Public Goods

Once a *pure public good* is provided, the additional resource cost of another person consuming the good is zero. The public good is “nonrival in consumption”. Examples:

- lighthouse
- national defense
- streets (if noncongested)
- radio broadcast
Public Goods

- public good $\neq$ **publicly provided good**
  Public provision: The good is paid for by the state. Schooling (up to high school) is a publicly provided good, but not a public good! (why?)

- Classification as a public good is not unchangeable; it depends on market conditions and technology. Example: Streets during rush hour/ streets off-peak

- Excludability
  Prerequisite for collecting payments: Public goods which are excludable could in principle be provided by the private sector (example: Pay-TV). Public goods which are nonexcludable certainly cannot (example: National defense).
Efficient provision of a public good

Indivisible public good (i.e. either provided completely or not; e.g., radio broadcast).

A’s willingness to pay: $ 20  
B’s willingness to pay: $ 10  
When should the public good be provided?  
→ If the cost of producing the good is smaller than A’s and B’s joint willingness to pay = $ 30.

Different from the case of private goods!  
Private goods: One unit should be provided to A if and only if the cost of producing one unit is smaller than 20$, and one unit should be provided to B if and only if it is smaller than 10$.

Both people can consume the same unit of the p.g. simultaneously.  
→ For public goods, we sum demand curves vertically.
Continuous public goods: Variable quantity (or quality)

Educated guess: efficient provision of a public goods requires that the sum of each person’s valuation of the last unit is equal to the marginal cost of production of the last unit:

\[ MB^A + MB^B = MC \]

General equilibrium framework with public goods.

\( g_i \): contribution of player \( i \) to the public good

\( G = g_1 + g_2 \) total amount of the public good provided.

\[ U_i(G, x_i) = U_i(g_1 + g_2, w_i - g_i) \]

Pareto optimum \( \rightarrow \) maximize weighted sum of the two individuals’ utilities:
Continuous public goods

\[
\max_{g_1, g_2} a_1 U_1(g_1 + g_2, w_1 - g_1) + a_2 U_2(g_1 + g_2, w_2 - g_2)
\]

First order conditions

\[
\begin{align*}
a_1 \frac{\partial U_1}{\partial G} - a_1 \frac{\partial U_1}{\partial x_1} + a_2 \frac{\partial U_2}{\partial G} &= 0 \\
a_1 \frac{\partial U_1}{\partial G} + a_2 \frac{\partial U_2}{\partial G} - a_2 \frac{\partial U_2}{\partial x_2} &= 0
\end{align*}
\]

\[\Rightarrow a_1 \frac{\partial U_1}{\partial x_1} = a_2 \frac{\partial U_2}{\partial x_2}.\] Rewrite FOC as

\[
\begin{align*}
a_1 \frac{\partial U_1}{\partial G} + a_2 \frac{\partial U_2}{\partial G} &= a_1 \frac{\partial U_1}{\partial x_1} \\
a_1 \frac{\partial U_1}{\partial G} + a_2 \frac{\partial U_2}{\partial G} &= a_2 \frac{\partial U_2}{\partial x_2}
\end{align*}
\]
Continuous public goods

\[ a_1 \frac{\partial U_1}{\partial G} + a_2 \frac{\partial U_2}{\partial G} = a_1 \frac{\partial U_1}{\partial x_1} \]

\[ a_1 \frac{\partial U_1}{\partial G} + a_2 \frac{\partial U_2}{\partial G} = a_2 \frac{\partial U_2}{\partial x_2} \]

Divide first equation by \( a_1 \frac{\partial U_1}{\partial x_1} \):

\[ \frac{\partial U_1}{\partial G} + \frac{a_2}{a_1} \frac{\partial U_2}{\partial G} = \frac{\partial U_1}{\partial x_1} + \frac{\partial U_2}{\partial x_2} = 1. \]

Sum of marginal rates of substitution must equal the marginal cost of the public good.
Suppose all of you are players in the following game: Everyone has two feasible actions, to “contribute” or “not to contribute”. If you contribute, you have to pay 1$, but for every player in the room (including yourself), there is a 80c benefit. Evidently, it would be very beneficial if all players would “contribute”, but the unique Nash equilibrium is that no one contributes (why?)

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<tr>
<td></td>
<td>contribute</td>
<td>don’t contr.</td>
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<tr>
<td><strong>Player 1</strong></td>
<td>contribute</td>
<td>(0.6,0.6)</td>
<td>(-0.2,0.8)</td>
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<tr>
<td></td>
<td>don’t contr.</td>
<td>(0.8,-0.2)</td>
<td>(0,0)</td>
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A’s marginal benefit: $MB_A = 10 - X$
B’s marginal benefit: $MB_B = 8 - X$.
The cost of providing one unit of the public good is 4. ($MC = 4$)
Efficient quantity: $MB_A + MB_B = 18 - 2X = 4 = MC$. $\Rightarrow X^* = 7$. 
Private provision of public goods, Example 2

What is the Nash equilibrium?

A’s marginal benefit: \( MB_A = 10 - X \)
B’s marginal benefit: \( MB_B = 8 - X \).

Claim: “A provides 6 units of the public good and B provides 0” is a NE.

Check: 1. Assume \( g_B = 0 \) ⇒ the best A can do is to buy X such that his own MB is equal to the marginal cost, hence to choose \( g_A = 6 \) as contribution.

2. Assume \( g_A = 6 \) ⇒ B’s marginal benefit is 2, and therefore he will not buy additional units for which he would have to pay 4 per unit.

This is in fact the unique NE. The player with the higher marginal benefit pays everything, the other player just benefits and pays nothing.
Problem: How could the state find out how much of the public good to supply, if individual demand functions are unknown (for the state; of course, people know their utility).

Indivisible good, costs 1 (if provided)
A: \( v_A \in (0, 1) \)
B: \( v_B \in (0, 1) \)

Efficiency: The good should be provided if and only if \( v_A + v_B > 1 \). Can the efficient allocation be implemented even if the state does not know the individuals’ WTP in the beginning?
A non-truthful mechanism

A possible mechanism:
1. Both people are asked about their type \((\rightarrow m_A, m_B)\)
2. If \(m_A + m_B > 1\), the good is provided; A pays \(\frac{m_A}{m_A+m_B}\), B pays \(\frac{m_B}{m_A+m_B}\)

If both people tell the truth, this mechanism implements the social optimum. However, will people tell the truth?
A non-truthful mechanism

Consider A with type $v_A$, and suppose that B tells the truth. If A reports to be of type $m$, A’s expected utility is

$$\int_{1-m}^{1} \left[ v_A - \frac{m}{m + v_B} \right] f(v_B) \, dv_B$$

Take the derivative with respect to $m$:

$$\left[ v_A - \frac{m}{m + 1 - m} \right] f(1 - m) - \int_{1-m}^{1} \frac{v_B}{(m + v_B)^2} f(v_B) \, dv_B$$

Evaluated at $m = v_A$, the first term is zero and hence the derivative is negative

$\Rightarrow$ It is better to set $m < v_A$. 
Clarke–Groves Mechanism

Mechanism

1. Both people announce $m_A$ and $m_B$ as their willingness to pay (they can, of course, lie).
2. If $m_A + m_B > 1$, the good is provided and A pays $(1 - m_B)$, B pays $(1 - m_A)$.
3. If $m_A + m_B < 1$, the good is not provided and no payments are made.

Observation: The report $m_A$ affects A’s payoff only if it changes whether the good is provided; the price A has to pay (if the good is provided) is independent of $m_A$ and depends only on B’s report!
Suppose $A$ knew $B$’s report, and $\nu_A + m_B > 1$. Then announcing $m_A = \nu_A$ is optimal for $A$ (why?).

Now suppose $\nu_A + m_B < 1$. Then announcing $m_A = \nu_A$ is again optimal for $A$ (why?).

This is true for every value of $m_B$: Announcing $m_A = \nu_A$ is a (weakly) dominant strategy for $A$! In particular, this is completely independent of whether $B$ told the truth.

The same argument holds for $B$. Under this mechanism, both people announce the truth and the efficient solution can be implemented.
Intuition

1 − \( m_B \): Net social cost of the public good for A (if B told the truth).
Do you want to buy the PG if you have to pay these net social cost?
Yes, if \( v_A > 1 − m_B \)
No, if \( v_A < 1 − m_B \)
Reporting \( m_A = v_A \) to the CG-mechanism implements exactly this policy.
Who pays?

Sum of the payments by A and B:

$$1 - m_B + 1 - m_A = 2 - (m_A + m_B).$$

Whenever the PG is provided, $$(m_A + m_B) > 1$$, so payments by A and B are never sufficient to cover the cost of the PG (“no budget balance”).

A third party (“state”) has to put in some money.

However: One could charge from both people an additional lump sum payment (i.e., the same amount, whether or not the good is provided) to offset this.
Externalities

Externality: decisions of one economic agent directly affect the utility of another economic agent.
Very much related to public goods

Distinction is unclear; sometimes based on whether the provision of the good in question is made consciously (public good), or whether the good arises as a by-product of some other activity (externality).

Also, public goods are usually “good” while externalities may be positive or negative.
“Pecuniary” vs. “non-pecuniary” externalities: Does the action affect another agent by changing market prices or directly?

Example:
Case 1: I pollute the environment by driving my car; my action harms other people directly
Case 2: I go to an auction; my presence will lead (in expectation) to higher prices and therefore harms the other bidders

In Case 1, the state corrects this externality by levying a tax on gas. Should they also levy a tax on auction bidders?

In the following, we restrict ourselves to “non-pecuniary externalities”
Example: Pollution

A’s factory pollutes a river and causes harm to B’s fishing firm

\[ p: \text{market price for A’s product} \]
\[ \text{MD: marginal damage caused to B’s firm.} \]
\[ \text{MC: A’s (private) marginal costs} \]
\[ \text{MSC: marginal social costs (MC + MD)} \]

$p$ is the market price for A’s product, and MD is the marginal damage caused to B’s firm. MC is A’s (private) marginal costs, and MSC is the marginal social costs, which is the sum of the private marginal costs and the marginal damage caused to B’s firm.
Possibilities to restore the optimum:

- **Mergers**: A and B merge their two firms and therefore “internalize” the externality: If the merged firm maximizes profits, it will choose the socially efficient level of output (why?)

Important reason why firms exist; however, it is apparently not beneficial to merge the whole economy into a single big firm, so there are limits to this solution (“Williamson’s puzzle”)
Example: Pollution

- **Pigou taxes**: State raises a unit tax on A’s production. This tax must be equal to the difference between the MSC and A’s private MC, i.e. to the MD (evaluated at the social optimum). The tax forces A to *internalize* his external effect.
Example: Pollution

- **Property assignment 1**: B receives the *property right* to have a clean river. As effective “owner” of the river, B can sell the right to emit a certain level of pollution to A.
Example: Pollution

Start from zero pollution:
If A gets the efficient amount of pollution rights, how much is willing to pay for that right?
How much does B need to be compensated for her losses?
⇒ A bilateral trade can realize all welfare gains.
Property assignment 2: A has the right to pollute the river.
Example: Pollution

Start from A’s privately optimal pollution level:
How much would B be willing to pay in order to convince A to only produce the efficient level of pollution?
Which payment would A at least require to accept that proposal?
⇒ A bilateral trade can realize all welfare gains.
Coase theorem

The equivalence of Property assignment 1 and 2 in terms of achieving an efficient outcome is known as the

Theorem (Coase Theorem)

*If property rights are clearly assigned to one party and can be enforced, then the efficient level of pollution will be realized; for this, it does not matter who (A or B) receives the property rights.*
Coase theorem, remarks

1. While efficiency will be achieved in both arrangements, both A and B clearly prefer property rights assignments to themselves (A prefers Property assignment 1, and B prefers Property assignment 2)

2. Limits to private deals: The example presents a very simple case of an externality, because there are only 2 parties involved. In more realistic pollution examples, there are many polluters and many people who suffer from pollution (for car pollution, these groups even largely coincide!). For large groups, it will be much more difficult to reach an efficient agreement through multilateral bargaining.

3. Using Pigou taxes and property assignments simultaneously to achieve efficiency is not a good idea! If the parties involved can bargain with each other, don’t use Pigou taxes.
Positive externalities: Someone else benefits from that action. Example: Research. Suppose that when a firm does research, other firms benefit by learning the results. (not patentable research)
The problem of the commons

“Commons”: in the middle ages, a meadow which belonged to all farmers of a community together; every farmer could decide how many cows to graze on the commons.
In general: Resource that is non-excludable, but rival.
We will show: Inefficient arrangement ⇒ commons disappeared later as an institution.
Example:
The price of a cow is 5.
Cows produce milk, which has a price of 1.
$x_i$: number of cows of farmer $i$. $X = \sum_{i=1}^{n} x_i$
A cow produces $20 - \frac{1}{10}X$ units of milk:
More cows $\rightarrow$ less grass per cow $\rightarrow$ less milk per cow.
The problem of the commons

Cooperative solution: Maximize the joint profit
\[ \max_X [20 - \frac{1}{10}X - 5]X \]
Condition for optimality:
\[ 15 - \frac{2}{10}X = 0, \quad X = 75. \]
Noncooperative solution (what actually happens): Given the other farmers’ decisions, farmer \( i \) maximizes his profit:
\[ \max_{x_i} [20 - \frac{1}{10}(x_1 + x_2 + \cdots + x_i + \cdots + x_n) - 5]x_i \]
Condition for an optimum (differentiation wrt \( x_i \)):
\[ 15 - \frac{1}{10}(x_1 + x_2 + \cdots + 2x_i + \cdots + x_n) = 0 \]
Symmetry: In an equilibrium, it is plausible that every farmer will have the same number \( x \) of cows on the meadow.
\[ 15 - \frac{1}{10}(n + 1)x = 0 \Rightarrow x = \frac{150}{n + 1} \]
The problem of the commons

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<th>4</th>
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PPC: Profit per cow $= 20 - \frac{1}{10}X - 5$.
TP: total profit

Other examples:
Congested streets Fishing in oceans
Greenhouse gases/global warming