensing and measuring power consumption from consumers with the integration of advanced information and communication technologies (ICT). Thus the power generation, distribution and consumption is efficient, more economical and more reliable in the smart grid network.

Demand Response Management (DRM), a key feature of the smart grid, is defined as changes in electric usage by end-users in response to changes in the price of electricity over time or across different energy sources. The importance of DRM can go far beyond reducing the electricity bills of consumers or the cost of generating power. It helps to balance the demand and supply in the power market through real-time pricing. It can also provide short-term reliability benefits as it can offer load relief to resolve system and/or local capacity constraints.

The recent studies on DRM can be categorized mainly into two areas: utility company (UC) oriented and end-user oriented. There has been considerable amount of work in power systems on supply-demand balance and market clearance [3], [4]. Such studies on power systems have focused on the economic aspects at the planning and generation level and have not considered user-utility as a significant component. On the other hand, the literature on user-utility has introduced schemes to maximize user utilities, without considering the power generation costs or the revenue of the UCs. This has motivated us to consider the issue of benefit maximization for users alongside with the revenue maximization for the UCs. Our work aims to bridge the gap between the existing two research directions. In addition, with increasing concerns towards environment, incorporating renewable energy resources becomes important in the smart grid. This has motivated us to include in our work renewable energy sources in addition to traditional fossil fuel based sources.

We study the interactions among multiple UCs and multiple consumers, who aim to maximize their own payoffs. The UCs maximize their revenues by setting appropriate unit prices. The consumers choose power to buy from UCs based on the unit prices. The payoff of each consumer depends on the prices set by all the sources. In turn, the price set by each UC also depends on the prices of other UCs. These complicated interactions motivate us to use a game theoretical framework in our analysis. We develop a Stackelberg game between the UCs and the users where the UCs play a non-cooperative game and the consumers find their optimal response to the UCs' strategies. The interactions between the UCs and the users are enabled by the bidirectional communications between them.

An advanced metering infrastructure (AMI) is a communication infrastructure that enables meters and utilities to exchange information such as power consumption, price update, or outage awareness. Smart meters play the key role of gateway between the customers' premises and the utility

network. Their functionality make them an interesting target for attackers [5]. Therefore it is important to assess possible consequences of attacks and develop mechanisms to maintain the reliability and resilience of the grid in the face of unanticipated events. We assess the impact of an attacker that can manipulate the price of the UCs, and propose a scheme to ensure the reliability of power supply in the presence of an attacker, thus making the smart grid a dependable system.

We have three major contributions in this work.

1) We establish an analytical model for the multiple-UCs multiple-consumers Stackelberg game and characterize its unique Stackelberg equilibrium (SE).
2) We propose a distributed algorithm which converges to the SE with only local information of the users and the UCs.
3) We propose a scheme based on a common reserve to improve the dependability of the smart grid. We also discuss reliability of the grid when one of the sources gets disconnected from the grid due to occurance of some physical incidents.

The rest of the paper is organized as follows. Related work is described in Section II. We introduce the system model and the communication model in Section III. In Section IV, we formulate the problem as a Stackelberg game and prove the existence and uniqueness of the SE. We propose a distributed algorithm for the game which converges to the SE, in Section V. In Section VI, we study the impact of an attacker as a possible threat to grid stability and propose a scheme based on maintaining a shared reserve power. We provide numerical results and discussion in Section VII. Section VIII concludes the paper.

II. RELATED WORK

There are several studies on DRM in the smart grid [6]-[10]. In [6], the authors have formulated the energy consumption scheduling problem as a non-cooperative game among the consumers for increasing strictly convex cost functions. In [7], the authors have considered a distributed system where price is modeled by its dependence on the overall system load. Based on the price information, the users adapt their demands to maximize their own utility. In [8], a robust optimization problem has been formulated to maximize the utility of a consumer, taking into account price uncertainties at each hour. In [9], the authors have exploited the awareness of the end-users and proposed a method to aggregate and manage end-users' preferences to maximize energy efficiency and user satisfaction. In [10], a dynamic pricing scheme has been proposed to incentivize costumers to achieve an aggregate load profile suitable for utilities, and the demand response problem has been investigated for different levels of information sharing among the consumers in the smart grid. In [11], the unit commitment scheduling problem in smart grid communications has been studied using a partially observable Markov decision process framework for stochastic power demand loads and renewable energy resources. However, the analyses in [6]-[11], are limited in the sense that there is either only one source or a number of sources/utilities treated as one entity. Differently

in our study, we include multiple UCs and consumers whose goal is to maximize their own payoffs, using the concept of Stackelberg game.

We note that there is rich literature using Stackelberg games in the context of congestion control, revenue maximization and cooperative transmission [14]-[15]. Our approach is similar to those in congestion control to model the behavior of end-users, but our study involves multiple UCs, and we adopt the non-cooperative game framework among UCs using the Stackelberg solution concept.

DRM enhances the reliability of the grid [16] when the data communications is perfect. However, the data communications in the smart grid may suffer attacks such as data manipulation or false data injection [17] from malicious nodes. In such cases, the UC or the users may incur economic loss or physical impact e.g., grid instability. In [18], the authors have studied the utility-privacy tradeoffs of smart meter data and shed light on the impact of leakage of the data on the utilities of both the users and the suppliers. In [19], the authors have proposed a secure routing protocol incorporating delay due to queue building. They have investigated the tradeoffs between efficiency, reliability and resilience in centralized and decentralized approaches for secure routing. [20] proposes a six-layer hierarchical security architecture for the smart grid, identifying the security challenges present at each layer and addressing security issues at three different layers. In [21] the authors have developed a formal model for the C12.22 standard protocol to guarantee that no attack can violate the security policy without being detected based on the concept of specification-based intrusion detection. It is observed that there is no work that addresses the impact of attacks from an outsider on DRM through the information exchange between the users and the UCs. Because of the communications between the consumers and the UCs, there are inherent vulnerabilities that attackers can exploit to harm the utilities of either side or to even cause physical damage on the system.

III. SYSTEM MODEL

We consider N end-users, which we also call customers, and K electricity UCs. Fig. 1 depicts an overview of the scenario. The utility side consists of the renewable and non-renewable energy sources. The fossil-fuel based energy generators have certain amount of power available all the time. The power generated by the fossil fuel generators creates pollution to the environment. On the other hand, the renewable energy sources can be seen as pollution free but they do not always have power available. When renewable energy sources are incorporated into the system, we add uncertainties to the utility side. There are many studies where discrete time Markov chain models have been used to model the availability of energy from the renewable sources (such as wind and solar energy) [11], [23], [24]. We incorporate the renewable energy sources too, and consider a stationary distribution for the states of the renewable energy generators. The end-user side consists of several consumers, which may be residential users, commercial users or industries. These different types of users have different needs for electricity. We differentiate them in terms of
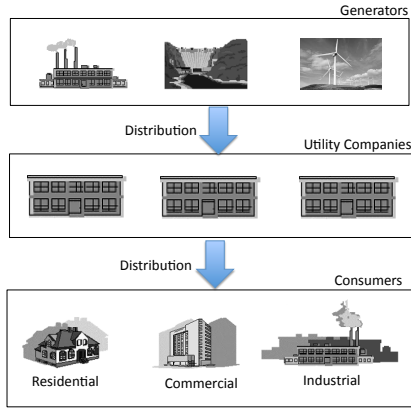
Fig. 1. Smart grid system model with multiple energy sources and end-users

Fig. 2. Illustration of communications between utility companies and consumers

available budget which is an upper bound on their affordability to buy power. We employ a utility function for each user that increases with the amount of electricity the user can consume. At the same time, we incorporate a cost constraint for each user. The UCs and the consumers have bidirectional communications for exchanging price and demand information as shown in Fig. 2. The UCs can also communicate with each other. The users receive the price information from the utility companies and transmit their demand to them. The data communication is carried out through the communication channel using wireless technologies, e.g., WiFi, WiMAX, or LTE.

In practice, the electricity generation, distribution and consumption can be decomposed into three layers as described in [22]: generators, aggregators or utility companies, and the end-users. The acquisition of power by the utility companies from the generators is a separate process. In this paper, we focus on the interactions between the UCs and the end-users. In practice, the unit price of a UC is determined through the market by the system operator. In this paper, the UCs play a non-cooperative game at the market level. Different from the traditional *perfect competitive market*, the UCs participate in an imperfect competition. In a perfectly competitive market, no market participant has the ability to influence the market price through its individual actions, i.e., the market price is a parameter over which firms have no control. Consequently, each firm should increase its production up to the point where its marginal cost equals the market price. This is valid when the number of market participants is large and none of the participants controls a large proportion of the production. However, in this paper, we consider a finite number of market participants (UCs) and each individual UC has non-infintesimal influence in the market. This leads to imperfect competition, where each firm determines its unit price based on its available power.

IV. Utility-User Interaction: Stackelberg Game

When there are multiple UCs with different energy prices, the cost to each user varies according to the prices set by each UC. In addition, the price set by a UC also depends on the
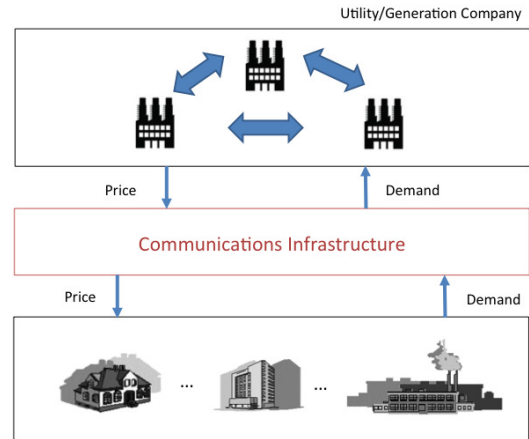
prices of other UCs. Game theory provides a natural paradigm to model the behavior of the end-users and of the UCs in this scenario. The UCs set the price per unit power and announce it to the users. The users respond to the price by demanding an optimal amount of power from the UCs. Since the UCs act first and then the users make their decision based on the prices, the two events are sequential. Thus, we model the interactions between the UCs and the end-users as a Stackelberg game [25]. In our proposed game, the UCs are the leaders and the consumers are the followers. It is a multi-leaders and multi-followers game. The demand of the users depends on the unit price set by the UCs as well as their own cost constraints. In turn, the UCs optimize their unit prices according to the response of the consumers.

A. User Side Analysis

Let $x_{n,k}$ be the demand of user n from UC k. We define the utility of user n, $U_{\text{user},n}$ as

$$U_{\text{user},n} = \alpha_n \sum_{k \in \mathscr{K}} \ln \left(\beta_n + x_{n,k} \right), \ \forall k \in \mathscr{K} \tag{1}$$

where α_n and β_n are constants. The ln function has been widely used in economics for modeling the preference ordering of users and for decision making [12], [13].

The motivation behind choosing the utility function for user n as in (1) is that it is closely related to the utility function $\alpha_n \sum \ln(x_{n,k})$ that leads to proportionally fair demand response [12] [14]. If we use the utility function $\alpha_n \sum \ln(x_{n,k})$, then a user gets a payoff of $-\infty$ with respect to (w.r.t.) UC k when $x_{n,k} = 0$. With β_n, when $x_{n,k} = 0$, its benefit with w.r.t. that UC becomes finite. A typical value of β_n is 1.

Let y_k be the unit price set by UC k and let $C_n > 0$ denote the budget of user n. For a given set of prices from the UCs $\{y_1, y_2, \ldots, y_K\}$, user $n \in \mathscr{N}$ calculates its optimal demand response by solving the user optimization problem (OP$_{\text{user}}$)

$$\max_{\mathbf{x}_n := \{x_{n,k}, \forall k \in \mathscr{K}\}} U_{\text{user},n} \tag{2}$$

$$\text{s.t.} \sum_{k \in \mathscr{K}} y_k x_{n,k} \leq C_n, \tag{3}$$

$$x_{n,k} \geq 0; \ \forall k \in \mathscr{K}. \tag{4}$$

OP_{user} is a convex optimization problem. Hence, the stationary solution is unique and optimal.

Let us start the analysis with N users and 2 UCs. We will later generalize the results to K UCs. The optimization problem for user n in this case is

$$\max_{\mathbf{x}_n := \{x_{n,1}, x_{n,2}\}} \alpha_n \sum_{k=1}^{2} ln(\beta_n + x_{n,k}) \tag{5}$$

$$\text{s.t. } y_1 x_{n,1} + y_2 x_{n,2} \leq C_n, \tag{6}$$

$$x_{n,1}, x_{n,2} \geq 0. \tag{7}$$

Using Lagrange's multipliers $\lambda_{n,1}, \lambda_{n,2}$ and $\lambda_{n,3}$ for constraints (6) and (7), we convert the constrained optimization problem (5) - (7) to the form

$$L_{user,n} = \alpha_n \sum_{k=1}^{2} ln\left(\beta_1 + x_{n,k}\right)$$
$$- \lambda_{n,1}\left(\sum_{k=1}^{2} y_k x_{n,k} - C_1\right) + \lambda_{n,2} x_{n,1} + \lambda_{n,3} x_{n,2} \tag{8}$$

and the complementarity slackness conditions

$$\lambda_{n,1}\left(\sum_{k=1}^{2} y_k x_{n,k} - C_n\right) = 0, \tag{9}$$

$$\lambda_{n,2} x_{n,1} = 0, \tag{10}$$

$$\lambda_{n,3} x_{n,2} = 0, \tag{11}$$

$$\lambda_{n,1} > 0, \lambda_{n,2}, \lambda_{n,3}, x_{n,1}, x_{n,2} \geq 0. \tag{12}$$

The first-order optimality condition for the maximization problem is $\nabla L_{user} = 0$, where $L_{user} = \{L_{user,n}, \forall n \in \mathcal{N}\}$. Since the only coupling between the users is through y_k, $\nabla L_{user} = 0$ leads to

$$\frac{\partial L_{user,n}}{\partial x_{n,k}} = 0, \forall n \in \mathcal{N}, k \in \mathcal{K}, \text{i.e.,},$$

$$\begin{cases} \frac{\alpha_n}{\beta_n + x_{n,1}} - \lambda_{n,1} y_1 + \lambda_{n,2} = 0, \\ \frac{\alpha_n}{\beta_n + x_{n,2}} - \lambda_{n,1} y_2 + \lambda_{n,3} = 0. \end{cases} \tag{13}$$

The optimal demands of users can take one of the following forms.

1) Case 1 : $x_{n,1}, x_{n,2} > 0$: In this case, $\lambda_{n,2} = \lambda_{n,3} = 0$. Substituting $\lambda_{n,2}$ and $\lambda_{n,3}$ into (13) yields

$$x_{n,k} = \frac{\alpha_n}{\lambda_{n,1} y_k} - \beta_n, \ \forall n \in \mathcal{N}, k = 1, 2. \tag{14}$$

Using (14) in (9) yields

$$\frac{1}{\lambda_{n,1}} = \frac{C_n + \beta_n \sum\limits_{k=1}^{2} y_k}{2\alpha_n}. \tag{15}$$

Now substituting (15) into (14) yields

$$x_{n,k} = \frac{C_n + \beta_n \sum\limits_{k=1}^{2} y_k}{2y_k} - \beta_n \ k = 1, 2. \tag{16}$$

2) Case 2 : $x_{n,1} > 0, x_{n,2} = 0$: This is the case when $\frac{C_n + \beta_n \sum\limits_{k=1}^{2} y_k}{2y_2} = \beta_n$. Equation (10) implies $\lambda_{n,2} = 0$ and

$$x_{n,1} = \frac{\alpha_n}{\lambda_{n,1} y_1} - \beta_n. \tag{17}$$

Substituting $x_{n,1}$ into (9), we get,

$$\lambda_{n,1}\left(\frac{\alpha_n}{\lambda_{n,1}} - \beta_n y_1 - C_n\right) = 0.$$

Since $\lambda_{n,1} > 0$, $\frac{\alpha_n}{\lambda_{n,1}} - \beta_n y_1 - C_n = 0$ which gives $\lambda_{n,1} = \frac{\alpha_n}{C_n + \beta_n y_1}$. Using $\lambda_{n,1}$ in (17), we obtain

$$x_{n,1} = \frac{C_n + \beta_n y_1}{y_1} - \beta_n = \frac{C_n}{y_1}. \tag{18}$$

Equation (18) can be written as $x_{n,1} = \frac{C_n + \beta_n(y_1 + y_2)}{2y_1} + \frac{C_n - \beta_n(y_1 + y_2)}{2y_1}$. From $\frac{C_n + \beta_n \sum\limits_{k=1}^{2} y_k}{2y_2} = \beta_n$ we get, $\beta_n(y_2 - y_1) = C_n$. Using this, we can write $x_{n,1} = \frac{C_n + \beta_n(y_1 + y_2)}{2y_1} + \frac{\beta_n(y_2 - y_1) - \beta_n(y_1 + y_2)}{2y_1}$. After simplifying we get,

$$x_{n,1} = \frac{C_n + \beta_n(y_1 + y_2)}{2y_1} - \beta_n. \tag{19}$$

3) Case 3 : $x_{n,1} = 0, x_{n,2} > 0$: Similar analysis can be performed as in case 2 to obtain

$$x_{n,2} = \frac{C_n}{y_2} = \frac{C_n + \beta_n(y_1 + y_2)}{2y_2} - \beta_n. \tag{20}$$

4) Case 4 : $x_{n,1} = 0, x_{n,2} = 0$: In this case, $\lambda_{n,1} = 0$ and $\lambda_{n,2}, \lambda_{n,3}$ can be any non-negative real value. This is an extreme case, which does not happen unless $C_n = 0$ or $y_k = \infty \ \forall k \in \mathcal{K}$. Note that in cases 1-3 discussed above, both power constraint and the cost constraint are satisfied as equalities.

Thus using (16), (19) and (20), the demands for the general case of N users and K UCs that covers cases $1 - 3$ for a given set of $\{y_k\}$, can be formulated as,

$$x_{n,k} = \frac{C_n + \beta_n \sum_{k \in \mathcal{K}} y_k}{K y_k} - \beta_n \tag{21}$$

where $x_{n,k} \geq 0, \forall k \in \mathcal{K}, n \in \mathcal{N}$. Since $x_{n,k} \geq 0 \ \forall n \in \mathcal{N}, \forall k \in \mathcal{K}$, (21) implies that

$$C_n + \beta_n\left(\sum_{g \in \mathcal{K}, g \neq k} y_g\right) \geq \beta_n(K-1)y_k, \ \forall n \in \mathcal{N}, \forall k \in \mathcal{K}. \tag{22}$$

Conversely, user n will demand $x_{n,k} \geq 0$ from UC k if

$$y_k \leq \left[\frac{C_n + \beta_n\left(\sum_{g \in \mathcal{K}, g \neq k} y_g\right)}{\beta_n(K-1)}\right]. \tag{23}$$

We will derive a closed form for the necessary condition for $x_{n,k} \geq 0, \forall k \in \mathcal{K}, \forall n \in \mathcal{N}$ to be satisfied, in Section IV-B2.

B. Utility Side Analysis

Let $P_k > 0$ denote the available power of UC k. Each UC aims to sell all the available power. If it had been a single UC case, it could have a set a very high unit price to maximize its revenue. In this case however, there are two factors that limit the unit price of the UCs. The first one is the budget of the users and the second one is the competition among the UCs. The UCs play a non-cooperative *price selection game* with each other to decide the optimal unit price. We assume that P_k is given for all $k \in \mathcal{K}$. For given P_k, since the cost of power generation is given, we define the revenue of UC k, $U_{\mathrm{gen},k}$ as

$$U_{\mathrm{gen},k}(y_k, \mathbf{y}_{-\mathbf{k}}) = y_k \sum_{n \in \mathcal{N}} x_{n,k}, \tag{24}$$

where $\mathbf{y}_{-\mathbf{k}}$ is the price of UCs other than k. Then, the optimization problem for a UC ($\mathrm{OP}_{\mathrm{gen}}$) is formulated as

$$\max_{\mathbf{y}:=\{y_k, \forall k \in \mathcal{K}\}} U_{\mathrm{gen},k}(y_k, \mathbf{y}_{-\mathbf{k}}) \tag{25}$$

$$\mathrm{s.t.} \sum_{n \in \mathcal{N}} x_{n,k} \leq P_k, \tag{26}$$

$$y_k > 0, k \in \mathcal{K}. \tag{27}$$

Since the revenue of a UC is an increasing function in terms of the amount of power for a fixed y_k, (26) can be taken as an equality constraint. Since we do not assume the availability of storage with the UCs, when the available power is given, each UC prefers to sell all its power. In order to solve $\mathrm{OP}_{\mathrm{gen}}$, we start by relaxing the positivity constraint (27) but will show that the solution of (25)-(26) will lead to positive vector $\{y_k, \forall k \in \mathcal{K}\}$. Let us define $L_{\mathrm{gen},k}$ as

$$L_{\mathrm{gen},k} = y_k \sum_{n \in \mathcal{N}} x_{n,k} - \mu_k \left(\sum_{n \in \mathcal{N}} x_{n,k} - P_k \right) \tag{28}$$

The first order optimality condition for the UCs leads to $\frac{\partial L_{\mathrm{gen},k}}{\partial y_k} = 0, \forall k \in \mathcal{K}$. Using (21) in $\frac{\partial L_{\mathrm{gen},k}}{\partial y_k} = 0$ for UC k, we obtain

$$(K-1)B y_k^2 - \mu_k \left[B \left(\sum_{g \in \mathcal{K}, g \neq k} y_g \right) + C \right] = 0, \tag{29}$$

where $B = \sum_{n \in \mathcal{N}} \beta_n$ and $C = \sum_{n \in \mathcal{N}} C_n$. Eqn. (29) gives K equations. Further, $\frac{\partial L_{\mathrm{gen},k}}{\partial \mu_k} = 0$ gives K equations, which are actually the original constraints: (26). Solving these $2K$ equations we can obtain $\mathbf{y}^* := \{y_1^*, y_2^*, \ldots, y_K^*\}$ and $\mu^* := \{\mu_1^*, \mu_2^*, \ldots, \mu_K^*\}$. Using $\mathbf{y}^*$, we can compute $\mathbf{x}^* := \{x_{n,k}^*\}$. Now using (21) in the equality form of (26), we get

$$y_k = \frac{C + B \left(\sum_{g \in \mathcal{K}, g \neq k} y_g \right)}{K P_k + B(K-1)} \tag{30}$$

Substituting y_k into (29), we arrive at

$$\mu_k = (K-1)B \left[\frac{C + B \left(\sum_{g \in \mathcal{K}, g \neq k} y_g \right)}{K(P_k + B)} \right] = (K-1)B y_k \tag{31}$$

Here, $B, C > 0$. when $K = 1$, $\mu_k = 0$ and $y_1 = \frac{C}{P_k + B}$. This means that there is no game when there is only one UC. Our interest is in the case where $K \geq 2$. Then, $\mu_k > 0$ if $y_k > 0$. Equation (30) can be represented in the matrix form as

$$\mathbf{A}\mathbf{y} = \mathbf{F}, \tag{32}$$

where

$$\mathbf{A} := \begin{bmatrix} P_1 + D & -E & .... & -E \\ -E & P_2 + D & .... & -E \\ .. & .. & .... & .. \\ -E & -E & .... & P_K + D \end{bmatrix}, \tag{33}$$

$\mathbf{y} := \{y_1, y_2, \ldots, y_K\}'$, $D := \frac{B(K-1)}{K}$, $E := \frac{B}{K}$ and $F := \frac{C}{K}$. Thus, provided that $\mathbf{A}$ is invertible, the solution of (32) is

$$\mathbf{y} = \mathbf{A}^{-1} \mathbf{F}. \tag{34}$$

In order to obtain the closed-form of $\mathbf{y}$, let us consider the following cases:

1) Homogeneous case: When $P_1 = P_2 = \ldots = P_K = P$: If all UCs have the same amount of power available, then, (32) can be solved to obtain

$$y_k = y := \frac{F}{-(K-1)E + D + P} = \frac{C}{KP} > 0 \ \forall k \in \mathcal{K}. \tag{35}$$

As P increases, y decreases and vice versa. Now using (35) in (22), the condition for the demand of user n from all UCs to be positive is

$$C_n > \beta_n(K-1)y - \beta_n(K-1)y, \tag{36}$$

i.e., $C_n > 0$, which is always true. This indicates that when all UCs are homogeneous in terms of the available power, they set the same unit price and all users will buy at least some power from them.

2) Heterogeneous case: When different UCs have different available power: In this case, it is difficult to judge the existence of the solution without knowing the nature of $\mathbf{A}^{-1}$ (if it exists). Interestingly, matrix $\mathbf{A}$ possesses some special properties. Let us state the following definitions and properties.

Definition 1. *A real matrix $A := \{a_{i,j} \ i,j = 1,2,\ldots,K\} \in \mathcal{R}^{K \times K}$ is said to be strictly diagonally dominant if it satisfies the following condition:*

$$|a_{i,i}| - \sum_{j \neq i} |a_{i,j}| > 0 \ \ i = 1,2,\ldots,K. \tag{37}$$

Property 1. *A strictly diagonally dominant matrix is non-singular.*

Property 2. *If a matrix is strictly diagonally dominant by rows and has positive diagonal entries, then, its determinant will be positive [26].*

For matrix $\mathbf{A}$, since, $P_k + D - (K-1)E = P_k + \frac{(K-1)B}{K} - \frac{(K-1)B}{K} = P_k > 0$, $\mathbf{A}$ is a strictly diagonally dominant matrix. From Property 1, $\mathbf{A}$ is invertible and (32) yields a unique solution to $\mathbf{y}$.

Theorem 1. *The unique solution obtained from (33) is positive.*

Proof: The solution of (33) is given by

$$y_k = \frac{K^{K-1}C}{|\mathbf{A}|} \sum_{g \in \mathcal{K}, g \neq k} (B + P_g) \ \ \forall k \in \mathcal{K}. \tag{38}$$

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where $|\mathbf{A}|$ is the determinant of $\mathbf{A}$. The numerator of (38) is always positive. Property 2 implies that the denominator is positive, and hence the solution from (38) always yields $y_k > 0$, $\forall k \in \mathscr{K}$. ∎

Theorem 2. *For the case of heterogeneous generators the necessary condition for the demands of all users from all UCs to be non-negative, is:*

$$C_n \geq \frac{K^{K-1}\beta_n C}{|\mathbf{A}|} \left[\left(\sum_{g \in \mathscr{K}, g \neq k} P_g \right) - (K-1)P_k \right] \forall k, \forall n. \quad (39)$$

Proof: The demand of user n from UC k is non-negative ($x_{n,k} \geq 0$) if $C_n + \beta_n \sum_{k \in \mathscr{K}} y_k \geq K\beta_n y_k, \forall n \in \mathscr{N}, \forall k \in \mathscr{K}$, i.e., if $C_n \geq \beta_n(Ky_k - \sum_{k \in \mathscr{K}} y_k)$. Using (38), the condition can be written as $C_n \geq \beta_n \frac{K^{K-1}C}{|\mathbf{A}|} \left(K \sum_{g \in \mathscr{K}, g \neq k} (B + P_g) - \sum_{k \in \mathscr{K}, g \neq k} \sum_{g \in \mathscr{K}, g \neq k} (B + P_g) \right)$ After simplification, the required condition takes the form given by (39).

∎

Theorem 3. *The values of price obtained from (34) maximize the revenue and are best responses of the UCs to other UCs' strategies.*

Proof: Suppose y_k is the solution obtained from (34) for UC k and it increases its price from y_k to $y_k' = y_k + \delta_{y_k}$ while the prices of other UCs remains the same. Let us assume that y_k, y_k' satisfy (23). The demand of users from this UC will change from $x_{n,k}$ to $x_{n,k}'$ given by

$$x_{n,k}' = \left[\frac{C_n + \beta_n \left(\sum_{g \in \mathscr{K}, g \neq k} y_g + y_k' \right)}{Ky_k'} - \beta_n \right].$$

The difference in the total demands from the users from UC k will be

$$x_{n,k} - x_{n,k}' = \left[\frac{C_n + \beta_n \left(\sum_{g \in \mathscr{K}, g \neq k} y_g \right)}{K} \right] \frac{(y_k' - y_k)}{y_k y_k'}.$$

Clearly, $(x_{n,k} - x_{n,k}' > 0)$. The users will demand less power than the power available with the UC. The difference in the revenue of the UC because of the increase in the price will be

$$U_{\text{gen},k}' - U_{\text{gen},k} = y_k' \sum_{n \in \mathscr{N}} x_{n,k}' - y_k \sum_{n \in \mathscr{N}} x_{n,k}$$
$$= -\frac{(K-1)}{K} B \delta_{y_k}. \quad (40)$$

From (40), it is clear that $U_{\text{gen},k}(y_k', \mathbf{y_{-k}}) < U_{\text{gen},k}(y_k, \mathbf{y_{-k}})$ if $\delta_{y_k} > 0$. If $\delta_{y_k} < 0$, $y_k' < y_k$, but P_k is given. Consequently, the revenue will decrease. Therefore, the prices calculated using (34) are the best responses to each other and are the prices that maximize the revenue of each UC. ∎

In addition to positivity, there are tighter limits on y_k such that $y_k \in [y_{k,\min}, y_{k,\max}]$. The lower limit $y_{k,\min}$ is due to the associated generation costs. The UCs will not reduce their price below $y_{k,\min}$. The upper bound $y_{k,\max}$ can be fixed according to government standards. The power available with the UCs implicitly take into account these price limits. Thus, the solution obtained by solving (34) should be within the specified limits. In case if the optimal prices obtained are outside the range, we propose the following algorithm to calculate the unit prices.

Algorithm 1:
1) If $\{y_k\}_{k \in \mathscr{K}}$ be the solution obtained from (38) but $y_i < y_{i,\min}$ or $y_i > y_{i,\max}$ for UC i, then its price will be set as $y_i = y_{i,\min}$ or $y_i = y_{i,\max}$.
2) The remaining UCs will use y_i as a known value in (30) to obtain a square matrix of size $K - K_i$, where K_i is the number of UCs for which the prices obtained are modified. Then, the matrix of the reduced dimension is solved to get the prices of the remaining $K - K_i$ UCs.
3) The process continues until all the prices come within the specified range.

C. Stackelberg Equilibirum

The UCs play the non-cooperative game with each other to set the unit price, which is at Nash equilibrium (NE) point, and announce the prices to the users. The equilibrium strategy for the followers in a Stackelberg game is any strategy that constitutes an optimal response to the one adopted (and announced) by the leader(s) [25].

Let $\Gamma_{\text{gen},k}$ and $\Gamma_{\text{user},n}$ be the strategy sets for UC k and user n respectively. Then, the strategy sets of all UCs and all users are $\Gamma_{\text{gen}} = \Gamma_{\text{gen},1} \times \Gamma_{\text{gen},2} \times \ldots \times \Gamma_{\text{gen},K}$ and $\Gamma_{\text{user}} = \Gamma_{\text{user},1} \times \Gamma_{\text{user},2} \times \ldots \times \Gamma_{\text{user},N}$, respectively. Then, $y_k^* \in \Gamma_{\text{gen},k}$ is a Stackelberg equilibrium strategy for UC k is

$$U_{\text{gen},k}(\mathbf{y}^*, \mathbf{x}(\mathbf{y}^*)) \geq$$
$$U_{\text{gen},k}(\mathbf{y_k}, \mathbf{y_{-k}^*}, \mathbf{x}(\mathbf{y_k}; \mathbf{y_{-k}^*})), \forall k \in \mathscr{K}, \quad (41)$$

where $\mathbf{y}^* = \{y_k^*\}$, $\mathbf{x} := \{\mathbf{x_1}; \mathbf{x_2}; \ldots \mathbf{x_N}\}$ is the strategy of all users $1, 2, \ldots, N$ such that $\mathbf{x} \in \Gamma_{\text{user}}$, $\mathbf{x}(\mathbf{y}^*)$ is the optimal response of all users. The optimal response of user n for given $(y_1, y_2, \ldots, y_K) \in \Gamma_{\text{gen},1} \times \Gamma_{\text{gen},2} \times \ldots \times \Gamma_{\text{gen},K}$ is

$$\mathbf{x}_n(\mathbf{y}) = \{\zeta_{\text{user},n} \in \Gamma_{\text{user},n}; U_{\text{user},n}(\mathbf{y}, \zeta_{\text{user},n}) \geq$$
$$U_{\text{user},n}(\mathbf{y}, \mathbf{x}_n)\}, \forall n \in \mathscr{N}. \quad (42)$$

where $\zeta_{\text{user},n} := \mathbf{x}_n^* \in \mathbf{x}_n(\mathbf{y}^*)$. The strategy $\mathbf{x_n^*}$ is a corresponding optimal strategy for user n, which is computed by using (21). The set $(\mathbf{y}^*, \mathbf{x}^*)$ is a Stackelberg equilibrium of the game between the UCs and the users.

D. Existence and Uniqueness of Stackelberg equilibrium

OP$_{\text{user}}$ has a unique maximum for a given $\mathbf{y}$. Therefore, the Stackelberg game possesses a unique SE if the price setting game among the UCs admits a unique NE.

Theorem 4. *A unique Nash equilibrium exists in the price selection game among the UCs, and thereby a unique Stackelberg equilibrium.*

Proof: A Nash equilibrium exists for the UCs in the price selection game if
1) $\mathbf{y}$ is a non-empty, convex, and compact subset of some Euclidean space $\mathscr{R}^K$.
2) $U_{\text{gen},k}(\mathbf{y})$ is continuous in $\mathbf{y}$ and concave in $y_k, \forall k \in \mathscr{K}$.

In the price selection game for the UCs, the strategy space $\Gamma_{\text{gen}} = \Gamma_{\text{gen},1} \times \Gamma_{\text{gen},2} \times \ldots \times \Gamma_{\text{gen},K}$ where $y_k \in \Gamma_{\text{gen},k} :=$

$[y_{k,\min}, y_{k,\max}]$, $\forall k \in \mathcal{K}$. Thus the strategy set is a nonempty, convex and compact subset of the Euclidean space $\mathcal{R}^K$. From (24), we see that $U_{\text{gen},k}$ is continuous in y_k. Next, the second order derivative of $U_{\text{gen},k}$ w.r.t. y_k is

$$\frac{\partial^2 U_{\text{gen},k}}{\partial y_k^2} = 0, \ \forall k \in \mathcal{K}. \tag{43}$$

Hence, $U_k(\mathbf{y})$ is concave in y_k. Therefore, NE exists in this game.

As proven in Section IV-B, there exists only one positive solution for the price selection game given by (38). Therefore the NE of the UCs' game is unique and hence the Stackelberg game also admits a unique equilibrium. ∎

V. DISTRIBUTED ALGORITHM

In the previous section, the consumers calculate their optimal demands based on the prices provided by the UCs but the UCs play the best response to other UCs' strategies. In order to calculate the unit prices, UCs need to know the power available with other UCs too. We design a distributed algorithm that leads to the SE of the game without each UC knowing the parameters of the other UCs.

Each UC starts with an arbitrary price $y_{k,1} > 0$ and all of them send their price information to the consumers. This communication is enabled by the smart grid communication infrastructure between the UCs and the consumers. Each user decides how much to buy from each UC $\{x_{n,k,1}, \forall k \in \mathcal{K}\}$ using (21). The UCs get this demand matrix from all users. Then, one of the UCs will calculate the difference between the available power and the total power demanded from it by all users. Then, it will update its unit price using

$$y_{k,t+1} = y_{k,t} + \frac{\sum_{n \in \mathcal{N}} x_{n,k,t} - P_k}{\sigma_k}, \tag{44}$$

where t is the iteration number and σ_k is the speed adjustment parameter of UC k, which is a sufficiently large number. Whenever a UC updates its price, it sends this information to the users. The users again, update their demands vectors and inform the UCs. Then, other UCs will update their prices sequentially at alternative turns between users and UCs. This process continues until the price values converge.

We name this algorithm as **Algorithm** 2, which is shown in Table I. In the table, $n = 1$ indicates user number 1.

Theorem 5. *Provided that* $\forall k \in \mathcal{K}, \forall n \in \mathcal{N}, t = 1,2,3,\dots,$

$$\sigma_k > \frac{(KP_k - \beta_n)y_{k,t} - \beta_n \sum_{g \in \mathcal{K}, g \neq k} y_{g,t} - C_n}{Ky_{k,t}^2}, \tag{45}$$

Algorithm 2 converges to the optimal solutions for both the users and the UCs as long as the individual strategies are updated sequentially.

Proof: The users' response (21) is the optimal response to given $\{y_k\}$. The demand array of each user will converge to a fixed set once the price set converges to a fixed point. Consequently, it is sufficient to show the convergence of the price vector to prove the convergence of Algorithm 2.

The algorithm will diverge only if $y_{k,t}$ acquires a negative value in any iteration. If $\sum_{n \in \mathcal{N}} x_{n,k,t} - P_k < 0$, for any

TABLE I
DISTRIBUTED ALGORITHM FOR PRICE AND DEMAND UPDATING

1: For t=1, arbitrarily choose $y_{k,1}$ $\forall k \in \mathcal{K}$
2: Repeat for t=2,3,.....
3: User **n=1,2,....N**
4: **Do**
5: Solve (2) - (4) for given $\mathbf{y}_t$ using (21).
6: Transmit $x_{n,k,t}$ to each k.
7: **end**
8: UC $k \in \mathcal{K}$, which has not updated the price for iteration $t+1$
9: Calculate $y_{k,t+1}$ using (44).
10: **If** $y_{k,t+1} - y_{k,i} = 0$
11: Send a no-change signal to all users.
12: Go to 8.
13: **else**
14: Send the new value of price to all users.
15: Go to 3.
16: **end**
17: **If** $y_{k,t+1} == y_{k,t} \forall k \in \mathcal{K}$,
18: stop.
19: **else**
20: Go to 2.
21: **end**.

$k \in \mathcal{K}, \forall n \in \mathcal{N}, t = 1,2,3,\dots$, then the sufficient condition for $y_{k,t+1}$ not to acquire a negative value is $|\frac{\sum_{n \in \mathcal{N}} x_{n,k,t} - P_k}{\sigma_k}| < y_{k,t}$. Since the condition should be satisfied only when $\sum_{n \in \mathcal{N}} x_{n,k,t} - P_k < 0$, we can rewrite the condition as $\sigma_k > \frac{P_k - \sum_{n \in \mathcal{N}} x_{n,k,t}}{y_{k,t}}$ $\forall k \in \mathcal{K}, \forall n \in \mathcal{N}, t = 1,2,3,\dots$. Using (21) and upon simplification, the sufficient condition takes the form (45).

Equation (44) implies that the price y_k of UC k increases if $\sum_{n \in \mathcal{N}} x_{n,k,t} - P_k$ is positive and the price decreases if $\sum_{n \in \mathcal{N}} x_{n,k,t} - P_k$ is negative. Equation (44) shows that when $\sum_{n \in \mathcal{N}} x_{n,k,t} - P_k = 0$, the price will remain unchanged. This is the fixed point to which Algorithm 2 converges. As discussed in section IV-B, the prices corresponding to stock clearance are the prices that maximize the UC revenue. Therefore, if $\{y_{k,T}\}$ are the prices at iteration T such that $\sum_{n \in \mathcal{N}} x_{n,k,T} - P_k = 0$, then, $U_{\text{gen},k}(y_{k,T}, \mathbf{y}_{-k,T}) \geq U_{\text{gen},k}(y'_{k,T}, \mathbf{y}_{-k,T})$ where $U_{\text{gen},k}(y_{k,T}, \mathbf{y}_{-k,T})$ and $U_{\text{gen},k}(y'_{k,T}, \mathbf{y}_{-k,T})$ are the utilities of UC k at price $y_{k,T}$ and $y'_{k,T}$ respectively. Thus the fixed point to which Algorithm 2 converges is the NE of the game among the UCs, i.e., SE of the game between the UCs and the consumers. Thereafter, the UCs will not deviate from this point. ∎

VI. DEPENDABILITY OF DRM

One important component of dependability of DRM is the vulnerability due to attacks from malicious agents. While the bidirectional communications between the UCs and the users facilitates demand response management in a more timely and effective manner, it also creates room for different kinds of threats to the system from attackers.

Next, we proceed to analyze this kind of attack. Let us assume that there is an attacker who is an outsider. The goal of the attacker is to harm the UCs to the largest extent possible. We constrain the attacker such that it can attack only one of the UCs at a time by injecting the data or by manipulating the price. This assumption is reasonable because the attacker has physical constraints to access the UCs.

A. Attack and Its Impact

An attacker can cause two kinds of harm to the UCs and the users: economic impact and physical impact. We consider the case when all UCs have the same power to supply. We assume that there is a range in which the price can vary. The users have the knowledge of this range. Users get this range during the service agreement. If the attacker manipulates the prices in such a way that it is out of the range, the manipulation will become obvious. So, the attacker will launch the attack such that the resulting prices are within the specified range. When the attacker does not manipulate any of the prices, the price set by each UC is

$$y_k = y = \frac{C}{KP}, \quad \forall k \in \mathcal{K}. \tag{46}$$

Using (46) in (21), the demand of user n from UC k is

$$x_{n,k} = \frac{C_n}{C} P.$$

The total demand of user n from all UCs is

$$\sum_{k \in \mathcal{K}} x_{n,k} = \frac{C_n}{C} KP.$$

Let the price of UC k, y_k, be changed by the attacker through data injection or indirect manipulation to $y'_k = y_k + \delta_y$ and all other prices are in-tact, where δ_y is a real number such that $|\delta_y| \in [0, \delta_{y,\max}]$. Positive value of δ_y means that the price is increased and negative value means the price is decreased, $\delta_y = 0$ corresponds to the case when the attacker does not act. When the price is changed by the attacker, the demand of users will change from $x_{n,k}$ to $x'_{n,k}$. The attacker can change y_k but will still keep it such that $x'_{n,k}$ is non-negative. It can be calculated by replacing y_k in (21) by $y_k + \delta_y$, which gives

$$x'_{n,k} = \frac{C_n P - \delta_y \beta_n (K-1) P}{(C + \delta_y KP)}. \tag{47}$$

Equation (47) shows that the demand of each user from UC k will decrease if δ_y is positive and it will increase if δ_y is negative. The total demand from UC k from all users will be

$$\sum_{n \in \mathcal{N}} x'_{n,k} = \frac{CP - \delta_y B(K-1)P}{(C + \delta_y KP)}. \tag{48}$$

Now, let us see if the change in the price of UC k affects other UCs. For UC $g \neq k$, the demand from user n will be

$$x'_{n,g} = \frac{C_n}{C} P + \delta_y \beta_n \frac{P}{C}. \tag{49}$$

Using (49), the total demand from UC $g \neq k$ takes the form

$$\sum_{n \in \mathcal{N}} x'_{n,g} = P \left(1 + \delta_y \frac{B}{C} \right). \tag{50}$$

Equation (48) indicates that the total demand from UC k will decrease (increase) if y_k is increased (decreased) by the attacker compared to the case when there is no attack. If the total demand decreases from UC k, a part of the available power will be wasted and it will loose in terms of its revenue. On the other hand, if the demand increases than the available power, serious problems such as black-out may occur to the users side. If the grid attempts to produce the shortage power

immediately, the grid might suffer physical damage. There is an interesting observation here. Equations (49), (50) indicate that if the price of UC k is changed, the other UCs will also be affected. If y_k increases, the demand from the other UCs increases. Consequently, because of the excessive demand from the UCs other than k, problems such as grid instability may arise at the generation side while the users will suffer black-out. On the other hand, when y_k decreases, the demand from the other UCs decreases. So, they will suffer economic loss while the generators that supply the demand of the users from UC k may suffer physical damage. In either case ($\delta_y > 0$ or $\delta_y < 0$), the UCs and the grid will suffer both monetary loss and physical damage.

B. Proposed Scheme with Individual Reserve Power

In a scenario where the UCs are aware that an attacker might attempt to create monetary or physical damage, although both kinds of loss are serious, the physical damage can make the grid unstable and it can affect the whole infrastructure on the UC side. Hence, we mainly focus on avoiding this problem, for maintaining the grid dependable. We propose that each UC should have certain reserve power in addition to the available power to sell. Let us denote the reserve power for UC k by $P_{k,res}$. The reserve power for each UC should be the difference of the available power and the total possible demand from all the users in presence of the attack. Using (48), for UC k, whose price has been changed, the reserve power is

$$P_{k,res} = \frac{-\delta_y P (B(K-1) + KP)}{(C + \delta_y KP)}. \tag{51}$$

If $\delta_y > 0$, then $P_{k,res}$ will be negative, since the demand from UC k will be less than the amount of power available. This means that no reserve power is needed. Hence, we have

$$P_{k,res}^+ = 0. \tag{52}$$

For other UCs $g \neq k$, the reserve power can be calculated as

$$P_{g,res}^+ = \frac{BP\delta_y}{C}. \tag{53}$$

Equations (52)-(53) indicate that the amount of reserve power for each UC depends on the value of δ_y. Theoretically, each UC can keep a large reserve so that there is always extra power available even if the users demand more than the power desired to be sold. In practice, this is not possible since the UCs have to pay the cost associated to buy or produce the reserve power. Therefore, all UCs prefer to keep as minimum reserve power to save the associated costs, but at the same time, it is extremely important for them to avoid the damage to their infrastructure.

In contrast to communication problems where usually the average performance guarantee measures are taken, we need to consider the worst case for the power-trade scenario. The attacker aims to maximize the harm to the UCs and/or to the grid but without making the demands become out-of-the-range values, e.g., negative demand from the users. The attacker therefore, tries to manipulate y_k in such a way that the demands from all UCs is non-negative even from the user with the lowest budget. Let us examine the following cases.

1) $\delta_y > 0$: If δ_y is positive, $P^+_{k,res} = 0$. The maximum harm that the attacker can create in this case would be by letting the user with the lowest C_n buy zero power from UC k, which corresponds to $x'_{n,k} = 0$.

$$\delta_{y\max} = \frac{C_{\min}}{\beta_{\min}(K-1)}. \tag{54}$$

where $C_{\min} = \min(C_n)$ and $\beta_{\min} = \min(\beta_n)$. If $y_{\max}$ is the maximum unit price allowed, the range of δ_y given by $\delta_{y\max}$ should still yield $y'_k \leq y_{\max}$. Thus, y'_k is given by

$$y'_k = \begin{cases} y_k + \delta_y & \text{if } \delta_y \leq y_{\max} - y_k, \\ y_{\max} & \text{otherwise.} \end{cases} \tag{55}$$

Therefore, $\delta_{y\max}$ can be calculated as

$$\delta_{y\max} = \min\left\{ \frac{C_{\min}}{\beta_{\min}(K-1)}, (y_{\max} - y_k) \right\}. \tag{56}$$

Hence, $0 < \delta_y \leq \delta_{y\max}$ and $P^+_{g,res}$ can be calculated by substituting $\delta_y = \delta_{y\max}$ into (53).

2) $\delta_y < 0$: When $\delta_y < 0$, the worst impact that the attacker can cause is by setting the prices so low that the user with the lowest budget buys zero power from all UCs other than k. In this case, solving $x'_{n,g} = 0 \; \forall g \neq k$ yields

$$\delta_{y\max} = \frac{C_{\min}}{\beta_{\min}},$$

where n corresponds to the lowest budget user. However, y'_k should not be less than $y_{k,\min}$. If $y'_k = y_k - \frac{C_{\min}}{\beta_{\min}} > y_{k,\min}$, then, substituting δ_y in (48) and (50), and substracting the available power of the UC P, the reserve power for UCs (using $\beta_1 = \beta_2 = ... = \beta_N = \beta$) are

$$\begin{cases} P^-_{k,res} = \frac{NC_{n,\min}(K-1)\beta + C_{n,\min}KP^2}{\beta C - C_{n,\min}KP}, \\ P^-_{g,res} = 0. \end{cases}$$

If $y_k - \frac{C_{\min}}{\beta_{\min}} \leq 0$ or if $y_k - \frac{C_{\min}}{\beta_{\min}} > 0$ but $y'_k < y_{\min}$, then, the minimum y_k that can be chosen by the attacker is $y'_k = y_{\min}$. Using this constraint on (47), then summing over $n \in \mathcal{N}$ and subtracting the available power P, we obtain

$$P^-_{k,res} = \frac{C + B(K-1)y_d - KPy_{\min}}{Ky_{\min}}.$$

where $y_d = \frac{C}{KP} - y_{\min}$. The UCs should avoid problems even in the worst case, $P_{k,res} = P^-_{k,res}$ and $P_{g,res} = P^+_{g,res}$. However, the UCs may not know whether the attacker will choose $\delta_y > 0$ or $\delta_y < 0$. So, for each UC, the reserve power is

$$P_{res} = \max\{P^+_{g,res}, P^-_{k,res}\}. \tag{57}$$

C. Proposed Scheme with Common Reserve Power

The reserve power given by (57) is the worst case reserve power if the UC is attacked and if its price is lowered to the least possible value. Since, each UC needs to have this reserve separately, while the actual number of UCs that will be attacked is limited to 1, there is definitely a huge wastage of the reserve power from all other UCs who are not attacked. In particular, the UCs have to buy the reserve power from some sources and it is obvious that each UC aims to minimize the

reserve power while ensuring that there is no power-outage. The total reserve power needed for the individual reserve scheme is

$$P_{tot,res} = KP_{res}. \tag{58}$$

From the analysis in section VI-B, we can see that the total demand from all the users from all the UCs will be

$$X_{tot} \leq KP + P_{res}. \tag{59}$$

From (58) - (59), the total power that will not be used is

$$P_{tot,unused} \geq (K-1)P_{res}. \tag{60}$$

The incentive for the UCs to use a common reserve is that sharing common reserve power saves a considerable amount of cost for each UC. Furthermore, it is also beneficial from the overall system perspective, as there are increasing concerns towards minimizing the waste of power.
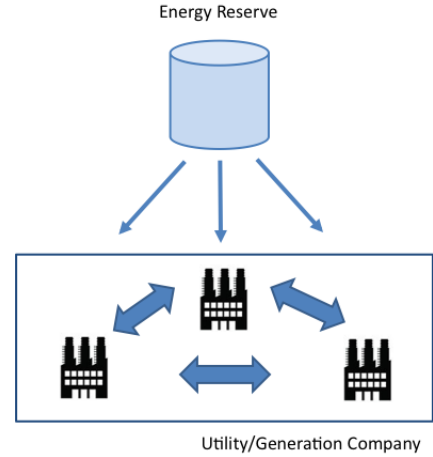

Fig. 3. Common reserve scheme: different utility/generation companies share a common energy reserve

Following the reasoning in Section VI-B, we can show that the total reserve power needed in this case will be

$$P_{tot,res} = P_{res}. \tag{61}$$

Employing this power sharing, we can see that the UCs can maintain a reliable supply of power to the users in the presence of the attacker with only a fraction of the reserve power needed for the individual reserve scheme. Thus, the supply side improves its dependability with this reserve power scheme.

D. Discussion

The impact on DRM when one (few) of the generator(s) is (are) unavailable is also important. In our proposed model, the UCs communicate with each other about their available power before the prices are decided. If one of the generators is not available, the prices will be calculated based on the power available with the UCs from the remaining generators. Our model is therefore resilient to the unavailability of one or few generators.

There is another possible type of reliability issue with the generators, even in the absence of an attacker. Even when all the generators have produced the power, one of the

generators can be disconnected from the grid after the UCs have transmitted the price information to the users. This kind of problem arises due to physical phenomena e.g., if one of the transmission lines suddenly gets disconnected. In this case, the users would already be using their appliances and thus this kind of failure is critical. The reliability can be improved in this case also by maintaining certain reserve power. This kind of reliability is referred to as $(K-1)$ reliability. On similar grounds as in Section VI, it can be inferred that a common reserve power equal to the available power of one UC (the maximum power in case of heterogeneous UCs) should maintain the system stable in the face of such incidents.

VII. NUMERICAL RESULTS

In this section, we examine how the users choose their optimal power based on the unit prices of the UCs and how the UCs optimize their unit prices based on their available power and the users' cost constraints. We also show the convergence of the distributed algorithm. The last part of this section shows the reserve power needed in the presence of an attacker. We consider 3 UCs and 5 users with parameters $\alpha_n = 1, \beta_n = 1 \ \forall n \in \mathcal{N}$. The cost limits of users are $C_1 = 5, C_2 = 10, C_3 = 15, C_4 = 20, C_5 = 25$ and the available power of the UCs are $P_1 = 10, P_2 = 15, P_3 = 20$, respectively, unless mentioned otherwise.

A. Stackelberg Game

Figs. 4 - 7 show how the change in the prices set by the UCs affects the users' utility, and in turn how the power available and the affordability of the users affect the UCs' equilibrium prices when the budget of user 1 varies from 2 to 42. Fig. 4 shows the total demand of each user at equilibrium. The demands of user 1 is increasing because its budget is increasing. The budget of other users do not change, but they ultimately demand less power because of the increase in the budget of user 1. Fig. 5 shows the utility of the users at equilibrium. Fig. 6 depicts the equilibrium prices of the UCs. The prices increase linearly as the purchasing capacity of the users increases. UC 1 charges the highest unit price, because it has the lowest amount of power available and the reverse is true for UC 3. Fig. 7 shows the revenue of each UC. UC 1 has the lowest utility despite its high unit price, because its available power is the lowest. UC 3 receives the highest revenue although its unit price is the lowest.

Next we show the equilibrium of the Stackelberg game for more UCs and a large number of users for a large variation in the budget of one of the users. Figs. 8(a)-8(c) show the user demands, user utilities and unit prices at equilibrium for 5 UCs and 100 electricity users. The budget of user 1 varies from 2 to 400, the budget of users 2 to 25, users 26 to 50, users 51 to 75 and users 76 to 100 are $10, 15, 20$ and 25, respectively. The available power of the UCs used for plots are $P_1 = P_2 = 150$ units, $P_3 = P_4 = 200$ units and $P_5 = 250$ units. The behaviors of the users and the UCs are similar to Figs. 4-6.

B. Distributed Algorithm

Figs. 9 - 12 show the performance of the distributed algorithm for $\sigma_k = 40, \forall k \in \mathcal{K}$. The UCs reach the equilibrium
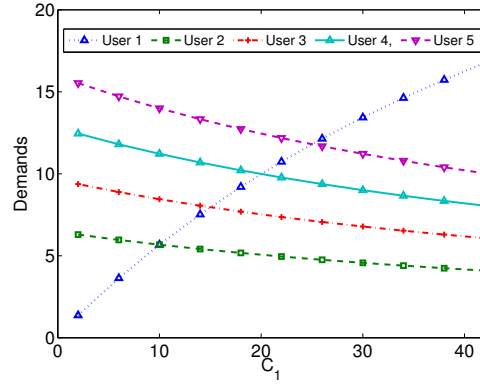

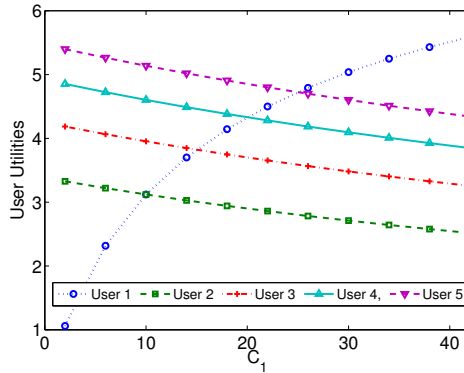Fig. 4. User demands at equilibrium
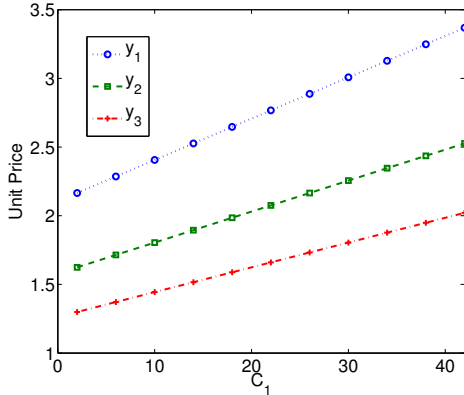

Fig. 5. User utilities at equilibrium

Fig. 6. Unit prices at equilibrium
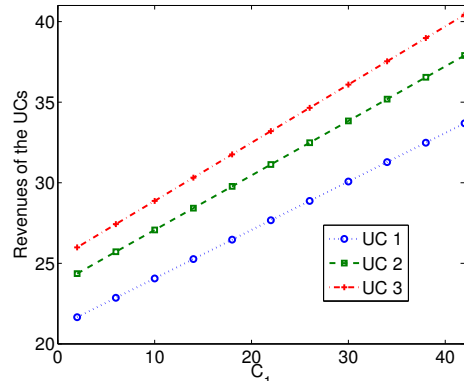
Fig. 7. UC revenues at equilibrium

(a) User demands   (b) User utilities   (c) Unit price
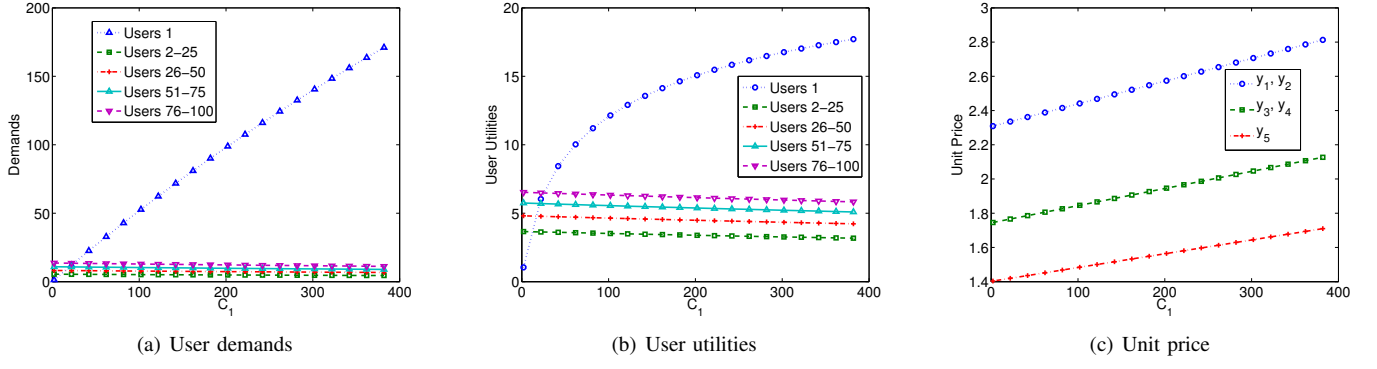
Fig. 8. Equilibrium for a large number of users and UCs for a large variation in user budget

(a) $y_{k,1} = 1; \sigma_k = 10$.   (b) $y_{k,1} = 5; \sigma_k = 50$.   (c) $y_{k,1} = 5; \sigma_k = 10$.
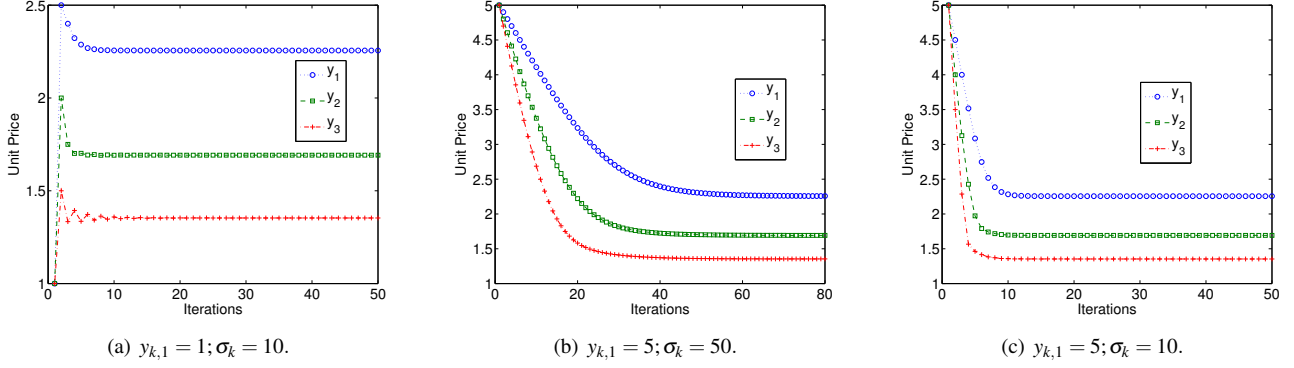
Fig. 13. The unit price of UCs for different initial starting points $y_{k,1}$ and different values of $\sigma_k, k = 1, 2, 3$.

price without communicating with each other. Consequently, the users reach their optimal demands based on the prices from the UCs. The results show that the equilibrium price and demand can be reached very quickly. Comparing Figs. 9 - 12 with Figs. 4 - 7, we can verify that the distributed algorithm converges to the optimal values.

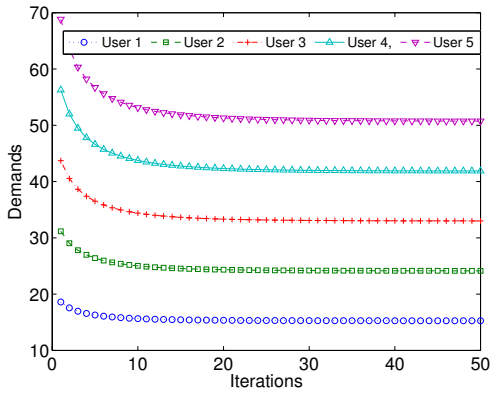Fig. 10. Reaching equilibrium using distributed algorithm: User utilities

Fig. 9. Reaching equilibrium using distributed algorithm: User demands

Next we evaluate the performance of the distributed algorithm for different values of $y_{k,1}$ and σ_k. Fig. 13(a) shows the performance when $\sigma_k = 10, k = 1, 2, 3, \ldots$. We see from the figure that the algorithm converges much faster (since σ_k is smaller) and the unit prices converge to the same values as in Fig. 11. Figs. 13(b) and 13(c) depict that the algorithm converges to the same values when we start from a different value for $y_{k,1}$, for $\sigma_k = 50$ and 10, respectively ,$\forall k \in \mathcal{K}$. We see that the convergence speed depends on the value
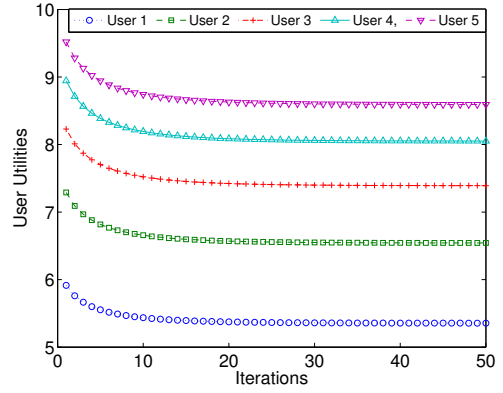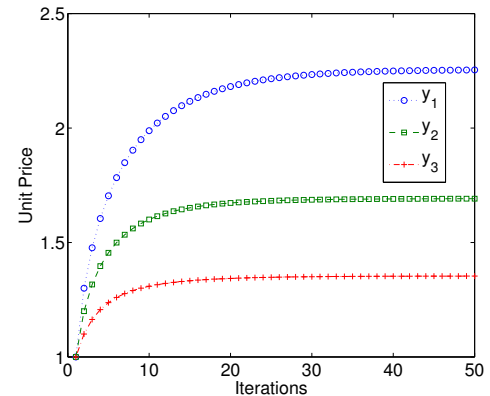

Fig. 11. Reaching equilibrium using distributed algorithm: Unit prices

of $\sigma_k, \forall k \in \mathcal{K}$ but the algorithm converges irrespective of
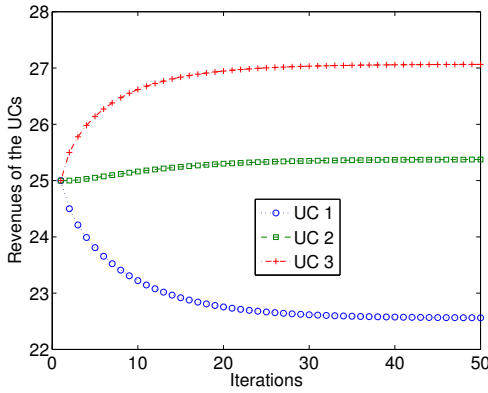
11

Fig. 12. Reaching equilibrium using distributed algorithm: UC revenues

different initial points.

C. Dependability of DRM

We show the impact of an attacker in terms of the reserve power needed under the two different schemes discussed in Sections VI-B and VI-C respectively in Fig.14. We consider both cases: when the price is increased and when the price is decreased. We use $P = 10$ and $[y_{\min}, y_{\max}] = [\frac{0.5C}{KP}, \frac{2C}{KP}]$ for the plot. For the sake of illustration, we suppose that the attacker
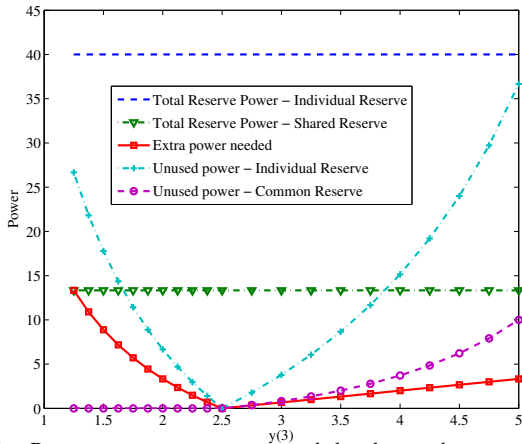

Fig. 14. Reserve power, extra power needed and unused power

manipulates the price of the third UC. In practice, the attacker can choose any of the UCs. This figure shows a number of interesting facts. It verifies that the UCs save a lot in terms of the reserve power needed if they share a common reserve. The solid line with square markers shows the extra power needed to cover the total demand of users. The reserve power from a common reserve is sufficient to cover the demands of all the users when the price is increased and also when it is decreased. In fact, this power is extra most of the times and there is a considerable amount of unused power in both cases (individual reserve and common reserve). The unused power is much more when each UC keeps its own reserve as shown by the dotted line with '+' markers. The only case where the reserve power is completely used is when the price is decreased to $y_{\min}$. From this figure, it might appear that the amount of power wasted is still significant even with the common reserve. That is because our analysis is based on the worst case reserve. This figure also

indicates how expensive an impact an attacker can create even if it has access to only one UC's price. The impact will be similar in case of the heterogeneous UCs with the difference in the scale of the power to be reserved.

VIII. CONCLUSION AND FUTURE WORK

In this paper, we have proposed a Stackelberg game between the electricity UCs for optimal price setting and the end-users for optimal power consumption. We have derived the SE of the game in closed form and have proved its existence and uniqueness. We have designed a distributed algorithm for convergence to the SE with only local information available to the UCs. We have introduced two types of reliability issues associated with the smart grid: reliability due to physical disturbance and dependability in the face of an attacker. We have investigated the impact of an attacker who manipulates the price information from the UCs. Furthermore, we have proposed a scheme based on the concept of shared reserve power to ensure the reliability and dependability of the grid. We have shown the validity of our concepts through analytical and numerical results.

This work opens the door to some interesting extensions. The DRM analysis incorporating the modeling of instability of the renewable energy sources is a potential direction. Here we have focused on a large time-scale one-period DRM scheme. A higher resolution multi-period scheme with inter-temporal constraints is another possible extension to this work.

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