A General Equilibrium Model of Capital Structure under Labor Market Search

Ping Liu*

University of Illinois at Urbana-Champaign

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Abstract

This paper develops a unified general equilibrium framework examining the joint relationships between firm capital structure choice and labor market outcomes in an economy featuring two-sided labor market search frictions. I nest a canonical asset pricing and capital structure model *à la* Leland (1994) into a competitive searching and bargaining environment in the spirit of Diamond-Mortensen-Pissarides. I obtain highly tractable solutions for optimal capital structure choices and equilibrium labor market outcomes in the presence of wage bargaining, capital structure posting and labor market search frictions. In particular, an increase in labor market search efficiency provokes the employers to adjust their leverage upward, which relieves the labor market congestions on the workers' side. This capital structure choice provides an important channel through which labor market search efficiency influences various aspects of labor market outcomes. For example, in the presence of optimal leverage choices, labor market search efficiency affects the wage of the new hires in a modest and non-monotonic way. Additionally, the endogenous capital structure choices by the employers are shown to influence the relationships between workers' bargaining power and labor market outcomes. Moreover, economic volatility influences the firms' optimal capital structure choices and labor market outcomes: most prominently, both firm leverage and the labor force participation rate climb up during turbulent economic times.

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1 Introduction

It is widely acknowledged that the labor market outcomes and firms' capital structure decisions are interdependent. A large volume of empirical research focuses on the joint relationship between labor market dynamics and corporate finance dynamics [1]. However, economic theories traditionally examine labor market dynamics and capital structure dynamics in isolated models[2]. This paper bridges the gap between empirical and theoretical research on joint dynamics of labor market outcomes and firms' capital structure choices. Specifically, I develop a general equilibrium framework answering the following questions: How do employers optimally choose their capital structures facing the frictional search in the labor market? How do the capital structure choices by the individual firms collectively feed back to the labor market and affect the labor market outcomes in the economy?

In this paper, I nest a standard dynamic asset pricing and capital structure model (Leland, 1994) to an equilibrium frictional labor market searching and matching framework in the spirit of Diamond-Mortensen-Pissarides (DMP hereafter), and examine how a firm in a frictional labor market designs its capital structure, and how these individual capital structure decisions collectively affect labor market outcomes in the economy. The framework captures two common themes in labor market models — wage bargaining and frictional search. The resulting model is highly tractable, featuring closed-form expressions of labor market outcomes. A simple numerical exercise generates novel and empirically testable implications regarding the influence of labor market characteristics, namely, workers' bargaining power and job market search efficiency, on employers' capital structure choices. One novel prediction is that an increase in labor market search efficiency provokes the employers to choose higher leverages, which relieves the congestions among searching workers. This capital structure decision provides an important channel through which labor market search efficiency influences various aspects of labor market

[1] One strand of empirical literature documents that employers' capital structure decisions influence the employment and wage dynamics (e.g., Hanka, 1998; Chemmanur, Cheng and Zhang, 2013). Meanwhile, firms' costly search for workers and workers' collective bargaining powers in wage negotiations affect the capital structure decisions on the firm side (e.g., Bronars and Deere, 1991; Cavanaugh and Garen, 1997; Klasa, Maxwell and Ortiz-Molina, 2009; Matsa, 2010; Bae, Kang and Wang, 2011; Agrawal and Matsa, 2013; Brown and Matsa, 2016).

[2] There are a few scholarly works that put labor market and capital structure under the same umbrella. However, this strand of research focuses on either frictionless labor market (e.g, Berk, Stanton Zechner, 2010), or simple debt instruments in a random matching framework (e.g., Monacelli, Quadrini and Trigari, 2011; Chugh, 2013; Petrosky-Nadeau, 2014).

outcomes. For example, in the presence of this capital structure choice, labor market search efficiency affects the wages of the new hires in a modest and non-monotonic way. This contrasts to the situation without consideration of firms' endogenous capital structure choices, in which the wages of new hires monotonically increase with the labor market search efficiency for obvious reasons: higher search efficiency increases the searching workers' outside option value, thereby increasing the required surplus they demand from a matching relationship. Moreover, the endogenous capital structure choices by the employers are shown to influence the relationships between workers' bargaining power and labor market outcomes. What is more, employers' endogenous capital structure choices in the frictional labor market provide a novel explanation for the empirically confirmed positive co-movement between economic volatility, aggregate leverages and labor market outcomes: both firm leverage and the labor force participation rate climb up during turbulent economic times. The baseline model is shown to be easily extended to two empirically prevalent environments: the environment featuring Bayesian learning about the matching quality and the environment with asymmetric information problem regarding the matching quality.

The model is motivated by two empirical observations in the relationship between capital structure choice and labor market characteristics. Firstly, several papers highlight the role of debt in strategic bargaining between firms and workers. Firms respond to higher bargaining power on the workers' side by employing a higher leverage (e.g., Bronars and Deere, 1991; Matsa, 2010). A more important conundrum comes from the second empirical observation: firms care about their employees' welfare. They are more conservative in debt usage when their employees are faced with higher unemployment risk or incur enormous loss upon unemployed (Agrawal and Matsa, 2013; Chemmanur, Cheng and Zhang, 2013). This is inconsistent with canonical view that a firm's sole objective is to maximize shareholder value[3]. To examine theoretically the role of debt in a strategic bargaining environment, as motivated by the first strand of empirical literature, I assume that the firm and the worker split the matching surplus induced by the labor market search friction, according to a generalized Nash bargaining rule based on the current state of the cash flow. I

[3] Recent empirical researches reveal the tip of the economic force behind the second empirical regularities. Brown and Matsa (2016) uses a proprietary data from a job matching platform and finds that job seekers have precise information about the employers' financial conditions for job vacancies they apply for. Moreover, they utilize such information and avoid applying for jobs posted by employers with higher leverage.

further assume that firms are able to issue debt to reduce the "size of the pie" shared with the workers. Specifically, firms issue debt against future cash flows from the match and the pay out the proceeds to the shareholders, immediately after the match is formed[4]. To examine theoretically the role of debt in the hiring practice, as motivated by the second strand of empirical literature, I develop a novel equilibrium concept — competitive search equilibrium with capital structure choices. Under this equilibrium concept, the firms compete for workers by posting the job vacancies and the associated debt level they intend to use. The firms commit to their posted capital structures. Job seekers observe all the job vacancies and have information regarding the leverage of each job vacancy. Job seekers apply for the jobs that give them the highest expected value of active searching. There are "congestions" on both the employer side and worker side of the labor market, preventing instantaneous matches between vacancies and workers[5]. In the equilibrium, the firm chooses the capital structure that maximizes the expected value of its job vacancy, subject to the constraint that it must provide the searching workers with the expected value comparable to other searching firms, in order to attract searching workers to apply for its job vacancy. As a result, two countervailing forces come into play in determining optimal leverage: a higher leverage enhances the shareholder value after the match, by expropriating a larger share of post-match cash flow in the form of debt issuance proceeds. Meanwhile, since workers have information regarding the leverage associated with each job vacancy, higher leverage choice leads to fewer job applications, thereby reducing the hiring rate. In the model, the former benefit is summarized by the elasticity of expected post-match shareholder value with respect to leverage choice, and the latter cost is captured by the elasticity of the expected hiring rate with respect to leverage choice. Individual firm optimally chooses its capital structure that balances the benefit and the cost associated with leverage. Mathematically, it equalizes the absolute value of the two elasticities. The expected value of a searching worker is pinned down by the free-entry condition of the firms. The labor market tightness is then determined by the searching worker's value function.

[4] The extant literature (e.g., Monacelli, Quadrini and Trigari, 2011) makes the same timing assumption regarding the payout of proceeds from debt issuance in the presence of wage bargaining.

[5] The searching friction demarcates my labor market from most of the competitive markets. For example, in a standard retail product market where the consumers search for the best price and suppliers post their prices, suppliers are able to satisfy any demand and consumers always visit the suppliers who announce the lowest price. Notice that in the absence of search frictions, my economy resembles the retail market economy, and the optimal leverage ratio is always zero.

Once I characterize the optimal leverage, expected value of a searching worker, and labor market tightness, other labor market outcomes can be solved in closed forms. I first solve for the optimal separation threshold of a matching relationship[6]. The individual firm's optimal choice of capital structure, together with the optimal separation threshold characterize the expected matching durations in the economy. The two optimal policies also characterize the stationary cross-sectional distribution of the wage rate, among the matches in the steady-state economy[7]. The steady-state cross-sectional distribution in turn gives rise to the equilibrium unemployment rate of the economy[8].

A simple numerical exercise, based on empirically confirmed matching function specifications and model parameters, generates rich and novel predictions regarding the comparative statics of optimal capital structure choice. Consistent with existing empirical research, the optimal debt level increases with the workers' bargaining power (e.g., Bronars and Deere, 1991; Matsa, 2010). Novel to the literature, the model is able to generate a positive relationship between the labor market search efficiency and firms' optimal leverage choices. The underlying logic is as follows: on one hand, the marginal benefit of a higher leverage on post-match shareholder value scales up with the labor market search efficiency. On the other hand, the negative impact of a higher leverage on the hiring rate is dampened when labor market search is more efficient. This is consistent with the recent literatures that document a negative relationship between unemployment risk of the workers and employers' debt usage (e.g., Agrawal and Matsa, 2013; Chemmanur, Cheng and Zhang, 2013). Another interesting fact is that the leverage increases as economic volatility increases. This is consistent with findings from other research on the relationship between leverage and aggregate volatility (e.g., Johnson, 2016). However, I provide a novel mechanism originated from labor market frictions[9]. To my best knowledge, this is the first theoretical research that tackles the positive leverage-volatility co-movement puzzle from a

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[6] The worker and firm in a matching relationship optimally choose the identical cash flow threshold to leave the matching relations, by the virtue of generalized Nash bargaining sharing rule.

[7] This stationary cross-sectional distribution can be conveniently characterized by an analytically solvable Fokker-Planck equation with proper boundary conditions. The resulting density function follows a Double Pareto form, which is similar to the literature on power laws in the stochastic growth models featuring population births and deaths (e.g., Gabaix, 2009). Refer to Section 3.5 for details.

[8] The equilibrium unemployment rate is represented by a probability mass of the density function of the stationary cross-sectional distribution.

[9] Johnson (2016) resorts to a deposit insurance mechanism to explain the positive leverage-volatility co-movement puzzle.

frictional labor market perspective. Lastly, the optimal leverage decreases with the cost of bankruptcy, which is again consistent with most of the extant corporate finance research (e.g., Leland, 1994).

The numerical exercise of the model also provides a rich set of empirically testable predictions regarding the impacts of the labor market search friction, workers' bargaining power and economic volatility on labor market consequences, through a novel channel of endogenous capital structure choice. One novel prediction is that in the presence of endogenous leverage decisions, labor market search efficiency affects the wage of the new hires in a modest and non-monotonic way: More efficient labor market search even suppresses the wage rate for a certain range of search efficiency levels, because of the higher leverage policy by the firms facing more efficient labor market. Moreover, the workers' bargaining power and labor market search frictions affect various other aspects of labor market outcomes, through the endogenous capital structure choice channel. For example, a lower workers' bargaining power or a lower search efficiency generates a fatter left tail of stationary cross-sectional cash flow distribution, thus wage distribution, in the economy. Unemployment rate increases with workers' bargaining power and decreases with the labor market search efficiency. Moreover, more efficient matching technology induces the workers to exert more job searching effort, in order to capitalize a more "productive" matching process. Lastly, the model opens up a novel explanation for the observed relationship between volatility and labor market outcomes. One prominent result is that a higher economic volatility elicits more searching effort by the workers. This finding is in line with the empirical regularities that the transition rate from out-of-labor-force to unemployment pool is countercyclical, ramping up during the recessions (e.g., Elsby, Hobijn and Sahin, 2015; Krueger, 2016). Although the economic recessions are characterized by both lower productivity and higher uncertainty, I have shown that the volatility certainly contributes to the observed countercyclical behavior of labor force participation, which is, to my best knowledge, novel to the literature.

I go on to extend the baseline model using alternative assumptions about the information structure of the productivities of the matches in the economy. First, I extend the model to an unobservable matching-specific productivity and Bayesian learning framework. The same set of equilibrium solutions goes through. Secondly, I assume that only the employer knows about its own productivity and it cannot credibly commit to a particular leverage choice. A High-productivity firm suffers from an asymmetric information and capital market undervaluation. Consequently, it has incentive to signal quality to the capital market through excessive debt issuance compared with the full-information first best scenario. I show a separating equilibrium always exists. Under the separating equilibrium, the high-productivity firm may issue more debt compared with its first best capital structure choice under symmetric information. In this case, the post-match shareholder value of a high-productivity firm is reduced by the asymmetric information problem. Therefore, high-productivity firms post fewer vacancies and the labor market is less tight. I also demonstrate that under certain restrictions on the model parameters, there also exist two types of pooling equilibria. This part of analysis takes the first step toward an understanding about the joint movement of capital market misvaluation and its impact on employment dynamics.

This paper contributes to several strands of literature. First of all, the modeling choice of this paper, i.e., bringing together the Leland-type capital structure model and the DMP labor market searching and matching model adds to the burgeoning macroeconomic literature that studies the relationship between financial market conditions and labor market conditions (e.g., Wasmer and Weill, 2004; Monacelli, Quadrini and Trigari, 2011; Chugh, 2013; Petrosky-Nadeau, 2014). The underlying mechanisms through which the labor market and financial market are interrelated demarcate this paper from most of extant literature (e.g., Chugh, 2013; Petrosky-Nadeau, 2014). The mechanism proposed in Chugh (2013) and Petrosky-Nadeau (2014) is the traditional credit channel where firms could be financially constrained and the financing cost of vacancy creations plays a central role in the transmission of shocks[10]. In this sense these papers share similar features to models proposed by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), which document the amplification of productivity shocks through financial constraints and depressed asset prices. In my model, the wage bargaining between firms and workers and the impact of debt on hiring rate jointly determine the optimal leverage choice[11]. A salient feature of my paper is the equilibrium concept, in which the firms internalize the effect of the leverage on the welfare of searching workers when choosing their capital structures, which is absent in the

[10] Similar channels also play a central role in Wasmer and Weil (2004), which considers an environment where bargaining is between entrepreneurs and financiers. In their model, financiers are needed to finance the cost of posting a vacancy and the surplus extracted by financiers is similar to the cost of financing investments.
[11] In Monacelli, Quardrini Trigari (2011), wage bargaining between firms and workers also plays a central role in determining the optimal leverage choice, but they do not consider the hiring role of debt.

extent models[12,13]. From a methodological point of view, the continuous time approach enables me to characterize the various aspects of labor market outcomes in closed forms. The optimal leverage, expected value of being unemployed, and labor market tightness are characterized by a simple system of equations.

Moreover, this paper also complements to the micro-economic level analyses on human capital and capital structure choices (Berk, Stanton and Zechner, 2010). In Berk, Stanton and Zechner (2010), firms compete for scarce labor force in a frictionless labor market. They only focus on the firm's optimal capital structure choice and do not consider the collective impact of individual firms' optimal capital structure choices on the aggregate labor market outcomes. On the contrary, my paper nests a dynamic capital structure model into a frictional labor market and is able to generate the individual firm's optimal capital structure choice in a frictional labor market searching and bargaining environment. More distinctively, my model is able to demonstrate the impact of the labor market search friction, workers' bargaining power and economic volatility on a rich set of aggregate labor market outcomes. Employers' optimal leverage decisions play a crucial role in determining such influences. More generally, several microeconomic analyses build models on the capital structure and debt maturity structure of firms facing frictional credit markets (e.g., He and Milbradt, 2014; Hugonnier, Malamund and Morellec, 2015). A common theme is that the imperfect credit market, featuring searching for financiers, can dramatically alter the firms' security issuance behaviors and default choices. My paper extends the literature by considering an alternative market friction, labor market friction, and its impact on firms' capital structure choices. A unique feature of my paper is the feedback from individual firms' optimal capital structure choices to the labor market consequences at macroeconomic level.

Lastly, the findings of this paper generate novel and empirically testable implications and call for a thorough welfare analysis of government labor market polices. For example, battling against the recent financial crisis, many countries from Europe, to name a few, UK, Germany and

[12] In most of the extent models (e.g., Monacelli, Quadrini and Trigari, 2011) the leverage is chosen to maximize the matching surplus only, since the leverage choice is determined only after the match is formed. This is similar to my last part of analysis, where the firms lack commitment power and are unable to credibly inform workers their capital structure choices early in workers' job hunting stage.

[13] My modelling of debt instrument is consistent with the classic dynamic corporate finance literature, in which debt is typically modeled as a perpetual coupon-bearing bond with endogenous bankruptcy threshold. My paper also embraces much richer features about the productivity shocks, default decisions, and information structure.

Ireland, expand current vocational training program and initiate new programs to reduce the labor market mismatches (Heyes, 2012). These active labor market programs that improve the labor market search efficiency are argued to swiftly increase the national welfare in the short run (Brown and Koettl, 2015). However, one subtlety is that employers might take advantage of these job creation programs by increasing their leverages. As a result, the employment rate might rise at a cost of lower wage. A complete welfare implication of these programs might yield more complex results than the original expectations.

The paper is organized as follows. The next section lays out some common structures of the model environment used throughout the paper. Section 3 considers the core model in which firms post their capital structures to job seekers under perfect information about matching productivity and gives a numerical example. Section 4 relaxes the assumption about the perfect information, and solves the model in the context of Bayesian learning about matching quality through cash flow performance. The next section considers the no-commitment case in which no capital structure posting is allowed. The first subsection deals with the perfect information case, followed by the subsection that concentrates on asymmetric information case and the resulting capital market signaling. Section 6 concludes the paper with some possible directions of future research.

2 Model environment

2.1 Labor market participants

Time is continuous. The labor market consists of a continuum of workers and a continuum of firms. The measure of workers is normalized to one. The measure of job vacancies is endogenously determined to ensure free entry on the firm side[14]. In the core model, the productivity of a match, θ, is deterministic and public knowledge. Firms post vacancies and the associated capital structure to the potential job seekers. A firm incurs a flow cost κ to keep the vacancy open. I assume that the labor market is so large and workers can only select a subset of job vacancies to apply for. The important assumption here is:

[14] I assume that each firm can only post one vacancy in the job market. However, this assumption only facilitates the expressions and has no material consequences.

Assumption 1 Workers have perfect information about the leverage of each job vacancy prior to their search, or at least at an early stage in the job search process.

Whether the workers' knowledge is perfect or with small noises is not crucial. For the expositional purpose, I assume that workers possess perfect knowledge on the leverage associated with each posted job vacancy they apply for. Both workers and firms are risk-neutral. They optimize and discount future cash flows at rate $r > 0$. The workers are ex-ante identical. All the benefits[15] accrued to an unemployed worker are summarized by a flow value b. I assume that b is small so that no matches are rejected by the workers and all the matches are socially efficient.

2.2 Production upon matching

The production starts immediately after the match is made, capital structure is set up, and the wage bargaining outcome is accepted by both parties. The matching-specific cash flow of a match i at time t is equal to $\theta X_{it} - f$. $f > 0$ represents a constant flow of operating costs[16]. In the remaining parts of the paper, except Section 4, the cash flow of the match is subject to two orthogonal sources of idiosyncratic noises. First of all, for each successful match i, X_{it} starts at X_0, and evolves according to a geometric Brownian motion process:

$$\frac{dX_{it}}{X_{it}} = \mu dt + \sigma dZ_{it} \quad \theta X_0 > f$$

where $\mu < \delta \coloneqq r + s$ and $\sigma > 0$. Moreover, there exists a Poisson process that governs the exogenous destruction rate of the matching relationship, with intensity[17] s. Upon exogenous match destruction, the salvage values for all financial claims are zero. I emphasize here that both sources of idiosyncratic noises are independent across matches.

2.3 Job search and match

Both the job search process and the labor hiring process are frictional. Specifically, the flow of new worker-firm matches is captured by the homogeneous-of-degree-one concave

[15] The benefits include, but not limited to, unemployment allowance, leisure, social welfare, and income from self-employment.
[16] My model implications are qualitatively unchanged if I assume that a fixed investment amount I is required to start the production after a match is formed, and the firm designs optimal capital structure to finance the fixed investment.
[17] The exogenous separation of a match is standard in literature (e.g., Pissarides, 2009; Moen and Rosen, 2011). This could reflect the risk of technological obsolescence, natural disasters and worker relocations, etc.

matching function $m(u, v)$. u and v denotes the unemployment rate and vacancy rate in the economy, respectively. Let g denote the matching rate of workers, representing the rate at which an unemployed worker meets a vacancy. Let h denote the matching rate of firms, representing the rate at which an idle firm meets an unemployed worker. Obviously, $g \coloneqq \frac{m(u,v)}{u} = m(1, \epsilon) \coloneqq g(\epsilon)$ and $h \coloneqq \frac{m(u,v)}{v} = m\left(\frac{1}{\epsilon}, 1\right) \coloneqq h(\epsilon)$[18], where $\epsilon = \frac{v}{u}$ stands for the labor market tightness. I assume that $\lim_{\varepsilon \to 0} g(\epsilon) = \lim_{\epsilon \to \infty} h(\epsilon) = 0$ and $\lim_{\varepsilon \to \infty} g(\epsilon) = \lim_{\epsilon \to 0} h(\epsilon) = \infty$. Sometimes it is useful to introduce the following expression: $h = h(\epsilon) = h\left(g^{-1}(g)\right) = h(g)$, where $h'(g) < 0$.

2.4 Debt contract

Consistent with Leland (1994), debt contract in this paper is represented by a consol bond with a constant coupon rate c. Consistent with Monacelli, Quadrini and Trigari (2011), a crucial assumption regarding the timing of the debt issuance and payment of proceeds to shareholders is:

Assumption 2 The proceeds of debt issuance are immediately distributed to shareholders, before the wage bargaining takes place.

Firms may declare bankruptcy at any time. If a bankruptcy occurs, a fraction $0 < \alpha \leq 1$ of net present value will be lost to the bankruptcy costs, leaving creditors with abandonment value net of bankruptcy costs, and shareholder with nothing. Upon bankruptcy, the match ends.

2.5 Wage bargaining

I assume that neither firms nor the workers have the commitment power to enter into long-term employment contracts. Either party can leave the match at any time and return to search. This reflect the fact that in the United States, most of the employment relationships are “at will”. A consequence of lack of commitment power is that the wage during a particular matching relationship is determined by continuous bilateral bargaining between the firm and the worker. Following the literature, unless otherwise specified, I take an axiomatic approach and use continuous generalized Nash bargaining solutions to characterize the bargaining outcome,

[18] Following conventions in mathematics, throughout the paper, " $=$ " means “equal to”, and " $\coloneqq$ " means “denoted as”

conditional on cash flow at time t. β stands for the bargaining power of the workers, and $1-\beta$ stands for the bargaining power of the firms.

2.6 Discussion

The key assumption is that searching workers have perfect information about the firm's intentional capital structure choice of each job vacancy. This assumption may be extreme at the first sight. However, this assumption has found empirical support recently (e.g., Brown and Matsa, 2016). With the help of newly available survey data from an online job search platform, Brown and Matsa (2016) finds that the job seekers' information on employers' financial conditions are consistent with employers' true financial conditions, such as indicated by their CDS prices. Moreover, job seekers act upon their information and are reluctant to apply job vacancies posted by firms with poor financial conditions and high leverages. Their findings corroborate my assumption here that workers have precise information about the leverages associated with job vacancies in the job market when searching for jobs. Another piece of evidence for predictable capital structure is that most of public firms often stick to particular capital structures over the course of many years (Lemmon, Roberts and Zender, 2008). This assumption is harmless even if one has strong prior that workers' information collection takes time. Consider the following thought experiment: Firms build up their reputation for leverage usage in the labor market through repeated matching and financing choices. Workers learn about each firm's reputation for leverage usage through observations. My analysis focuses on the economy at the steady state. Without loss of generality, I may still assume that workers have perfect knowledge about the firms' leverage choices and firms do not have incentives to deviate from their long-term leverage targets. Lastly, I conjecture that the insights from this paper will be qualitatively unaffected as long as the workers can glean some information regarding the capital structure choices by the potential employers in the labor market.

3 Baseline model — Perfect knowledge about deterministic θ

In the baseline model, the match productivity θ is deterministic and both the firms and the workers have the perfect knowledge about it. I begin with derivation of post-match values of debt $D(X)$, equity $E(X)$, worker's compensation $W(X)$. Then I define submarket in the economy, after which I present the asset values of unemployed workers, U, and asset values of idle vacancies, V.

Equation for U plays a central role in individual firm's equilibrium expectation about the unique relationship between the leverage choice and the probability of matching with workers. I continue to introduce and the key definition of this section: the competitive search rational expectation equilibrium. This section culminates with characterization of equilibrium leverage, separation threshold and stationary cross-sectional distribution of cash flow states. I use matching surplus $S(X)$ to obtain solutions.

3.1 post-match Asset values

It is convenient to introduce the following notations. Let the de facto discount rate, $\delta \coloneqq r + s$, the present value of operating cost, $F \coloneqq \int_0^\infty e^{-\delta t} f dt = \frac{f}{\delta}$, and the expected present value of a perpetual streams of value X starting at $X_0 = x$:

$$\Pi(x) \coloneqq E[\int_0^\infty e^{-\delta t} X_t dt | X_0 = x] = \frac{x}{\delta - \mu}$$

3.1.1 Debt

For a given coupon rate c and unemployment value U, the debt value $D(X)$ of a matched firm-worker pair satisfies the following Hamilton-Jacobi-Bellman (HJB hereafter) equation

$$rD(X) = c + \mu X D'(X) + \frac{1}{2}\sigma^2 X^2 D''(X) - sD(X) \tag{1}$$

The boundary condition are standard value-matching conditions[19]:

$$D(\underline{X}) = D^B = (1-\alpha)\left(\theta \Pi(\underline{X}) - F - \frac{rU}{\delta}\right); \ \lim_{X\to\infty} D(X) = \frac{c}{\delta} \tag{2}$$

By standard results from dynamic capital structure literature (e.g., Goldstein, Ju and Leland, 2001). The solution of the above boundary value problem is

$$D(X) = \frac{c}{\delta} - \left(\frac{c}{\delta} - (1-\alpha)\left(\theta \Pi(\underline{X}) - F - \frac{rU}{\delta}\right)\right)\left(\frac{X}{\underline{X}}\right)^{\nu} \tag{3}$$

[19] According to the specification of abandonment value, the abandonment value drops to zero following exogenous separation, while equal to the abandonment value of the firm net of default costs in case of endogenous default by the firms. This specification reflects the fact that exogenous separation, for example, a natural disaster, often wipes out the entire equipment and premise of the firms, rendering zero recovery value of the firm. Monacelli, Quadrini and Trigari (2011) has used the same specification.

where ν is the negative root of the equation $\nu(\nu-1)+\frac{2\mu}{\sigma^2}\nu-\frac{2\delta}{\sigma^2}=0$.

$$\nu=\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)-\sqrt{\left(\frac{1}{2}-\frac{\mu}{\sigma^2}\right)^2+\frac{2\delta}{\sigma^2}} \tag{4}$$

3.1.2 Equity

Similarly, for a given coupon rate c, wage rate w, and vacancy value V, the equity value obeys the following HJB equation

$$rE(X)=\theta X-f-c-w+\mu XE'(X)+\frac{1}{2}\sigma^2X^2E''(X)-s(E(X)-V) \tag{5}$$

The boundary conditions are:

$$E\left(\underline{X}^E\right)=V(value\ matching);$$
$$E'(X)|_{X=\underline{X}^E}=0\ (smooth\ pasting);\ \lim_{X\to\infty}\left(\frac{E}{X}\right)<\infty\ (no\ bubble) \tag{6}$$

$\underline{X}^E$ denotes the optimal bankruptcy threshold X for the firm.

3.1.3 Employed worker

For a given wage rate w and unemployment value U, an employed worker's value $W(X)$ satisfies the following HJB equation:

$$rW(X)=w+\mu XW'(X)+\frac{1}{2}\sigma^2X^2W''(X)-s(W(X)-U) \tag{7}$$

The boundary conditions are:

$$W\left(\underline{X}^W\right)=U(value\ matching);$$
$$W'(X)|_{X=\underline{X}^W}=0(smooth\ pasting);\ \lim_{X\to\infty}\left(\frac{W}{X}\right)<\infty(no\ bubble) \tag{8}$$

$\underline{X}^W$ denotes the optimal separation threshold X for the worker.

The optimal separation threshold for a matched firm-worker pair merits some additional explanation. Unlike the standard dynamic capital structure models, like Leland (1994), the no-commitment assumption on both sides of the match enables both parties of the matched pair to walk away at any time at his/her will. Therefore, the match lasts until X hits $\max\{\underline{X}^E,\ \underline{X}^W\}$. The party with higher valuation of the match might be tempted to make side payments to the other party after X hits the other party's separation threshold, only to hope that the other party stay in the matching relationship for a longer time. Such considerations significantly complicate the optimal stopping problem. Fortunately, as will be shown below, under generalized Nash bargaining, the worker and the firm always agree with each other on the separation threshold.

To facilitate the intuition behind my equilibrium concept, I first introduce a notion of submarket in the labor market[20]:

Definition 1 (Submarket) A submarket with coupon c_i, which I call it submarket i, consists all firms posting job vacancies with coupon c_i and all the workers applying for the job vacancies with this coupon.

3.2 Unemployed worker

I focus on a searching worker's behavior in the steady state labor market with I nonempty submarkets indexed by $i \in \{1,2, \dots, I\}$. Each submarket i is characterized by a coupon choice c_i, posted by a measure of m_i firms.

Let U_i denote the value of being unemployed, in other words, the value of active searching for jobs in submarket i, the HJB equation for an actively searching worker in submarket i with coupon choice c_i is:

$$rU_i = b + g(\epsilon_i)[W_i(X_0) - U_i] \tag{9}$$

Since workers are ex-ante identical, and they have perfect information regarding the leverage associated with all the job vacancies in the labor market. They will enter the submarket that provide them with the highest expected value of active job search. All the submarkets with nonempty job applicants must grant the same level of expected value to the unemployed workers, which I denote this value as U. Bringing U into (9), the value of an unemployed worker satisfies the following HJB equation:

$$rU = b + g(\epsilon_i)[W_i(X_0) - U] \tag{10}$$

Simple algebraic manipulation gives:

[20] This is similar to submarket concept in Moen (1995) on wage posting in labor market.

$$g(\epsilon_i) = \frac{rU - b}{W_i(X_0) - U} \tag{11}$$

For a given U, (11) defines a unique relationship between the coupon rate c and the labor market tightness ϵ in each submarket i. In other words, ϵ is a function, specified by (11), of U and c.

Note that U only depends on the aggregate debt level in the labor market. Since in the baseline model, all the firms have the same productivity and face the same optimization problem for coupon rate, all the firms choose the same coupon c in equilibrium. There is only one submarket.

3.3 Idle vacancies

Denote $V(c;U)$ as the expected value of a vacancy for which the firm chooses coupon rate c, given unemployment value U. Then $V(c;U)$ obeys the following HJB equation:

$$rV(c;U) = -\kappa + h^e\big(\epsilon(c;U)\big)[E(X_0) + D(X_0) - V] \tag{12}$$

where $h^e(\epsilon(c;U))$ is a firm's belief about relationship between the announced coupon choice c and the arrival rate of workers, given U. In equilibrium, the firm's expectation is always equal to the true relationship between announced capital structure and the arrival rate of workers, with $\epsilon(c;U)$ is an implicit function of c given by (11). This identity holds even for off-equilibrium coupon announcements[21]. By free-entry condition, in equilibrium, $V(c;U) = 0$.

3.4 Competitive search rational expectation equilibrium

Now I am ready to introduce the definition of competitive search rational expectation equilibrium (CSREE).

Definition 2 (CSREE) A competitive search rational expectation equilibrium consists of a coupon rate c, a separation threshold $\underline{X}$, a vector of asset values (D, E, U, V, W), a labor market tightness ϵ, an unemployment rate u, and the firm's belief h^e such that the following holds:

I. Profit-maximization: Given U, c solves the following profit-maximization problem:

[21] Moen (1995) has shown that such belief restriction is also consistent with a stable equilibrium concept first introduced by Gale (1992), in which impact of deviating coupon choices associated with a subset of job vacancies on the equilibrium converges to zero as the measure of deviating job vacancies approaches to zero.

$$c = argmax_{c \geq 0} V(c;U) \quad (13)$$
$$subject\ to: (3)\ (5)\ (6)(7)(8)\ (10)\ (11)\ (12)$$

and

$$\underline{X} = max\{argmax_{\underline{X}'} E(X_0), argmax_{\underline{X}''} W(X_0)\} \quad (14)$$

II. Asset values: Given the optimal $(c, \underline{X})$ and U, D satisfies (3); E satisfies (5) and (6); W satisfies (7) and (8), where w is determined by generalized Nash bargaining.

III. Free entry: Given the optimal $(c, \underline{X})$, U is such that $V(c;U) = 0$

IV. Labor market tightness: Given the optimal $(c, \underline{X})$, w and U, ϵ solves

$$g(\epsilon) = \frac{rU - b}{W(X_0) - U}$$

V. Belief consistency: Given the labor market tightness ϵ,

$$h^e\big(\epsilon(c;U)\big) = h\big(\epsilon(c;U)\big) \quad (15)$$

VI. Stationary labor market: An unemployment rate u characterized by the stationary cross-sectional distribution density function $\not{f}(X)$ such that outflow from the unemployment population is equal to the inflow to the unemployment population in every dt, which is equivalent to the requirement that the inflow to employment population is equal to the outflow from the employment population in every dt.

3.5 Solve the equilibrium

3.5.1 Wage function

Notice that according to Definition 2, the equilibrium separation threshold for a given match is the higher value of the worker's and the firm's optimal separation thresholds. In this subsection, I will show that the two separation thresholds always coincide with each other, thereby greatly simplifying my subsequent analyses. As a byproduct, I also present a wage function linear in current cash flow state X.

After a match is created and the debt is issued, the worker and the firm split the remaining matching surplus through continuous bilateral bargaining according to a generalized Nash bargaining rule. The generalized Nash bargaining selects the wage:

$$w(X) \in \underset{w}{\operatorname{argmax}}[W(X) - U]^{\beta}[E(X) - V]^{1-\beta}$$

As repeated shown in labor market search literature, this maximization yields as a necessary and sufficient first-order condition:

$$\beta[E(X) - V] = (1 - \beta)[W(X) - U]$$

In equilibrium, $V = 0$. The worker's outside option value is U. Therefore, I have

$$\beta E(X) = (1 - \beta)[W(X) - U] \tag{16}$$

Taking derivatives of both sides of (16) with respect to X. I have:

$$\beta E'(X) = (1 - \beta)W'(X) \tag{17}$$

and

$$\beta E''(X) = (1 - \beta)W''(X) \tag{18}$$

One direct consequence of equation (17) is that the matched firm and worker agree to separate the matching relationship and return to search when X hits the same threshold, i.e., $\underline{X}^E = \underline{X}^W \coloneqq \underline{X}$. Therefore, the asset values in the economy have similar expressions as in Leland (1994), which greatly simplifies my analyses. I also obtain the following lemma with regard to the wage function, which is linear in current cash flow state X.

Lemma 1 (Wage function) In equilibrium, under generalized Nash bargaining, the wage function is linear in X

$$\begin{aligned} w(X) &= \beta(\theta X - f - c) + (1 - \beta)b + \beta g(\epsilon)E(X_0) \\ &= \beta(\theta X - f - c) + (1 - \beta)b + (1 - \beta)g(\epsilon)[W(X_0) - U] \end{aligned} \tag{19}$$

Proof: Appendix A1.

3.5.2 Matching surplus

It is easier to work with the matching surplus than to derive the expected discounted values of equity and wage. First, I define matching surplus as $S \coloneqq E + W - V - U$. Then by generalized Nash bargaining:

$$E - V = (1 - \beta)S$$

and

$$W - U = \beta S$$

Denote $D^0 \coloneqq D(X_0)$, $S^0 \coloneqq S(X_0), g \coloneqq g(\epsilon), and\ h \coloneqq h(\epsilon)$. The value function of being unemployed (10) can be expressed in terms of S:

$$rU = b + g\beta S^0 \tag{20}$$

Similarly, the value function of an idled vacancy becomes

$$rV = -\kappa + h[(1 - \beta)S^0 + D^0] \tag{21}$$

By the definition of S, the HJB equation for S is as follows:

$$\delta S(X) = \theta X - f - c - b + \kappa - [g\beta + h(1 - \beta)]S^0 - hD^0 + \mu X S'(X) + \frac{1}{2}\sigma^2 X^2 S''(X) \tag{22}$$

Using (20) and (21),

$$\delta S(X) = \theta X - f - c - rU - rV + \mu X S'(X) + \frac{1}{2}\sigma^2 X^2 S''(X) \tag{23}$$

with boundary conditions:

$$S(\underline{X}) = 0\ (value\ matching);$$
$$S'(X)|_{X=\underline{X}} = 0(smooth\ pasting);\ \lim_{X\to\infty}(\frac{S}{X}) < \infty\ (no\ bubble) \tag{24}$$

In the appendix A2, I obtain a closed-form solution of the boundary problem (22), (23) and (24). Taking derivative of $S(X)$ with respect to X and setting this expression equal to zero at $X = \underline{X}$, I achieve Proposition 1 regarding the optimal separation threshold:

Proposition 1 (Optimal separation threshold) Given U and c, the matching surplus $S(X)$ is given by

$$S(X) = \theta\Pi(X) - F - \frac{c + b - \kappa + [g\beta + h(1-\beta)]S^0 + hD^0}{\delta} -$$
$$\left[\theta\Pi(\underline{X}) - F - \frac{c + b - \kappa + [g\beta + h(1-\beta)]S^0 + hD^0}{\delta}\right]\left(\frac{X}{\underline{X}}\right)^{\nu}$$
$$= \theta\Pi(X) - F - \frac{c + rU + rV}{\delta} - \left[\theta\Pi(\underline{X}) - F - \frac{c + rU + rV}{\delta}\right]\left(\frac{X}{\underline{X}}\right)^{\nu} \quad (25)$$

The optimal separation threshold $\underline{X}$ is

$$\underline{X} = \frac{-\nu}{1-\nu}\frac{\delta - \mu}{\theta}\left[F + \frac{c + b - \kappa + [g\beta + h(1-\beta)]S^0 + hD^0}{\delta}\right]$$
$$= \frac{-\nu}{1-\nu}\frac{\delta - \mu}{\theta}\left[F + \frac{c + rU + rV}{\delta}\right] \quad (26)$$

In equilibrium, the matching surplus becomes

$$S(X) = \theta\Pi(X) - F - \frac{c + rU}{\delta} - \left[\theta\Pi(\underline{X}) - F - \frac{c + rU}{\delta}\right]\left(\frac{X}{\underline{X}}\right)^{\nu} \quad (27)$$

The equilibrium optimal separation threshold $\underline{X}$ is

$$\underline{X} = \frac{-\nu}{1-\nu}\frac{\delta - \mu}{\theta}\left[F + \frac{c + rU}{\delta}\right] \quad (28)$$

The optimal separation threshold $\underline{X}$ is decreasing in productivity θ, and is increasing in c and U.

Proof: Appendix A2.

The fact that the optimal separation threshold is increasing in c echoes the finding from risky debt and capital structure literature, for example, Leland (1994). The optimal separation threshold and U move in the same direction is new to the literature. The separation threshold can be triggered by either party of the firm-worker match. Therefore, the optimal separation threshold incorporates the worker's outside option value U.

3.5.3 Optimal coupon c^*

The optimal coupon rate c^* solves the following constrained maximization problem:

$$rV^*(U) = \max_{c \geq 0} -\kappa + h[(1-\beta)S^0 + D^0] \quad (29)$$

subject to the following constraints: D^0 is specified by (3) with $X = X_0$; S^0 is specified by (27) with $X = X_0$; $\underline{X}$ is specified by (28); $h = h\big(\epsilon(c;U)\big)$ is such that $\epsilon h(\epsilon) = g(\epsilon) = \frac{rU-b}{\beta S^0}$

In the appendix, I show that the first order condition for the above problem is characterized by the following two equations[22,23]:

Proposition 2 (Optimal coupon rate c) In equilibrium, the first order condition for optimal coupon rate c satisfies the following first order condition:

$$h\big(\epsilon(U,c)\big)\left[\frac{\beta}{\delta}+\frac{1}{\delta}\left(\alpha\nu\frac{c}{\delta F+c+rU}-\beta\right)\left(\frac{X_0}{\underline{X}}\right)^{\nu}\right]+$$
$$h^{(c)}\big(\epsilon(U,c)\big)\left\{\begin{array}{l}(1-\beta)\left[\theta\Pi(X_0)-F-\dfrac{rU}{\delta}\right]+\beta\dfrac{c}{\delta}\\+\left(\dfrac{\alpha-\beta}{1-\nu}\dfrac{\delta F+c+rU}{\delta}-\dfrac{\alpha c}{\delta}\right)\left(\dfrac{X_0}{\underline{X}}\right)^{\nu}\end{array}\right\}=0 \qquad (30)$$

where $h^{(c)}\big(\epsilon(U,c)\big) = h^{(g)}g^{(c)} < 0$ and $g^{(c)} = -g\frac{S^{0\,(c)}}{S^0} > 0$. A sufficient condition for optimal c defined by (30) is the solution of constrained optimization problem defined by (29) are: $\frac{S^{0\,(c)}}{S^0}$ is decreasing in c and $h^{(gg)} < 0$[24].

Proof: Appendix A3.

(30) gives me an intuitive result regarding the optimal coupon choices of individual firms. When posting coupon rate to workers, the firm balances three opposing forces that c imposes to the expected shareholder surplus. All three forces are consistent empirical regularities. First of all, larger coupon rate increases post-match shareholder surplus, because the "size of the pie" divided between shareholders and workers shrinks, and workers cannot get their hands on the proceeds of debt issuance. A similar effect has been derived in Monacelli, Quadrini and Trigari (2011), under a discrete time setting featuring one-period short term debt. This so-called "strategic role of debt"

[22]To conserve space, I use c to denote the optimal coupon hereafter unless explicitly specified otherwise.

[23] In order to confirm optimality, I need to consider the second order condition at the optimal coupon rate c. In the appendix, I give sufficient conditions for the second order derivative to be negative. However, a complete characterization of optimal coupon rate depends on the specific matching functional form and model parameters.

[24] For any function l, $l^{(.)}$ denotes the partial derivative of l with respect to ., $\frac{\partial l}{.}$, and $l^{(..)}$ denotes the second order partial derivative of l with respect to ., $\frac{\partial^2 l}{.^2}$.

is empirically proved by, for example, Matsa (2010), which finds that firms respond to stronger pro-union state laws by using higher leverage. The second effect is the classic cost of financial distress. Since higher debt issuance triggers bankruptcy earlier and bankruptcy is costly by the model assumption, a higher c reduces the equity value by forcing premature separation of a firm-worker match. This effect is absent in the traditional labor economics literature, since most scholarly works focus on all-equity financed firms. The cost of financial distress associated with high leverage is widely documented in the tradeoff theory of capital structure with risky debt, for example, Leland (1994). The last effect, which is novel to the theoretical literature on labor market search, is that a higher coupon rate c reduces the arrival rate of the applicants to the posted job vacancy, thus reduces the probability of the matching formation in the first place. This effect has met great empirical success recently. For example, Brown and Matsa (2016) uses newly available data from an online job search platform and finds that job vacancies posted by firms with poor financial conditions and higher leverage result in fewer applicants. In equilibrium, individual firm optimally chooses its coupon rate, that balances the benefit of leverage, the strategic role of debt, and two costs of leverage, the cost of financial distress and the hiring role of debt. Mathematically, the firm chooses optimal c that equalizes the following two absolute values of elasticities with respect to c: the elasticity of expected post-match shareholder surplus, $(1-\beta)S^0 + D^0$, and the elasticity of before-match hiring rate, $h\big(\epsilon(U,c)\big)$.

$$|\eta_c[(1-\beta)S^0 + D^0]| = |\eta_c[h(\epsilon(U,c))]| \tag{31}$$

where $|\eta_c(.)|$ stands for the absolute value of respective elasticity with respect to c.

3.5.4 Expected job tenure

My model settings allow me to derive a closed-form representation of the expected remaining job tenure, i.e., the expected match duration, when current cash flow state is X. Specifically, $T(X)$ stands for the expected remaining duration of a match when current cash flow state is X. Standard results from stochastic process literature (e.g., Karlin and Taylor, 1981, 15.3) shows that $T(X)$ solves the following boundary value ODE problem.

$$\mu X T'(X) + \frac{1}{2}\sigma^2 X^2 T''(X) - sT(X) = -1 \tag{32}$$

The boundary conditions are:

$$T(\underline{X}) = 0 \; and \; \lim_{X\to\infty} T(X) = \frac{1}{s} \tag{33}$$

Heuristically, the remaining tenure is zero if X hits the separation threshold, $\underline{X}$. Meanwhile, if X is very large, only event that could end the match is the exogenous match destruction event, with arrival intensity s. Solving explicitly the boundary value problem (32) and (33) for the expression of $T(X)$, we have the following proposition:

Proposition 3 (Expected tenure) Given the expected value of a searching worker U and optimal coupon rate c, the expected job tenure in equilibrium is

$$T(X) = \frac{1}{s}\left[1 - \left(\frac{X}{\underline{X}}\right)^{\rho}\right] \tag{34}$$

where $\rho = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2s}{\sigma^2}}$. $T(X)$ is decreasing in coupon rate c and is increasing in the current cash flow state X and vacancy productivity θ.

Proof: Appendix A4.

Recall that wage is also increasing in the current cash flow state X, therefore I obtain a positive relationship between job tenure and wage. Many empirical labor economists find a robust positive relationship between seniority and wage, in both the United States (e.g., Topel, 1991) and Europe (e.g., Dustmann and Meghir, 2005).

3.5.5 Stationary distributions of cash flow state X

In this section, I characterize the stationary cross-sectional distribution of the cash flow state X in the economy. This exercise serves two purposes. First of all, the steady-state unemployment rate u is expressed in terms of stationary cross-sectional distribution density function, since the total mass of workers is one. Moreover, as seen from Lemma 1 and Proposition 3, wages and expected job tenures are deterministic function of cash flow state X. By deriving the stationary cross-sectional distribution of X, I am able to pin down the stationary cross-sectional distribution of wages and expected job tenure in the economy.

My economy is a stochastic growth economy featuring matching pair deaths and births[25], with labor market search being the only friction. The stochastic process governing the dynamic evolution of the cash flow state X, by assumption, is a geometric Brownian process with drift. Obviously, X belongs a class of Kolmogorov-Feller diffusion process[26]. Let $\tilde{f}(X; X_0)$ be the transition probability density function for X in the economy with the starting value X_0. From the classic treatment (e.g., Karatzas and Shreve, 1991, Chapter 5, Section 1), the dynamics of $\tilde{f}(X)$ follows a Fokker-Planck equation, also known as Kolmogorov forward equation of the process X, $\forall X \in [\underline{X}, \infty] \setminus \{X_0\}$,

$$\frac{d\tilde{f}(X)}{dt} = -\frac{d}{dX}\left[\mu X\tilde{f}(X)\right] + \frac{1}{2}\frac{d^2}{dX^2}\left[\sigma^2 X^2 \tilde{f}(X)\right] - s\tilde{f}(X) \tag{35}$$

Let $f(X)$ denote the stationary $\tilde{f}(X)$, I have the following boundary value problems governing $f(X)$:

$$-\frac{d}{dX}[\mu X f(X)] + \frac{1}{2}\frac{d^2}{dX^2}[\sigma^2 X^2 f(X)] - sf(X) = 0 \tag{36}$$

with boundary conditions[27]:

$$f(\underline{X}+) = 0 \tag{37}$$

$$\frac{1}{2}\sigma^2 X_0^2 [f'(X_0 -) - f'(X_0 +)] = s\int_{\underline{X}}^{\infty} f(X)dX + \frac{1}{2}\sigma^2 \underline{X}^2 f'(\underline{X}+) \tag{38}$$

$$g\left[1 - \int_{\underline{X}}^{\infty} f(X)dX\right] = s\int_{\underline{X}}^{\infty} f(X)dX + \frac{1}{2}\sigma^2 \underline{X}^2 f'(\underline{X}+) \tag{39}$$

The boundary conditions, despite their complexities, are intuitive under scrutiny. First, once the post-match performance is poor and the cash flow state X reaches the equilibrium endogenous default threshold, $\underline{X}$, separation occurs immediately. In other words, X spends no time at $\underline{X}$[28]. Mathematically, this requires $\frac{1}{2}\sigma^2\underline{X}^2 f(\underline{X}+) = 0$, since $\frac{1}{2}\sigma^2\underline{X}^2 \neq 0$, I have (37). Secondly,

[25] For the application of power laws to city and population growth, refer to Gabaix (2009).
[26] For the definition of Kolmogorov-Feller diffusion process, please consult to Karatzas and Shreve (1991), Chapter 5, Definition 1.1.
[27] $X+ := \lim_{X' \downarrow X} X'$ and $X- := \lim_{X' \uparrow X} X'$
[28] Mathematically, $\underline{X}$ is an attainable boundary that can be hit by the process in finite time period with positive probability. Moreover, attainable boundaries are either absorbing or reflecting. In my case, it is absorbing.

(38) has an economic meaning as follows: at steady state, the total flows into the employment must commensurate the total flows out of the employment. The left hand side is the total flows into the employment. The density $\not{f}(X)$ is not differentiable at X_0, corresponding to the inflow of workers to the employment and all new matches starting at X_0. The right hand side is the total flows out of the employment. The first term is intuitive. For the last term, the flow of matching separation at $\underline{X}$ is given by $\frac{1}{2}\sigma^2\underline{X}^2\not{f}'(\underline{X}+)$. Intuitively, over a small enough interval of time Δ, the diffusion term in $dX_t = \mu X_t dt + \sigma X_t dZ_t$ dominates, and half of the measure[29] of $\not{f}(\underline{X} + \sigma\underline{X}\sqrt{\Delta}) \times \sigma\underline{X}\sqrt{\Delta}$ matched firm-worker pairs near the boundary $\underline{X}$ will exit the production. Finally, (39) is the standard restriction in labor market search models (e.g., Mortensen and Pissarides, 1994), which yields the Beveridge curve. The left hand side is the outflow from the unemployment population, and the right hand is the inflow to the unemployment population, which, by definition, is also the outflow from the employment population.

The solution technique of the boundary problem (36) subject to (37) — (39) is similar to those continuous time cases in the power law literature (e.g., Gabaix, 2009; Achdou, Han, Lasry, Lions and Moll, 2015). For $X \in [\underline{X}, \infty] \setminus \{X_0\}$, the following proposition characterizes the stationary cross-sectional distribution density function of X, in the equilibrium:

Proposition 4 (Stationary cross-sectional distribution of X), Given U, g and c, for $X \in [\underline{X}, \infty] \setminus \{X_0\}$, the stationary cross-sectional distribution density function of X in equilibrium is:

$$\not{f}(X) = \begin{cases} \zeta X^{-m_1-1}, & X > X_0 \\ \tilde{\zeta} X^{-m_0-1}\left[1 - \left(\frac{\underline{X}}{X}\right)^{m_1-m_0}\right], & \underline{X} \le X < X_0 \end{cases} \tag{40}$$

where $m_0 = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2s}{\sigma^2}}$ and $m_1 = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2s}{\sigma^2}}$; ζ and $\tilde{\zeta}$ are positive and uniquely determined by boundary conditions (38) and (39).

Proof: Appendix A5.

[29] This expression is derived by using Taylor expansion at $X = \underline{X}$.

The expression of the stationary cross-sectional distribution density function $\mathscr{f}(X)$ of cash flow state X takes the form of Double-Pareto distribution density, as repeatedly shown in the stochastic growth literature (e.g., Gabaix, 2009; Achdou, Han, Lasry, Lions and Moll, 2015).

We thus complete the solution of a Competitive Search Rational Expectation Equilibrium defined in Definition 2. The solutions for optimal coupon rate c, unemployment value U, and labor market tightness ϵ, are characterized by a system algebraic equations. The separation threshold $\underline{X}$ and the labor market aggregates: the wage function w, expected job tenure T, and the stationary cross-sectional distribution of cash flow state in the economy $\mathscr{f}$, are all in analytical forms.

3.6 Labor force participation rate

The labor force participation rate (LFPR hereafter) is counter-cyclical. The empirical consensus is that the transition from out of labor force to unemployment goes up when the economy is sliding into recession (Elsby, Hobijn and Sahin, 2015; Krueger, 2016). In this subsection, I try to extend the model to allow for workers' job searching intensity decisions. Higher job searching intensity indicates more active labor force participation.

Theoretically, a rigorous treatment of labor force participation decisions requires three states of workers— unemployed, employed and out of the labor force, and three value functions for workers, one for each state. However, my model only offers two-state value functions for workers. I bypass the modeling difficulties by incorporating an endogenous job searching effort variable to my baseline model, and shed some light on the role of capital structure choice in affecting the worker's labor force participation choice, which is the job searching effort he/she optimally expends.

Specifically, let e_j denote the job searching effort an unemployed worker j exerts in the labor market. Without loss of generality, I restrict $e_j \in [0, \bar{e}], \bar{e} < \infty$. Higher e_j denotes for more active labor force participation, with $e_j = \bar{e}$ standing for full labor force participation. The cost of labor searching effort is $l(e_j)$, with the usual convex assumptions: $l'(e_j) > 0$ and $l''(e_j) > 0$.

The matching rate for searching worker j is $g(e_j, \epsilon) \coloneqq \frac{m(e_j u, v)}{u} = m(e_j, \epsilon)$. Again, u, v and ϵ denotes the unemployment rate, vacancy rate and labor market tightness in the economy, respectively. Similarly, the matching rate for the firm is $\frac{\int_0^u m(e_j, \epsilon) dj}{v}$.

The value function of an unemployed worker becomes[30]

$$rU(e_j) = b + g(e_j, \epsilon)\beta S^0 - l(e_j) \tag{41}$$

and the first order condition for the optimal effort e_j is[31]

$$g^{(e_j)}(e_j, \epsilon)\beta S^0 - l'(e_j) = 0 \tag{42}$$

In the equilibrium, since workers are ex-ante identical, they face the same job searching effort optimization problem and choose the same optimal amount of effort, denoted as e, and they obtain the same value of being unemployed. I omit the subscripts of worker indices. (41) and (42) become:

$$rU = b + g(e, \epsilon)\beta S^0 - l(e) \tag{43}$$

and

$$g^{(e)}(e, \epsilon)\beta S^0 - l'(e) = 0 \tag{44}$$

The value function for an idle vacancy is:

$$rV = -\kappa + h(e, g)[(1 - \beta)S^0 + D^0] \tag{45}$$

which is similar to (29), except that the matching rate h now depends on the job searching effort in the economy, e, in addition to the labor market tightness.

The first order condition for optimal coupon rate c is similar to (30), except that $h^{(c)}(e, g) = h^{(e)} e^{(c)} + h^{(g)} g^{(c)}$, in the appendix A6, I show the explicit expressions of $e^{(c)}$ and $g^{(c)}$. The following proposition characterizes the optimal coupon rate c in the presence of endogenous searching effort.

[30] Note that as in the baseline case, a searching worker j's U does not depend on the individual coupon choice. Meanwhile, U does depend on the individual j's searching effort.

[31] Note that the internal solution always exists because (41) is concave in e.

Proposition 5 (Optimal coupon rate c in the presence of job searching effort e) In equilibrium with optimal searching effort e, the optimal coupon rate c satisfies the following first order condition:

$$h\big(e(U,c),g(U,c)\big)\left[\frac{\beta}{\delta}+\frac{1}{\delta}\left(\alpha\nu\frac{c}{\delta F+c+rU}-\beta\right)\left(\frac{X_0}{\underline{X}}\right)^{\nu}\right]+$$
$$h^{(c)}\big(e(U,c),g(U,c)\big)\left\{\begin{aligned}&(1-\beta)\left[\theta\Pi(X_0)-F-\frac{rU}{\delta}\right]+\beta\frac{c}{\delta}\\&+\left(\frac{\alpha-\beta}{1-\nu}\frac{\delta F+c+rU}{\delta}-\frac{\alpha c}{\delta}\right)\left(\frac{X_0}{\underline{X}}\right)^{\nu}\end{aligned}\right\}=0 \qquad (46)$$

where $h^{(c)}\big(e(U,c),g(U,c)\big)=h^{(e)}e^{(c)}+h^{(g)}g^{(c)}$. $g^{(c)}$ and $e^{(c)}$ is defined in $(\mathrm{A}35)$ and $(\mathrm{A}36)$, respectively. A sufficient condition for optimal c defined by (46) is the solution of constrained optimization problem defined by (45) is $h^{(cc)}(e,g)<0$.

Proof: Appendix A6.

The solutions of expected duration of a matching relationship in the economy and stationary cross-sectional distribution of the cash flow states X are similar to the baseline case, which I omit here.

3.7 A numerical example

In this section, I demonstrate the model implications for joint relationship between optimal leverage choices and labor market dynamics for different model parameters. Specifically, I focus on three sets of model parameters: the workers' bargaining power, β, the matching efficiency, A, and indicators of economic downturns, α and σ. Consistent with the empirical findings on functional form of the labor market matching technology, I use a constant-return-to-scale, Cobb-Douglas matching function[32] $m(u,v)=Au^{\iota}v^{1-\iota}$ (Petrongolo and Pissarides, 2001). The benchmark parameter values are presented in Table 1 and are in line with extant research on aggregate labor market dynamics.

[Insert Table 1 here]

[32] In the presence of job searching effort, the matching function becomes: $m(eu,v)=A(eu)^{\iota}v^{1-\iota}$, in which e is searching worker's job searching effort.

3.7.1 Optimal leverage

In this subsection, I examine the impacts of model parameters on firms' optimal leverage choices. The results are illustrated in Figure 1. Several robust patterns are revealed. First of all, debt level c is increasing in workers' bargaining power, β, as illustrated in Figure 1A. This is consistent with recent empirical and theoretical findings that firms utilize higher leverage to discourage the workers' stronger wage demand (e.g., Matsa, 2010; Monacelli, Quadrini and Trigari, 2011). More interesting facts about the leverage choice is that it increases with the labor market search efficiency parameter, A, for an empirical plausible range of estimates[33]. The underlying logic is as follows: on one hand, the marginal benefit of a higher leverage on post-match shareholder value scales up with the labor market search efficiency. On the other hand, recall that the marginal cost of posting a larger c for the firm is lowering the labor market matching probability. As labor market search becomes more efficient, this marginal cost of c is decreasing. The two forces induce the firm to lever up[34] as labor market search efficiency improves. This relationship is supported by recent empirical literature, which finds that firms choose lower leverage when their workers face greater unemployment risk (e.g., Agrawal and Matsa, 2013; Chemmanur, Cheng and Zhang, 2013). Another interesting fact is that the leverage increases as economic volatility mounts up. This is consistent with findings from other research on the relationship between leverage and aggregate volatility (e.g., Johnson, 2016). However, the underlying mechanism is different. Johnson (2016) resorts to a deposit insurance mechanism. I provide an alternative mechanism originated from labor market search frictions. I further decompose the marginal benefits and marginal cost to various levels of c, at high and low volatility levels. The result confirms that marginal cost of choosing a higher coupon rate c decreases rapidly with the volatility. Notice that the productivity does not change in the core model. Therefore, as volatility increases, the value of a match deteriorates[35]. Consequently, the marginal cost of a higher leverage c decreases. This is because as the value of a successful match to the firm is lower,

[33] Most of the labor economics papers estimate A between 4 and 5.

[34] The relationship between c and A is not monotonic. A further examination reveals that as A becomes very large, the optimal leverage jumps down to zero. Notice that the labor market matching process becomes almost frictionless as A becomes very large. The labor market is analogues to a retail market, in which firms provide homogeneous product—job vacancies to workers, and workers always go to the highest valued vacancies, i.e., vacancies with zero leverage. The searching workers' choices arise from the fact that as A becomes very large, the firms can instantaneously fulfill any amounts of searching workers' job demands.

[35] This is because bankruptcy is costly and the matching relationship has higher chance to hit the bankruptcy boundary.

increasing matching probability through lower c becomes less desirable[36]. Therefore, the firm responds to a higher economic volatility by employing a higher leverage policy. To my best knowledge, this is the first theoretical research that tackles the positive leverage-volatility co-movement puzzle from a frictional labor market perspective. Lastly, as seen from Figure 1D, the optimal coupon rate decreases with the cost of bankruptcy α. This finding is consistent with most of the extant corporate finance research (e.g., Leland, 1994).

[Insert Figure 1 here]

3.7.2 Expected tenure

Notice that from (34) of Proposition 3, the expected matching duration decreases with the separation threshold, which in turn, increases with the optimal debt usage by the firm. Consistent with findings regarding the comparative statics of optimal leverage in Figure 1, the expected job tenure in the economy is decreasing in the worker's bargaining power, the labor market search efficiency and the cash flow volatility. It increases with the bankruptcy cost parameter.

[Insert Figure 2 here]

3.7.3 Stationary cross-sectional density function of X

I compare the stationary cross-sectional density function of the cash flow state X, between high and low values of workers' bargaining power, and high and low values of labor market search efficiency. As shown in Figure 3, lower value of workers' bargaining power generates a fatter left tail of stationary cash flow distribution among matches, so does a lower value of search efficiency parameter. These results are intuitive. Since the separation threshold increases with the worker's bargaining power and the search efficiency[37], matches are endogenously destroyed at higher cash flow level. Therefore, the stationary cross-sectional cash flow distributions in the economy with lower workers' bargaining power and lower matching efficiency are more dispersed, compared

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[36] Volatility also affects the marginal benefit of c. Notice that as volatility increases, the firm has higher chance to generate large cash flow. By Nash bargaining, the worker will take a larger share of the cash flow under this high profitability scenario. Therefore, the firm has more incentive to use debt to reduce the worker's wage demand.

[37] Two forces contribute to this. First of all, the optimal leverage increases with worker's bargaining power and matching efficiency, which elevates the separation threshold. A second and subtler force is as follows: increases in worker's bargaining power and search efficiency elevate the expected value of a searching worker, thereby raising the required cash flow threshold to keep the matching relationship valuable to both parties.

with an otherwise identical economy characterized by higher workers' bargaining power and higher search efficiency. Since wage is a linear function of cash flow state X, as shown in (19) of Lemma 1, the wage distribution in the former economy also has a fatter left tail, compared with that of the latter economy. As far as I am concerned, this is the first research relating the wage bargaining and labor market search efficiency to the dispersion of the wage and cash flows in the economy, through an endogenous capital structure choice channel on the employer side.

[Insert Figure 3 here]

3.7.4 Unemployment rate u

In this subsection, I examine how the wage bargaining, the labor market search efficiency and the economy-wide volatility affect the steady-state unemployment rate u. This practice differs from traditional labor market search models because an important underlying channel is the optimal capital structure choice by the employers in the economy. First of all, as shown in Figure 4A, the unemployment rate increases with the workers' bargaining power. Intuitively, a higher bargaining power induces the termination of the matching relationship at a higher cash flow level, as firms employ higher leverages to prevent workers from scooping large share of matching surplus. As a result, more workers go back to unemployment pool during each time period. Regarding the labor market search efficiency, despite higher leverage choice as a response to a higher search efficiency, the unemployment rate drops as the search efficiency improves, as shown in Figure 4B. Regarding the effect of bankruptcy cost, a higher bankruptcy cost constrains the firm's ability to grab a larger share of matching surplus by levering up, thereby reducing its incentive to post a vacancy. Moreover, the more conservative leverage policy as a response to a higher bankruptcy cost elicits more job applicants, thereby further reducing the matching probability of the individual worker. The two effects collectively drive up the unemployment rate. This is consistent with the empirical findings that unemployment rate is higher during collateral crisis. One counterintuitive result is the negative relationship between economic volatility and unemployment rate. Unemployment rate is well known as a countercyclical variable while higher economic volatility is often accompanied by an economic recession. However, one equally widely known fact about unemployment rate is that unemployment rate is less volatile than the gross domestic product, so called "Okun's law". A common explanation is that a large component of unemployment rate is unrelated with business cycles (Hall, 2005; Hall, 2016). My model sheds new light to this old conundrum. In my economy, the productivity is constant over time. Therefore, I could isolate the effect of volatility change on unemployment rate from the overall business cycle effect. The result highlights one silver lining of higher leverage choice during more volatile times: Higher leverage choices reduce the "congestion effect" among the searching workers. In other words, the higher leverage policy unintendedly creates a positive externality on the workers' job searching process. This "congestion reduction" effect dominates "surplus reduction" effect during turbulent times, thereby leading to a negative relationship between economic volatility and unemployment rate[38]. This positive externality of high leverage choice in reducing the "congestion" on workers' job searching is overlooked by the previous literature.

[Insert Figure 4 here]

3.7.5 Initial wage w_0

In this subsection, I fix the cash flow state at X_0, and examine how the wage bargaining, the labor market search efficiency, and the economy-wide volatility affect the initial wage w_0. Several patterns emerge. First and foremost, as seen from Figure 5B, labor market search efficiency affects the wage of the new hires in a modest and non-monotonic way. From un-tabulated analyses, for very inefficient matching technology, an increase in search efficiency rapidly boosts the expected value of an unemployed worker, thereby elevating the wage for the new hires. However, as the search efficiency continue to improve, the positive effect of search efficiency on unemployment value dwindles. The firm's higher optimal leverage policy as a response of improved search efficiency dominates and wears down the starting wage of a matching relationship. This contrasts to traditional labor market search models without consideration of employers' leverage choices, in which the wage of new hires monotonically increases with the search efficiency for obvious reasons: higher search efficiency increases the workers' expected value of searching, thereby increasing the required surplus they demand from a matching relationship. Moreover, this non-monotonic and modest relationship between search efficiency and wage dynamics calls for a thorough cost-benefit analysis of government policies aimed at promoting labor market search efficiency. For example, battling against the recent financial crisis,

[38] Of course, allowing for a multi-state Markov process of productivity θ will reduce both the debt and the surplus values of the matching, which, in turn, overturns the positive relationship between economic volatility and leverage.

many countries from Europe, to name a few, UK, Germany and Ireland, expand current vocational training program and initiate new programs to reduce the labor market mismatches (Heyes, 2012). These active labor market programs that improve the labor market search efficiency are argued to swiftly increase the national welfare in the short run (Brown and Koettl, 2015). However, one subtlety is that employers might take advantage of these job creation programs by adjusting upward their leverage ratios. As a result, the new employments might arise at a cost of lower wages. A complete welfare implication of these programs might yield more complex results than the original expectations. Lastly, as seen from Figure 5A, Figure 5C and Figure 5D, the optimal capital structure choice as responses to the worker's bargaining power and macroeconomic condition plays a dominant role in determining the initial wages of a matching relationship. Specifically, higher bargaining power on the worker side and more volatile economy elicit higher leverage choices by the firms, which in turn cuts back the initial wages. An opposite effect of higher bankruptcy cost on wage holds analogously. An important empirical implication drawn from Figure 5C and Figure 5D is that collateral crisis and volatility spikes might alter wages in opposite directions, even if they are often concomitant with each other during economic recessions.

[Insert Figure 5 here]

3.7.6 Labor force participation e

As shown in Section 3.6, I interpret workers' labor force participation rate as workers' job search intensity. Two model parameters come into play when I examine the comparative statics of LFPR. First of all, as shown in Figure 6A, a more efficient matching technology induces the workers to exert more job searching effort, in order to capitalize a more "productive" matching process. It is intuitive because by the matching function specified in Section 3.6, an additional searching effort yields a larger increase in matching probability when search efficiency is higher. Another important model parameter is the economic volatility. A higher economic volatility elicits more job searching effort by the workers. This is because the positive externality of higher leverage in reducing the congestion among searching workers. As a result, workers have higher incentives to participate in the labor market, because the return of such effort, in terms of job matching probability, is higher. This finding is in line with the empirical regularities that the transition rate from out-of-labor-force to the unemployment pool is countercyclical, ramping up during the recessions (e.g., Elsby, Hobijn and Sahin, 2015; Krueger, 2016). Although the economic recessions are characterized by both lower productivity and higher uncertainty, I have shown that the volatility certainly contributes to the observed counter-cyclical behavior of labor force participation, which is, to my best knowledge, novel to the literature.

[Insert Figure 6 here]

4 Learning the random μ

In this section, I relax the model assumption about the deterministic and publicly observable match-specific quality. I assume that the match-specific productivity can be either high or low and is unobservable to both parties of the match. Such a setting meets with great empirical success[39]. I begin this section by specifying the modified environment of the model. The characterization of the competitive search rational expectation equilibrium is very similar to that derived in Section 3. Therefore, I delegate the details to Appendix B.

4.1 Searching and learning environment

The environment is the same as Section 2, except for the following changes. I change the cash flow specification to maintain the model's tractability. Specifically, the match-specific cumulative cash flow process evolves according to a standard Brownian motion with unknown drifts. For a match i,

$$dX_{it} = \mu_i dt + \sigma dZ_{it}, where\ \mu_i = \begin{cases} \mu_H , & p_0 \\ \mu_L , & 1 - p_0 \end{cases} \tag{47}$$

μ is a match-specific quality measure, and p_0 represents the common prior belief that the matching quality is high. I assume that all the agents have the same prior beliefs regarding the matching quality. Furthermore, I also assume that $b \in [\mu_L, p_0\mu_H + (1-p_0)\mu_L]$. In other words, learning is nontrivial in my economy. It is socially inefficient to keep a match with productivity μ_L.

According to classic treatment on optimal nonlinear filtering (e.g., Liptser and Shiryaev, 2001, Chapter 9), the steady-state posterior p_t about the matching quality evolves according to:

$$dp_t = p_t(1-p_t)\phi d\bar{Z}_t \tag{48}$$

[39] Inspired by the influential work by Jovanovic (1979), micro-labor economics models treat the firm-worker match as an experienced good, whose quality is initially unknown and is gradually revealed through a noisy cash flow process. For details, please refer to an excellent survey by Lazear and Oyer (2009).

$\phi := \frac{\mu_H - \mu_L}{\sigma}$ is the signal-to-noise ratio, which measures the informativeness of the cumulative cash flow process regarding the unobserved matching quality. $d\bar{Z}_t = \frac{1}{\sigma}\,[dX_t - p_t\mu_H dt - (1 - p_t)\mu_L dt]$ is a Brownian motion process with respect to the filtration $\{\mathcal{F}_t^X\}$. Intuitively, $d\bar{Z}_t$ is an innovation process from the perspectives of both parties of a match. In Appendix B, I follow the same procedure as Section 3, to characterize the asset values, optimal separation threshold and coupon rate, in terms of p_t. The stationary cross-sectional distribution of posterior belief p_t again takes the Double-Pareto form.

5 No coupon-posting

In this section, I relax two important assumptions of the baseline model in Section 3. First, I relax the assumption of credible capital structure posting. There is no reliable way that firms could credibly signal their intentional capital structure choice to potential job applicants[40]. Furthermore, I introduce heterogeneity in job productivity types. More specifically, there are two large measures of firms, different in the productivities of the job vacancies they post. Each measure is determined endogenously through free-entry conditions for both types of the firms. Workers are agnostic about the employers' productivity types during job search. The above model environments are consistent with real-world labor market observations. For majority of non-publicly listed firms, it is generally impossible to find out reliable information about their capital structure choices. Empiricists have shown that there exist considerable productivity discrepancies across firms within the same industry, and across industries[41]. I begin this section with a formalization of the aforementioned two relaxations. Then I consider two cases regarding the information structure about firm-specific productivity: the case in which the worker, the firm and the capital provider know the firm-specific productivity after the match is formed, and the other case in which only the firm is savvy about its own productivity after the match is formed.

5.1 Model environment

[40] I could equivalently keep the credible capital structure posting assumption, but instead assume that the firm is always attempted to deviate from the pre-committed capital structure to an ex-post optimal one after a match is formed.
[41] For a survey regarding the determinants and cross-sectional distributions of firm productivity in U.S. economy, please refer to Syverson (2011).

The cash flow process and flow operating cost are the same as in Section 3, except that $\theta \in \{\theta_H, \theta_L\}$. The financial contract space is the same as in Section 3. A Firm chooses its capital structure by issuing perpetual debt after the match is formed, but before the wage negotiation begins. Proceeds of debt issuance are distributed to shareholders immediately. Let p denote the common prior belief that productivity of the job vacancy is high, which is endogenously determined in equilibrium. Furthermore, I assume that job vacancies of both productivity levels are accepted by the workers. This occurs if the difference between θ_H and θ_L is not large, or the "efficiency" of labor market matching function $m(u, v)$ is sufficiently low[42]. Throughout the section, I use subscript $k \in \{H, L\}$ to denote respective quantities for firms of a certain type, either high or low productivity.

5.2 Full information about θ

On one hand, the HJB equations and boundary conditions for $D_k(X)$ and $S_k(X)$ remain the same as corresponding equations in Section 3, expect one expression for each type. Therefore, the expressions for debt value $D_k(X)$, match surplus $S_k(X)$, and optimal separation threshold $\underline{X_k}$ are analogous to the respective asset value equations in Section 3[43]. On the other hand, the unemployment value satisfies the following HJB equation:

$$rU = b + g(\epsilon)[pW_H^0 + (1-p)W_L^0] = b + g(\epsilon)\beta[pS_H^0 + (1-p)S_L^0] \tag{49}$$

Under the case that firms choose debt issuance only after the matches are formed. The optimal coupon c_k solves the optimization problem:

$$\max_{c'_k}(1-\beta)S_k^0 + D_k^0 \tag{50}$$

The first order condition for c_k is:

$$\frac{\beta}{\delta} - \frac{\beta - \alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_k}}\right)^{\nu} = 0$$

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[42] For example, if $m(u, v) := Au^{\iota}v^{1-\iota}$, then A is sufficiently small.

[43] I modify the default value of the firm to $D(\underline{X}) = D^B = (1-\alpha)\theta\Pi(\underline{X}) - F - \frac{rU}{\delta}$ to make the calculation less cumbersome. It is not crucial and does not change any model predictions in the section.

After simplifying, the optimal coupon c_k for the firms with productivity type θ_k under symmetric information is

$$c_k = \delta \left[\frac{1-\nu}{-\nu} \left(\frac{\beta}{\beta - \alpha\nu} \right)^{-\frac{1}{\nu}} \theta_k \Pi(X_0) - F \right] - rU \tag{51}$$

Bringing (51) to (28), the optimal separation threshold $\underline{X_k}$ is

$$\underline{X_k} = \left(\frac{\beta}{\beta - \alpha\nu} \right)^{-\frac{1}{\nu}} X_0 \tag{52}$$

Notice that the optimal separation threshold is independent of the productivity parameter θ_k. On one hand, ceteris paribus, both parties of a match are willing to separate at a later time when the productivity of the match is higher; on the other hand, more productive firms optimally choose larger coupon rate c_k, which leads to earlier defaults. In equilibrium, the two effects exactly offset each other.

Equipped with the optimal coupon rate c_k and optimal default threshold $\underline{X_k}$, I am ready to simplify the debt value and matching surplus, $D_k(X)$ and $S_k(X)$. Similar to steps in Section 3, I have:

$$S_k(X) = \theta_k \Pi(X) + \left(\frac{\beta}{\beta - \alpha\nu} \right)^{-\frac{1}{\nu}} \theta_k \Pi(X_0) \left[\frac{1-\nu}{\nu} - \frac{1}{\nu} \frac{\beta}{\beta - \alpha\nu} \left(\frac{X}{X_0} \right)^{\nu} \right] \tag{53}$$

At the start of the match, $X = X_0$,

$$S_k(X_0) = \theta_k \Pi(X_0) + \left(\frac{\beta}{\beta - \alpha\nu} \right)^{-\frac{1}{\nu}} \theta_k \Pi(X_0) \left(\frac{1-\nu}{\nu} - \frac{1}{\nu} \frac{\beta}{\beta - \alpha\nu} \right) \tag{54}$$

The debt value is

$$D_k(X) = \frac{1-\nu}{-\nu} \left(\frac{\beta}{\beta - \alpha\nu} \right)^{-\frac{1}{\nu}} \theta_k \Pi(X_0) \left[1 - \left(1 + (1-\alpha) \frac{\nu}{1-\nu} \right) \frac{\beta}{\beta - \alpha\nu} \left(\frac{X}{X_0} \right)^{\nu} \right] - F - \frac{rU}{\delta} \tag{55}$$

At the start of the match, $X = X_0$,

$$D_k(X_0) = \frac{1-\nu}{-\nu} \left(\frac{\beta}{\beta - \alpha\nu} \right)^{-\frac{1}{\nu}} \theta_k \Pi(X_0) \left[1 - \left(1 + (1-\alpha) \frac{\nu}{1-\nu} \right) \frac{\beta}{\beta - \alpha\nu} \right] - F - \frac{rU}{\delta} \tag{56}$$

Since the matches of high and low productivities have the same separation threshold, the stationary cross-sectional distribution density function $\mathscr{f}(X)$ is defined similarly to Proposition 4.

5.3 Asymmetric information about θ

In this subsection, I consider the cases in which the productivity θ is only observable by the firm. To make it more interesting, I assume that the firm cares about market value of its securities, as well as the intrinsic values. Specifically, I define that a firm's objective function after the match is formed as:

$$\max_{c} \omega[M^0(c;\theta_m)] + (1-\omega)[(1-\beta)S^0(c;\theta_k) + D^0(c;\theta_k)] \tag{57}$$

where $M^0(c;\theta_m) = \sum_{k=L,H} p_k[(1-\beta)S^0(c;\theta_k) + D^0(c;\theta_k)]$, where $p_k \coloneqq \Pr[\theta = \theta_k]$ for $k \in \{L, H\}$. In other words, $M^0(c;\theta_m)$ is the market valuation of the firm's financial claims, including debt and equity, when the current cash flow state is X_0, the coupon rate is c, and the market belief about the firm's productivity type is θ_m.

The specification of (57) is consistent with the "capital-market driven" corporate finance models (e.g., Baker, 2009; Baker and Wurgler, 2011), in which the firm cares about the intrinsic value of its marketable securities, but at the same time is well aware of any misvaluation. The objective function is also consistent with the fact that a firm has to sell financial claims against future cash flows to investors, and become a sole custodian of the firm's productive assets[44]. Informed capital providers are often capital-constrained. Therefore, the firm is forced to go to arm's-length capital market agnostic as to the firm's type. θ_m is the market belief about the firm's productivity type, and θ_k is the firm's true type. ω measures the firm's dependence to arm's-length capital market. If the firms of both productivity types in the arm's-length market issue the same amount of debt, then the market's belief about θ is simply $p_k = p$, where p is the common prior belief that the productivity of the job vacancy is high. In this case, a firm of high productivity suffers from undervaluation in capital market while a firm of low productivity enjoy overvaluation[45]. Therefore, I face a situation of capital market signaling through debt issuance[46].

[44] There are numerous reasons for the selling of securities, for example, liquidity reasons (e.g., DeMarzo and Duffie, 1999).

[45] It is straightforward from (54) and (56) that both debt and equity value increase in θ.

[46] There is a notable uniqueness in my setting. The asymmetric information between the firm and the worker renders the generalized Nash bargaining solution inappropriate. However, since wage bargaining occurs after the security

To make the model recursively stable, I assume that the amount and valuation of security issuance, as well as the wage bargaining are private information among the firm, the current employed worker and the current capital provider. I also assume that capital market is atomless so that it is impossible for the firm to meet the same capital provider more than once. I begin by considering the separating equilibrium, followed by two categories of pooling equilibria.

5.3.1 Separating equilibrium

In this section, I first prove the existence of a separating equilibrium, in which the more productive firm deviates from its full-information optimal coupon choice, in order to differentiate itself from the less productive firm, who always chooses its full-information optimal coupon rate. Having observed the debt issuance, the matched worker can perfectly infer the employer's productivity type from its debt issuance choice. The ensued wage bargaining outcome is the same as full-information case and is dictated by the generalized Nash bargaining solution.

First, I show a sufficient condition for the existence of a separating equilibrium[47]. As repeatedly shown in the signaling game literature, a sufficient condition for the existence of a separating equilibrium in a two-player signaling game is the "single-crossing" condition (e.g., Sobel, 2007). Specifically, I have the following proposition.

Proposition 6 (Single-crossing condition)

$$\frac{\partial}{\partial \theta_k}\left(\frac{\partial M}{\partial c}\right) < 0 \tag{58}$$

Thus a separating equilibrium always exists.

Proof: Appendix C1.1.

Intuitively, signaling through excessive debt issuance is costly, which requires additional reward from capital providers by assigning higher valuations of the firm's financial securities, in

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issuance and the issuing amount is observable to the matched worker by assumption. The worker is able to infer the firm quality from its security issuance outcome, and make her wage demand accordingly. As I show later, worker's inference gives rise to two types of pooling equilibria.

[47] Throughout this section, I focus on the case that high-type firms signal their qualities via additional debt issuance compared with their full-information first-best levels. This assumption greatly simplifies my analysis on the existence and characteristics of the separating equilibrium, and is consistent with capital market signaling and security design literature (e.g., Noe, 1988; Nachman and Noe, 1994; DeMarzo and Duffie, 1999).

order for the firm to remain on the same indifference curve. However, the high-type firm requires less increase in capital market valuation than the low-type firm to stay on the same indifference curve. Therefore, there always exists a debt level that the low-type firm would rather issue its full-information debt amount and enjoy a utility level corresponding to its full-information first best level.

I am ready to characterize the separating equilibrium. First, I present the incentive compatibility for the low-productivity firm. Let c^s be the debt level chosen by the high-productivity firm in the separating equilibrium, then c^s must satisfies:

$$(IC.L)\ \omega[(1-\beta)S^0(c^s;\theta_H)+D^0(c^s;\theta_H)]+(1-\omega)[(1-\beta)S^0(c^s;\theta_L)+D^0(c^s;\theta_L)] \leq (1-\beta)S^0(c_L;\theta_L)+D^0(c_L;\theta_L) \quad (59)$$

Intuitively, the (IC.L) (59) requires that the utility for the low-type firm from mimicking the coupon choice of the high-type firm is lower than that for the low-type firm from sticking with its full-information first best coupon choice. Let c^{s*} the coupon rate such that the left hand side of (59) is equal to the right hand side. In the appendix C1.2, I show that such c^{s*} always exists. Let $\bar{c}$ be some large but finite coupon rate that is never optimal for both type of the firms[48]. Then any value of $c^s \in [c^{s*}, \bar{c}]$ satisfies the incentive compatibility condition for the low-type firm (59). I have the following lemma.

Lemma 2 (Incentive compatibility for low-productivity firms) There always exists a finite c^{s*} such that (59) holds with identity. any value of $c^s \in [c^{s*}, \bar{c}]$ satisfies the incentive compatibility for the low-productivity firm, (59).

Proof: Appendix C1.2.

Next, I characterize the incentive compatibility constraints for the high-type firm, which is:

$$(IC.H)\ (1-\beta)S^0(c^s;\theta_H)+D^0(c^s;\theta_H) \geq \omega[(1-\beta)S^0(c_L;\theta_L)+D^0(c_L;\theta_L)]+(1-\omega)[(1-\beta)S^0(c_L;\theta_H)+D^0(c_L;\theta_H)] \quad (60)$$

[48] One candidate $\bar{c} = \delta\left[\frac{1-\nu}{-\nu}\theta_H\Pi(X_0)-F\right]-rU$,i.e., $\underline{X_H} = X_0$. Immediate default occurs. I assume that default cost is such that $(1-\alpha)\theta_H < \theta_L$. Under such assumption, the left hand side of (59) is strictly smaller than the right hand side.

Intuitively, in order for a separating equilibrium to exist, the utility for the high-type firm from signaling its type must be higher than the utility for the high-type firm from pooling with the low-type firm in its coupon choice. Let c^{s**} be the coupon rate such that the left hand side of (60) is equal to the right hand side. In the appendix C1.3, I show that such c^{s**} always exists and $c^{s**} > c^{s*}$. In sum, I have the following lemma.

Lemma 3 (Incentive compatibility for high-productivity firms) There always exists a finite c^{s**} such that (60) holds with identity. Any value of $c^{s} \in [c_H, c^{s**}]$ satisfies the incentive compatibility condition for the high-productivity firm.

Proof: Appendix C1.3.

Therefore, I have the following proposition regarding the characterization of c^{s} that enforces a separating equilibrium.

Proposition 7 (c^{s} in separating equilibrium) Any $c^{s} \in [c^{s*}, c^{s**}]$ enforces a separating equilibrium.

Proof: from Lemma 2 and Lemma 3.

In equilibrium, whenever $c^{s*} > c_H$, the values accrued to the high-productivity firms upon matches are smaller compared with the full-information first best case, because the high-productivity firms have to issue additional debt to signal their types. As a consequence, the high-productivity firms post fewer job vacancies and the economy suffers from lower employment compared with the full-information case. The stationary cross-sectional distribution density function $\mathscr{f}(X)$ can be derived similarly to Proposition 4, and is omitted here.

5.3.2 Pooling equilibrium

Under pooling equilibrium, firms of high and low productivities issue the same amount of debt in the arm's length capital market. Matched worker cannot infer his/her employer's productivity type from its capital structure choice. Like any other signaling games, the equilibrium suffers from multiplicity. By assuming that all firms use one particular coupon rate regardless of their productivity levels, and that the capital market punishes all other coupon choices with the least attractive valuation upon observing deviating coupon choices, I could have infinite number of pooling equilibria. However, according to the equilibrium refinement in Maskin and Tirole (1992), in the game in which an informed principal (the firm in my case) offers contracts to outside agents (arm's length capital market in my case), the pooling equilibria that survives from the refinement are those at least weakly Pareto-dominate the least-cost separating equilibrium, which corresponds to the equilibrium characterized by the coupon choice $(c^{s,LC}, c_L)$ in my capital-raising game, where $c^{s,LC} = \max(c^{s*}, c_H)$. Meanwhile, a unique feature of my signaling game is that under asymmetric information about matching surplus, I cannot apply generalized Nash bargaining solution to characterize the wage negotiation outcome[49]. Fortunately, Myerson (1984) has characterized the so-called neutral bargaining solutions for two-person bargaining game that can be applied to the cases in which the bargaining parties have incomplete information about value-relevant parameters. This bargaining solution can be implemented by a random-dictator mechanism[50]. In my case, the wage bargaining takes place at the beginning of the match, after the firm's capital raising, but before the production begins. With probability β, the worker makes a wage demand, and firm could choose to accept the demand and starts the production, or could choose to reject it and dissolves the match. In case that the match is dissolved, both parties return to search. With probability $1-\beta$, the firm makes a wage offer, and if the worker accepts, the production begins; if she/he rejects it, the match dissolves and both parties return to search. Obviously, if it is the firm's turn to make wage offers, regardless of its productivity type, it will offer the worker a compensation package with expected value equal to the worker's outside option, i.e., the value of being unemployed, U. Meanwhile, if the worker gets the chance to make a wage demand, she/he has two choices: Firstly, the worker could demand a compensation with expected value equal to the high productivity matching surplus, which I term as "screening demand", exposing herself/himself to the risk of matching dissolution from the rejection by the low-productivity firms. The probability of the match continuation is equal to the proportion of highly productive job vacancies in the economy. Meanwhile, the worker could demand a compensation with expected value equal to the low productivity matching surplus, which I term as "pooling demand", leaving the high-productivity firm an information rent with the amount equal to the

[49] The three axioms that generalized Nash bargaining solution satisfies are silent about the bargaining outcomes under the scenario in which there exists asymmetric information between bargaining parties about the surplus value.
[50] Kennan (2010) applies neutral bargaining solution in a classic DMP labor market match model. However, that paper only focuses on the pooling wage demand by the worker, without considering the screening wage demand by the worker.

difference in expected matching surplus value between the high and low productivity firms. I examine the two types of wage demands in turn in the next two subsections[51,52].

5.3.2a "Screening demand"

This case arises if the expected value to the worker from making a screening wage demand is higher than that from making a pooling wage demand, i.e., $pS^0(c^p, \theta_H) > S^0(c^p, \theta_L)$, where c^p denotes the coupon rate in the pooling equilibrium. In the appendix C2.1, I demonstrate the incentive compatibility conditions for both types of firms to pool their capital structure choices in the capital market, and show that the pooling equilibrium exists under certain parameter restrictions. Moreover, let c_1^{p*} be the optimal pooling coupon rate for high-type firms under "screening demand". Then $c_1^{p*} < c_H$, where c_H is the full-information first best coupon choice for the high-productivity firm.

5.3.2b "Pooling demand"

This case arises if the expected value to the worker from making a screening wage demand is lower than that from making a pooling wage demand, i.e., $pS^0(c^p, \theta_H) < S^0(c^p, \theta_L)$, where c^p denotes the coupon rate in the pooling equilibrium. In the appendix C2.2, I demonstrate the incentive compatibility conditions for both types of firms to pool their capital structure choices in the capital market, and show that the pooling equilibrium exists under certain parameter restrictions. Moreover, let c_2^{p*} be the optimal pooling coupon rate for high-type firm under "pooling demand", I have $c_L < c_2^{p*} < c_H$, between the full-information first best coupon choices for the low-productivity and high-productivity firms.

6 Conclusion

This paper outlines a highly tractable labor market search model, which encompasses the capital structure choice on the firm side *à la* Leland (1994). Novel to the literature, this paper has shown that under competitive search rational expectation equilibrium, individual firms optimally choose their capital structures that equalize the absolute value of the elasticity of expected post-

[51] Notice that under "screening demand" case, the neutral bargaining solution coincides with generalized Nash bargaining solution.
[52] A complete characterization of the conditions for the existence of each type of pooling equilibrium is analytically impossible. They can only be full characterized via numerical methods, which I leave for future research.

match shareholder value with respect to capital structure choice, to the absolute value of the elasticity of ex-ante hiring rate with respect to the capital structure choice. Aggregate outcomes in labor markets can be conveniently expressed as functions of firms' optimal capital structure choices. A simple numerical illustration of the baseline model generates rich and empirically testable predictions regarding the impact of labor market search frictions, workers' bargaining power, and aggregate economic performance on firms' optimal capital structure choices and labor market outcomes, such as wage dispersions and unemployment rate. It calls for a thorough welfare analysis on the government policies aimed to reduce the labor market frictions. Specifically, any careful cost-benefit analysis of these programs should take into consideration the employers' optimal capital structure adjustments in response to the changes in labor market conditions. The equilibrium solution is similar to those from the burgeoning continuous time macroeconomic models on heterogeneous agents (e.g., Brunnermeier and Sannikov, 2014; Achdou, Han, Lasry, Lions and Moll, 2015). The continuous time approach delivers a more tractable framework compared with discrete time modelling choice.

To keep the tractability, the paper overlooks some potentially interesting modelling choice. Firstly, this paper assumes that in a given match, the firm only has one opportunity to choose its capital structure, at the beginning of the matching relationship. Starting from Goldstein, Ju and Leland (2001), and recently addressed in Hugonnier, Malamund and Morellec (2015), allowing the firm to repeatedly tap capital market greatly alters its capital structure choice. A direct extension would be to examine how the model fares if the employer is allowed to adjust the capital structure over the course of matching relationship. Moreover, a drastic assumption in this paper is that the searching worker has perfect information about the capital structure associated with every posted job vacancy. A more realistic assumption would be that a firm's past capital structure choices have a reputational effect on the worker's perception about the firm's future capital structure choice. With the continuous-time approach on reputation game (e.g., Faingold and Sannikov, 2011) at my toolbox, I could incorporate reputational effects in my model. I leave the aforementioned and other interesting extensions for the future research.

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Appendix A Deterministic and publicly observable θ

A1 Proof of Lemma 1

I speculate $w(X)$ is linear in X. Notice that U and V are independent of X. From the HJB equations in (5), (7), (10) and free-entry condition, multiplying both sides of (5) by β, I have

$$\beta r E(X) = \beta\left[(\theta X - f - c - w) + \mu X E'(X) + \frac{1}{2}\sigma^2 E''(X) - sE(X)\right] \qquad (A1)$$

and multiplying the difference between (7) and (10) by $1-\beta$, I have

$$(1-\beta)r[W(X)-U] = (1-\beta)\begin{Bmatrix} w + \mu X W'(X) + \frac{1}{2}\sigma^2 X^2 W''(X) - s[W(X)-U] - \\ b - g(\epsilon)[W(X_0) - U] \end{Bmatrix} \qquad (A2)$$

By (16), (A1) is equal to (A2), and using (17) and (18) in the main text and simplifying, I have proved (19). ||

A2 Proof of Proposition 1

Notice that the homogeneous part of (22) and (23) is a Cauchy-Euler equation, and the general solution takes the form:

$$S_{HG}(X) = AX^{\nu} + \tilde{A}X^{\tilde{\nu}}$$

where ν *and* $\tilde{\nu}$ are negative and positive solutions of the equation $\nu(\nu-1) + \frac{2\mu}{\sigma^2}\nu - \frac{2\delta}{\sigma^2} = 0$.

The general solution of (22) and (23) takes the form[53]:

$$\begin{aligned} S(X) &= \theta\Pi(X) - F - \frac{c + b - \kappa + [g\beta + h(1-\beta)]S^0 + hD^0}{\delta} + AX^{\nu} \\ &= \theta\Pi(X) - F - \frac{c + rU + rV}{\delta} + AX^{\nu} \end{aligned} \qquad (A3)$$

By *"value-matching"* condition:

$$-A\underline{X}^{\nu} = \theta\Pi(\underline{X}) - F - \frac{c + rU + rV}{\delta}$$

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[53] Notice that $\tilde{A} = 0$ by "no-bubble" condition.

$$A = -\underline{X}^{-\nu}\left[\theta\Pi(\underline{X}) - F - \frac{c + rU + rV}{\delta}\right] \qquad (A4)$$

Bringing (A4) to (A3), I have obtained (25) in Proposition 1.

$$S'(X)|_{X=\underline{X}} = 0$$

From (25), taking derivatives with respect to X and by "smooth-pasting" condition, I have

$$\frac{\theta}{\delta - \mu} - \left[\theta\Pi(\underline{X}) - F - \frac{c + b - \kappa + [g\beta + h(1-\beta)]S^0 + hD^0}{\delta}\right]\left(\frac{\nu}{\underline{X}}\right) = 0$$

$$(1-\nu)\theta\Pi(\underline{X}) = -\nu\left[F + \frac{c + b - \kappa + [g\beta + h(1-\beta)]S^0 + hD^0}{\delta}\right]$$

$$\underline{X} = \frac{-\nu}{1-\nu}\frac{\delta - \mu}{\theta}\left[F + \frac{c + b - \kappa + [g\beta + h(1-\beta)]S^0 + hD^0}{\delta}\right]$$

$$= \frac{-\nu}{1-\nu}\frac{\delta - \mu}{\theta}\left[F + \frac{c + rU + rV}{\delta}\right] \qquad (A5)$$

The last line gives (26) in Proposition 1. In equilibrium, by free-entry condition of the firms, $V = 0$, bringing the free-entry condition to (25) and (26) yields (27) and (28) in Proposition 1. ||

A3 Proof of Proposition 2

A3.1 First order condition

The optimal coupon rate c solves the constrained maximization problem defined by (29). First, I solve several quantities that will facilitate the calculation of the first order condition. The derivative of $\underline{X}$ with respect to c, $\underline{X}^{(c)}$, from (28), is

$$\underline{X}^{(c)} = \frac{-\nu}{1-\nu}\frac{\delta - \mu}{\theta}\frac{1}{\delta} \qquad (A6)$$

It is convenient to calculate some quantities that I use repeatedly in this appendix. From (28),

$$\theta\Pi(\underline{X}) = \theta\frac{\underline{X}}{\delta - \mu} = \frac{\theta}{\delta - \mu}\frac{-\nu}{1-\nu}\frac{\delta - \mu}{\theta}\frac{\delta F + c + rU}{\delta} = \frac{-\nu}{1-\nu}\frac{\delta F + c + rU}{\delta} \qquad (A7)$$

$$\theta\Pi(\underline{X}) - \frac{\delta F + c + rU}{\delta} = \frac{-\nu}{1-\nu}\frac{\delta F + c + rU}{\delta} - \frac{\delta F + c + rU}{\delta} = -\frac{\delta F + c + rU}{(1-\nu)\delta} \qquad (A8)$$

From (28) and (A6),

$$\frac{\partial}{\partial c}\left(\frac{X}{\underline{X}}\right)^{\nu}=\left(\frac{X}{\underline{X}}\right)^{\nu}(-\nu)\frac{1-\nu}{-\nu}\frac{\theta}{\delta-\mu}\frac{\delta}{\delta F+c+rU}\frac{-\nu}{1-\nu}\frac{\delta-\mu}{\theta}\frac{1}{\delta}=\left(\frac{X}{\underline{X}}\right)^{\nu}\frac{-\nu}{\delta F+c+rU} \quad \text{(A9)}$$

From (27) and (A8), the matching surplus function $S(X)$ becomes

$$S(X)=\theta\Pi(X)-F-\frac{c+rU}{\delta}+\frac{\delta F+c+rU}{(1-\nu)\delta}\left(\frac{X}{\underline{X}}\right)^{\nu} \quad (A10)$$

and from (A9) and (A10), in equilibrium, its derivative with respect to c is

$$\begin{aligned}S^{(c)}(X)&=-\frac{1}{\delta}+\frac{\delta F+c+rU}{(1-\nu)\delta}\left(\frac{X}{\underline{X}}\right)^{\nu}\frac{-\nu}{\delta F+c+rU}+\frac{1}{(1-\nu)\delta}\left(\frac{X}{\underline{X}}\right)^{\nu}\\&=-\frac{1}{\delta}+\frac{1}{\delta}\left(\frac{X}{\underline{X}}\right)^{\nu}=-\frac{1}{\delta}\left(1-\left(\frac{X}{\underline{X}}\right)^{\nu}\right) \quad (A11)\end{aligned}$$

From (3) and (A8), the value of debt contract $D(X)$ is

$$\begin{aligned}D(X)&=\frac{c}{\delta}-\left(\frac{c}{\delta}-(1-\alpha)\left(\theta\Pi(\underline{X})-F-\frac{rU}{\delta}\right)\right)\left(\frac{X}{\underline{X}}\right)^{\nu}\\&=\frac{c}{\delta}-\left(\frac{\alpha c}{\delta}-(1-\alpha)\left(\theta\Pi(\underline{X})-\frac{\delta F+c+rU}{\delta}\right)\right)\left(\frac{X}{\underline{X}}\right)^{\nu}\\&=\frac{c}{\delta}-\left(\frac{\alpha c}{\delta}+(1-\alpha)\frac{\delta F+c+rU}{(1-\nu)\delta}\right)\left(\frac{X}{\underline{X}}\right)^{\nu} \quad (A12)\end{aligned}$$

Bringing (A9) into (A12), the derivative of $D_k(X)$ with respect to c_k, $D_k^{(c)}(X)$, is

$$\begin{aligned}D^{(c)}(X)&=\frac{1}{\delta}-\left(\frac{\alpha}{\delta}+\frac{1-\alpha}{(1-\nu)\delta}\right)\left(\frac{X}{\underline{X}}\right)^{\nu}-\left(\frac{\alpha c}{\delta}+(1-\alpha)\frac{\delta F+c+rU}{(1-\nu)\delta}\right)\frac{-\nu}{\delta F+c+rU}\left(\frac{X}{\underline{X}}\right)^{\nu}\\&=\frac{1}{\delta}-\frac{1}{\delta}\left(\alpha+\frac{1-\alpha}{1-\nu}-\frac{\nu(1-\alpha)}{1-\nu}\right)\left(\frac{X}{\underline{X}}\right)^{\nu}+\frac{\alpha\nu}{\delta}\frac{c}{\delta F+c+rU}\left(\frac{X}{\underline{X}}\right)^{\nu}\\&=\frac{1}{\delta}-\frac{1}{\delta}\left(\frac{X}{\underline{X}}\right)^{\nu}+\frac{\alpha\nu}{\delta}\frac{c}{\delta F+c+rU}\left(\frac{X}{\underline{X}}\right)^{\nu}\\&=\frac{1}{\delta}\left[1-\left(1-\alpha\nu\frac{c}{\delta F+c+rU}\right)\left(\frac{X}{\underline{X}}\right)^{\nu}\right] \quad (A13)\end{aligned}$$

From (29), the first order condition of the maximization problem (29) with respect to c is

$$h(\epsilon(U,c))\left[(1-\beta)S^{0\,(c)}+D^{0\,(c)}\right]+h^{(c)}(\epsilon(U,c))\left[(1-\beta)S^{0}+D^{0}\right]=0 \quad (A14)$$

where S^0, D^0, $S^{0(c)}$ and $D^{0(c)}$ is specified by (A10), (A12), (A11), and (A13), respectively, with $X = X_0$, and $h\big(\epsilon(U,c)\big)$ is such that $\epsilon h(\epsilon) = g(\epsilon) = \frac{rU-b}{\beta S^0}$. Simple algebraic manipulation gives (30).

From Section 2.3, $h = h(\epsilon) = h\big(g^{-1}(g)\big) = h(g)$, where $h'(g) < 0$. Differentiating (10) with respect to c on both sides[54], I have

$$g^{(c)}\beta S^0 + g\beta S^{0\,(c)} = 0 \qquad (A15)$$

thereby proving the first order condition in Proposition 2. ||

A3.2 The second order condition for optimal coupon rate c

It is obvious that the objective function in (29) is continuously differentiable, and the set of viable coupon rate c is a closed interval in R^+, i.e., $c \in [0, \bar{c}]$, which is closed and bounded. Obviously, one candidate of $\bar{c}$ is value of c such that $\underline{X} = X_0$. In other words, default occurs immediately after matching. Therefore, the maximization problem is well defined and a maximizer c exists.

The second order derivative on the left hand side of (30) with respect to c at the optimal c is

$$\begin{aligned} h&\left[\frac{1}{\delta}\left(\alpha\nu\frac{\delta F + rU - \nu c}{(\delta F + c + rU)^2} + \beta\frac{\nu}{\delta F + c + rU}\right)\left(\frac{X_0}{\underline{X}}\right)^{\nu}\right] \\ &+2h^{(g)}g^{(c)}\left[\frac{\beta}{\delta} + \frac{1}{\delta}\left(\alpha\nu\frac{c}{\delta F + c + rU} - \beta\right)\left(\frac{X_0}{\underline{X}}\right)^{\nu}\right] \\ &+\left[h^{(g)}g^{(cc)} + h^{(gg)}\left(g^{(c)}\right)^2\right][(1-\beta)S^0 + D^0] \end{aligned} \qquad (A16)$$

Notice that the first two terms of (A16) are obviously negative. Take the second-order derivative of $g\big(\epsilon(c,U)\big)$ with respect to c:

$$g^{(cc)} = -g^{(c)}\frac{S^{0\,(c)}}{S^0} - g\frac{S^{0\,(cc)}S^0 - (S^{0(c)})^2}{(S^0)^2}$$

[54] Notice that in equilibrium, U only depends on the aggregate debt level.

The first term of $g^{(cc)}$ is greater than zero. In order to have $g^{(cc)} > 0$, the second term must be smaller than zero[55], i.e., $\frac{S^{0\,(c)}}{S^0}$ is decreasing in c. To ensure $h^{(gg)}(g^{(c)})^2 < 0$, I need $h^{(gg)} < 0$. In sum, I have proved Proposition 2. ||

A4 Proof of Proposition 3

Again, the homogenous part of ODE (32) is a Cauchy-Euler equation, which admits a general form of solution:

$$T_{HG}(X) = HX^{\rho} + \widetilde{H}X^{\tilde{\rho}}$$

where ρ and $\tilde{\rho}$ is negative and positive solution to $\frac{1}{2}\sigma^2\bar{\rho}(\bar{\rho}-1) + \mu\bar{\rho} - s = 0$, respectively.

$$\rho = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2s}{\sigma^2}} \quad \text{and} \quad \tilde{\rho} = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2s}{\sigma^2}}$$

Therefore, the general solution of the boundary problem (32) and (33) takes the form

$$T(X) = HX^{\rho} + \widetilde{H}X^{\tilde{\rho}} + \frac{1}{s} \qquad (A17)$$

Since $\lim_{X\to\infty} T(X) = \frac{1}{s} < \infty$, I have $\widetilde{H} = 0$. H can be determined by another boundary condition $T(\underline{X}) = 0$.

$$H\underline{X}^{\rho} + \frac{1}{s} = 0 \Rightarrow H = -\frac{1}{s}\,\underline{X}^{-\rho} \qquad (A18)$$

Bringing (A18) to (A17) leads to (34). ||

A5 Proof of Proposition 4

From Gabaix (2009), the general solution of (36) is

$$f^{i}(X) = \zeta_{-}^{i} X^{-m_0-1} + \zeta_{+}^{i} X^{-m_1-1} \quad X \neq X_0 \qquad (A19)$$

where $i \in \{0,1\}$, in which $f^0(X)$ represents the probability density function for $X \in [\underline{X}, X_0)$ and $f^1(X)$ represents the probability density function for $X \in (X_0, \infty]$. In (A19), m_0 and m_1 is

[55] Notice that $h^{(g)} < 0$ by assumption.

negative and positive root for the equation $\frac{\sigma^2}{2}m(m-1)+\mu m - s = 0$, respectively, from which I get the expressions for m_0 and m_1 in Proposition 4.

By the definition of probability density function, the expression $\int_{\underline{X}}^{\infty} \not{f}(X)dX$ must be integrable. From which, I have:

$$\int_{\underline{X}}^{\infty} \not{f}(X)dX = \not{f}(X_0) + \int_{\underline{X}}^{X_0} \not{f}^0(X)dX + \int_{X_0}^{\infty} \not{f}^1(X)dX < \infty \qquad (A20)$$

From (A19), I have $\zeta_-^1 = 0$, otherwise $\int_{X_0}^{\infty} \not{f}^1(X)dX$ explodes as $X \to \infty$[56].

From (37), I have $\zeta_-^0 \underline{X}^{-m_0-1} + \zeta_+^0 \underline{X}^{-m_1-1} = 0$, i.e.,

$$\begin{aligned} \zeta_+^0 &= -\zeta_-^0 \underline{X}^{-m_0-1} \underline{X}^{m_1+1} \\ &= -\zeta_-^0 \underline{X}^{m_1-m_0} \end{aligned}$$

The two remaining unknown coefficients ζ_+^1 and ζ_-^0 are determined by (38) and (39). Letting $\zeta \coloneqq \zeta_+^1$ and $\tilde{\zeta} \coloneqq \zeta_-^0$, we have (40) in Proposition 4. In what following, I will solve ζ and $\tilde{\zeta}$ explicitly. The solution procedure consists of several steps.

Firstly, using (40), I obtain the left and right derivatives of $f(X)$ with respected to X at $X = X_0$.

$$f'(X_0-) = -\tilde{\zeta}(m_0+1)X_0^{-m_0-2} + \tilde{\zeta}(m_1+1)X_0^{-m_0-2}\left(\frac{\underline{X}}{X_0}\right)^{m_1-m_0} \qquad (A21)$$

$$f'(X_0+) = -\zeta(m_1+1)X_0^{-m_1-2} \qquad (A22)$$

Secondly, using (40), I obtain the integral of $f(X)$ over the domain of viable X.

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[56] $\int_{X_0}^{\infty} \not{f}^1(X)dX = -\frac{\zeta_-^1}{m_-}X^{-m_-}\,|_{X_0}^{\infty} - \frac{\zeta_+^1}{m_+}X^{-m_+}\,|_{X_0}^{\infty}$, from which I could see that if $\zeta_-^1 \neq 0$, the first term explodes as $X \to \infty$.

$$\int_{\underline{X}}^{X_0} f(X)dX = \int_{\underline{X}}^{X_0} \left(\tilde{\zeta} X^{-m_0-1} - \tilde{\zeta} \underline{X}^{m_1-m_0} X^{-m_1-1}\right) dX$$

$$= \tilde{\zeta}\left(-\frac{1}{m_0} X^{-m_0} + \underline{X}^{m_1-m_0} \frac{1}{m_1} X^{-m_1}\right)|_{\underline{X}}^{X_0}$$

$$= \tilde{\zeta}\left[\frac{1}{m_1}\underline{X}^{-m_0}\left(\left(\frac{\underline{X}}{X_0}\right)^{m_1} - 1\right) + \frac{1}{m_0}\left(\underline{X}^{-m_0} - X_0^{-m_0}\right)\right] \qquad (A23)$$

Similarly,

$$\int_{X_0}^{\infty} f(X)dX = \int_{X_0}^{\infty} \zeta X^{-m_1-1} dX = \zeta\left(-\frac{1}{m_1}\right) X^{-m_1}|_{X_0}^{\infty} = \frac{\zeta}{m_1} X_0^{-m_1} \qquad (A24)$$

Thirdly, I obtain the right derivatives of $f(X)$ with respected to X at $X = \underline{X}$.

$$f'\left(\underline{X}+\right) = \tilde{\zeta}(m_1 - m_0)\underline{X}^{-m_0-2} \qquad (A25)$$

Fourthly, I express boundary conditions (38) and (39) from (A21) — (A25). (38) becomes

$$\tilde{\zeta}\Lambda_1 - \zeta\Lambda_2 = s\tilde{\zeta}\Lambda_3 + s\zeta\Lambda_4 + \tilde{\zeta}\Lambda_5$$

$$\Leftrightarrow (\Lambda_1 - s\Lambda_3 - \Lambda_5)\tilde{\zeta} - (\Lambda_2 + s\Lambda_4)\zeta = 0 \qquad (A26)$$

and (39) becomes

$$g - g\left(\tilde{\zeta}\Lambda_3 + \zeta\Lambda_4\right) = s\tilde{\zeta}\Lambda_3 + s\zeta\Lambda_4 + \tilde{\zeta}\Lambda_5$$

$$\Leftrightarrow [(g+s)\Lambda_3 + \Lambda_5]\tilde{\zeta} + (g+s)\Lambda_4\zeta = g \qquad (A27)$$

The parameters $\Lambda_i, i = \{1, 2, 3, 4, 5\}$ are defined as follows:

$$\Lambda_1 = \frac{1}{2}\sigma^2\left[-(m_0+1) + (m_1+1)\left(\frac{\underline{X}}{X_0}\right)^{m_1-m_0}\right] X_0^{-m_0} \qquad (A28)$$

$$\Lambda_2 = -\frac{1}{2}\sigma^2(m_1+1)X_0^{-m_1} \qquad (A29)$$

$$\Lambda_3 = \int_{\underline{X}}^{X_0} X^{-m_0-1}\left[1 - \left(\frac{\underline{X}}{X}\right)^{m_1-m_0}\right] dX = \frac{1}{m_1}\underline{X}^{-m_0}\left(\left(\frac{\underline{X}}{X_0}\right)^{m_1} - 1\right) + \frac{1}{m_0}\left(\underline{X}^{-m_0} - X_0^{-m_0}\right) (A30)$$

$$\Lambda_4 = \int_{X_0}^{\infty} X^{-m_1-1} dX = \frac{1}{m_1} X_0^{-m_1} \qquad (A31)$$

$$\Lambda_5 = \frac{\sigma^2}{2}(m_1 - m_0)\underline{X}^{-m_0} \qquad (A32)$$

(A26) and (A27) pin down ζ and $\tilde{\zeta}$. It can be shown that $f(X_0+) = f(X_0-)$, in other words, $f(X)$ is continuous at X_0. Therefore, ζ and $\tilde{\zeta}$ are positive and unique. ||

A6 Proof of Proposition 5

A6.1 First order condition

The proof of the first order condition (46) is the same as the proof of first order condition (30) in the baseline case. Therefore, I focus on the solutions of $g^{(c)}$ and $e^{(c)}$ in this appendix section.

Taking the derivative with respect to c on both sides of (43), notice that U only depends on c through e, by the envelope theorem

$$g\beta S^{0(c)} + g^{(c)}\beta S^0 - l'(e)e^{(c)} = 0 \tag{A33}$$

Taking the derivative with respect to c on both sides of (44),

$$g^{(e)}\beta S^{0(c)} + \left(g^{(ee)}e^{(c)} + g^{(eg)}g^{(c)}\right)\beta S^0 - l''(e)e^{(c)} = 0$$

Collecting terms yields

$$g^{(e)}\beta S^{0(c)} + g^{(eg)}\beta S^0 g^{(c)} + \left(g^{(ee)}\beta S^0 - l''(e)\right)e^{(c)} = 0 \tag{A34}$$

Solving $g^{(c)}$ and $e^{(c)}$ from (A33) and (A34) we have

$$g^{(c)} = \frac{\Gamma_3\Gamma_5 - \Gamma_2\Gamma_6}{\Gamma_1\Gamma_5 - \Gamma_2\Gamma_4} \tag{A35}$$

and

$$e^{(c)} = \frac{\Gamma_1\Gamma_6 - \Gamma_3\Gamma_4}{\Gamma_1\Gamma_5 - \Gamma_2\Gamma_4} \tag{A36}$$

where $\Gamma_1 = \beta S^0$, $\Gamma_2 = -l'(e)$, $\Gamma_3 = -g\beta S^{0(c)}$, $\Gamma_4 = g^{(eg)}\beta S^0$, $\Gamma_5 = g^{(ee)}\beta S^0 - l''(e)$ and $\Gamma_6 = -g^{(e)}\beta S^{0(c)}$. ||

A6.2 Second order condition

The first term of the second order condition is similar to that of (A16), except for the fact that h now also depends on e in addition to g. The second term of the second order condition is similar to that of (A16) except that $h^{(g)}g^{(c)}$ is replaced by $h^{(e)}e^{(c)} + h^{(g)}g^{(c)}$. The first two terms are smaller than zero. The third term of the second order condition is the same as that of (A16) except that $h^{(g)}g^{(cc)} + h^{(gg)}\left(g^{(c)}\right)^2$ is replaced by

$$\begin{aligned} h^{(cc)}(e,g) &= h^{(ee)}\left(e^{(c)}\right)^2 + h^{(eg)}g^{(c)}e^{(c)} + h^{(e)}e^{(cc)} + h^{(gg)}\left(g^{(c)}\right)^2 + h^{(ge)}e^{(c)}g^{(c)} + h^{(g)}g^{(cc)} \\ &= h^{(ee)}\left(e^{(c)}\right)^2 + h^{(gg)}\left(g^{(c)}\right)^2 + 2h^{(eg)}e^{(c)}g^{(c)} + h^{(e)}e^{(cc)} + h^{(g)}g^{(cc)} \end{aligned} \qquad (A37)$$

The third term is smaller than zero if $h^{(cc)}(e,g) < 0$. The remaining part of this subsection calculates $g^{(cc)}$ and $e^{(cc)}$.

Take derivative with respect to c on both sides of (A33)

$$g^{(c)}\beta S^{0(c)} + g\beta S^{0(cc)} + g^{(cc)}\beta S^0 + g^{(c)}\beta S^{0(c)} - l''(e)\left(e^{(c)}\right)^2 - l'(e)e^{(cc)} = 0$$

Collecting terms yields

$$-\beta S^0 g^{(cc)} + l'(e)e^{(cc)} = g\beta S^{0(cc)} + 2g^{(c)}\beta S^{0(c)} - l''(e)\left(e^{(c)}\right)^2 \qquad (A38)$$

Take derivative with respect to c on both sides of (A34)

$$\begin{gathered} g^{(e)}\beta S^{0(cc)} + \left(g^{(ee)}e^{(c)} + g^{(eg)}g^{(c)}\right)\beta S^{0(c)} + g^{(eg)}g^{(c)}\beta S^{0(c)} + g^{(eg)}g^{(cc)}\beta S^0 + \\ \left(g^{(ege)}e^{(c)} + g^{(egg)}g^{(c)}\right)\beta S^0 g^{(c)} + \\ \left(g^{(eee)}e^{(c)}\beta S^0 + g^{(eeg)}g^{(c)}\beta S^0 + g^{(ee)}\beta S^{0(c)} - l'''(e)e^{(c)}\right)e^{(c)} + \\ \left(g^{(ee)}\beta S^0 - l''(e)\right)e^{(cc)} = 0 \end{gathered}$$

Collecting terms yields

$$\begin{gathered} -g^{(eg)}\beta S^0 g^{(cc)} + \left(l''(e) - g^{(ee)}\beta S^0\right)e^{(cc)} = \\ g^{(e)}\beta S^{0(cc)} + 2\left(g^{(ee)}e^{(c)} + g^{(eg)}g^{(c)}\right)\beta S^{0(c)} + \left(g^{(ege)}e^{(c)} + g^{(egg)}g^{(c)}\right)\beta S^0 g^{(c)} + \\ \left(g^{(eee)}e^{(c)}\beta S^0 + g^{(eeg)}g^{(c)}\beta S^0 - l'''(e)e^{(c)}\right)e^{(c)} \end{gathered} \qquad (A39)$$

Solving $g^{(cc)}$ and $e^{(cc)}$ from (A38) and (A39) we have

$$g^{(cc)} = \frac{\Delta_3\Delta_5 - \Delta_2\Delta_6}{\Delta_1\Delta_5 - \Delta_2\Delta_4} \qquad (A40)$$

$$e^{(cc)} = \frac{\Delta_1\Delta_6 - \Delta_3\Delta_4}{\Delta_1\Delta_5 - \Delta_2\Delta_4} \qquad (A41)$$

where $\Delta_1 = -\beta S^0$, $\Delta_2 = l'(e)$, $\Delta_3 = RHS\ of$ (A38), $\Delta_4 = -g^{(eg)}\beta S^0$, $\Delta_5 = l''(e) - g^{(ee)}\beta S^0$ and $\Delta_6 = RHS\ of$ (A39). ||

A7 Numerical example

Table 1 Model parameters for baseline calibration

Parameters	Interpretation	Values	Reference
β	Worker's bargaining power	0.75	Shimer (2005)
α	Bankruptcy cost	0.5	Leland (1994)
b	Unemployment benefit	0.4	Shimer (2005)
s	Exogenous separation rate	0.15	Monacelli, Quadrini and Trigari (2011)
κ	Cost of maintaining a vacancy	0.4	Moen and Rosen (2011)
X_0	Initial cash flow state	1	
θ	Productivity	1	
r	Interest rate	0.05	Brunnermeier Sannikov (2014)
μ	Drift	-0.022	He and Milbradt (2014)
σ	Volatility	0.25	He and Milbradt (2014)
f	Flow operational cost	0	
F	Present value of f	0	
ι	Matching elasticity on u	0.3	Petrongolo and Pissarides (2001)
A	Matching efficiency	4	Mortensen and Pissarides (1994)

A7.1 An illustration of solution procedure

Firstly, the optimal coupon rate c, value of being unemployed U and matching rate of workers g are jointly determined by the worker's and firm's value function (20) and (21), respectively and the first order condition for firm's optimal coupon choice (30). After checking the second order condition for the optimality of coupon choice obtained from the first step, the expected tenure is derived according to (34) of Proposition 3. The stationary cross-sectional density function of cash flow state X, $\oint(X)$, is derived according to (40) of Proposition 4. The unemployment rate $u = 1 - \int_{\underline{X}}^{\infty} \oint(X)dx$.

Appendix B Bayesian learning about the unknown match quality $\boldsymbol{\mu}$

B1 Asset values and Wage function

Let $\Sigma(p) \coloneqq \frac{1}{2}p^2(1-p)^2\phi^2$ and $\bar{\mu}(p) \coloneqq p\mu_H + (1-p)\mu_L$. Throughout Section 4 and Appendix B, I assume that the flow operating cost $f = 0$[57].

[57] This assumption is innocuous.

B1.1 Debt

Given a coupon rate c and value of being unemployed U, the bellman equation for a debt contract is:

$$rD(p) = c + \Sigma(p)D''(p) - sD(p) \tag{B1}$$

subject to the boundary conditions[58]:

$$D\left(\underline{p}\right) = D_B = (1-\alpha)\frac{\bar{\mu}\left(\underline{p}\right)}{\delta} - \frac{rU}{\delta} \text{ and } \lim_{p\to 1} D(p) < \frac{\mu_H}{\delta} < \infty \tag{B2}$$

From Polyanin and Zaitsev (2003, 2.1.7 — 6, equation 216), the homogeneous part of (B1) has a general solution of the form:

$$\gamma_1 p^{\widehat{m}_1}(1-p)^{\widehat{m}_2} + \gamma_2 p^{\widehat{m}_2}(1-p)^{\widehat{m}_1}$$

where $\widehat{m}_1$ and $\widehat{m}_2$ is negative and positive root of the equation $\widehat{m}(\widehat{m}-1) - \frac{2\delta}{\phi^2} = 0$.

$$\widehat{m}_1 = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2\delta}{\phi^2}} \text{ and } \widehat{m}_2 = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2\delta}{\phi^2}} \tag{B3}$$

The general solution for (B1) is

$$D(p) = \frac{c}{\delta} + \gamma_1 p^{\widehat{m}_1}(1-p)^{\widehat{m}_2} + \gamma_2 p^{\widehat{m}_2}(1-p)^{\widehat{m}_1}$$

Since $\lim_{p\to 1} D(p) < \frac{\mu_H}{\delta} < \infty$, I have $\gamma_2 = 0$. Let $\gamma \coloneqq \gamma_1$. Using $D\left(\underline{p}\right) = D_B = (1-\alpha)\frac{\bar{\mu}\left(\underline{p}\right)}{\delta} - \frac{rU}{\delta}$, I have $\gamma = \frac{(1-\alpha)\bar{\mu}\left(\underline{p}\right) - rU - c}{\delta \underline{p}^{\widehat{m}_1}\left(1-\underline{p}\right)^{\widehat{m}_2}}$. Then the debt value $D(p)$ is given by

$$D(p) = \frac{c}{\delta} - \frac{c - (1-\alpha)\bar{\mu}\left(\underline{p}\right) + rU}{\delta}\left(\frac{p}{\underline{p}}\right)^{\widehat{m}_1}\left(\frac{1-p}{1-\underline{p}}\right)^{\widehat{m}_2} \tag{B4}$$

||

B1.2 Other asset values, wage function and match surplus

The equity value follows the HJB equation

[58] The bankruptcy cost definition is slight different from the benchmark case to make the calculation less cumbersome. It is not crucial.

$$rE(p) = \bar{\mu}(p) - c - w + \Sigma(p)E''(p) - s(E(p) - V) \qquad (B5)$$

subject to boundary conditions

$$E\left(\underline{p}\right) = V;\ E'(p)|_{p=\underline{p}} = 0;\ \lim_{p\to 1} E(p) < \infty \qquad (B6)$$

Similarly, the value of being employed satisfies the HJB equation

$$rW(p) = w + \Sigma(p)W''(p) - s(W(p) - U) \qquad (B7)$$

subject to boundary conditions

$$W\left(\underline{p}\right) = U;\ W'(p)|_{p=\underline{p}} = 0;\ \lim_{p\to 1} W(p) < \infty \qquad (B8)$$

The value of being unemployed satisfies the HJB equation

$$rU = b + g(\epsilon)[W(p_0) - U] \qquad (B9)$$

The value of an idle vacancy obeys the HJB equation

$$rV = -\kappa + h(\epsilon)[E(p_0) + D(p_0) - V] \qquad (B10)$$

Before I move on to calculate the matching surplus $S(p)$, I derive a wage function that is linear in posterior belief p. Specifically, I have the following lemma.

Lemma B1 In equilibrium, under generalized Nash bargaining, the wage function is linear in X

$$\begin{aligned} w(p) &= \beta(\bar{\mu}(p) - c) + (1-\beta)b + \beta g(\epsilon)E(p_0) \\ &= \beta(\bar{\mu}(p) - c) + (1-\beta)b + (1-\beta)g(\epsilon)[W(p_0) - U] \end{aligned} \qquad (B11)$$

Proof: Similar to Lemma 1. ||

By the virtue of continuous generalized Nash bargaining solution, I have

$$\beta E(p) = (1-\beta)[W(p) - U] \qquad (B12)$$

Taking derivatives of both sides of (B12) with respect to p. I have:

$$\beta E'(p) = (1-\beta)W'(p) \qquad (B13)$$

and

$$\beta E''(p) = (1-\beta)W''(p) \qquad (B14)$$

Again I have proved that the worker and the firm of a given matching pair agree to separate the matching relationship and return to search when p hits the same threshold, i.e., $\underline{p}^E = \underline{p}^W := \underline{p}$. ||

B1.3 Solve for the matching surplus $S(p)$, $\underline{p}$ and c

Similar to Section 3, I define matching surplus $S \coloneqq E + W - V - U$. Then by Nash bargaining:

$$E - V = (1-\beta)S$$

and

$$W - U = \beta S$$

Denote $D^0 \coloneqq D(p_0)$, $S^0 \coloneqq S(p_0)$, $g \coloneqq g(\epsilon)$ and $h \coloneqq h(\epsilon)$. The value function of being unemployed (B9) can be expressed in terms of S:

$$rU = b + g\beta S^0 \tag{B15}$$

Similarly, the value function of an idled vacancy becomes

$$rV = -\kappa + h[(1-\beta)S^0 + D^0] \tag{B16}$$

By the definition of S, the HJB equation for S is as follows:

$$\delta S(p) = \bar{\mu}(p) - c - b + \kappa - [g\beta + h(1-\beta)]S^0 - hD^0 + \Sigma(p)S''(p) \tag{B17}$$

with boundary conditions:

$$S\left(\underline{p}\right) = 0;\ S'(p)|_{p=\underline{p}} = 0;\ \lim_{p \to 1} S(p) < \infty \tag{B18}$$

Similar to derivation of debt value function, I have

$$S(p) = \frac{\bar{\mu}(p) - c - b + \kappa - \left(g\beta + h(1-\beta)\right)S^0 - hD^0}{\delta} - \frac{\bar{\mu}\left(\underline{p}\right) - c - b + \kappa - \left(g\beta + h(1-\beta)\right)S^0 - hD^0}{\delta}\left(\frac{p}{\underline{p}}\right)^{\widehat{m}_1}\left(\frac{1-p}{1-\underline{p}}\right)^{\widehat{m}_2} \tag{B19}$$

where $\widehat{m}_1$ and $\widehat{m}_2$ are the same as those in (B3).

The optimal separation threshold $\underline{p}$ is determined by smooth-pasting condition: $S'(p)|_{p=\underline{p}} = 0$

$$\frac{\mu_H - \mu_L}{\delta} - \frac{\bar{\mu}\left(\underline{p}\right) - c - b + \kappa - \left(g\beta + h(1-\beta)\right)S^0 - hD^0}{\delta}\left(\frac{\widehat{m}_1}{\underline{p}} - \frac{\widehat{m}_2}{1-\underline{p}}\right) = 0 \qquad (B20)$$

Using the fact that $\widehat{m}_1 + \widehat{m}_2 = 1$ and denoting $\Delta \coloneqq c + b - \kappa + \left(g\beta + h(1-\beta)\right)S^0 + hD^0$ give rise to the expression for optimal separation threshold $\underline{p}$.

$$\underline{p} = \frac{(\mu_L - \Delta)\widehat{m}_1}{\widehat{m}_2\mu_H + \widehat{m}_1\mu_L - \Delta} \qquad (B21)$$

In equilibrium, by free entry, $V = 0$, then

$$S(p) = \frac{\bar{\mu}(p) - c - rU}{\delta} - \frac{\bar{\mu}\left(\underline{p}\right) - c - rU}{\delta}\left(\frac{p}{\underline{p}}\right)^{\widehat{m}_1}\left(\frac{1-p}{1-\underline{p}}\right)^{\widehat{m}_2} \qquad (B22)$$

and the optimal separation threshold $\underline{p}$ is

$$\underline{p} = \frac{(\mu_L - c - rU)\widehat{m}_1}{\widehat{m}_2\mu_H + \widehat{m}_1\mu_L - c - rU} \qquad (B23)$$

$$\underline{p}^{(c)} = \frac{-\widehat{m}_1\widehat{m}_2(\mu_H - \mu_L)}{(\widehat{m}_2\mu_H + \widehat{m}_1\mu_L - c - rU)^2} > 0 \qquad (B24)$$

$$(1-\beta)S^0 + D^0 =$$
$$(1-\beta)\frac{\bar{\mu}(p_0) - rU}{\delta} + \frac{\beta c}{\delta} - \frac{(\alpha - \beta)\bar{\mu}\left(\underline{p}\right) + \beta(c + rU)}{\delta}\left(\frac{p_0}{\underline{p}}\right)^{\widehat{m}_1}\left(\frac{1-p_0}{1-\underline{p}}\right)^{\widehat{m}_2} \qquad (B25)$$

$$\frac{\partial}{\partial c}[(1-\beta)S^0 + D^0] = \frac{\beta}{\delta} - \frac{\beta + (\alpha-\beta)(\mu_H - \mu_L)\underline{p}^{(c)}}{\delta}\left(\frac{p_0}{\underline{p}}\right)^{\widehat{m}_1}\left(\frac{1-p_0}{1-\underline{p}}\right)^{\widehat{m}_2} -$$
$$\frac{(\alpha - \beta)\bar{\mu}\left(\underline{p}\right) + \beta(c + rU)}{\delta}\left(\frac{p_0}{\underline{p}}\right)^{\widehat{m}_1}\left(\frac{1-p_0}{1-\underline{p}}\right)^{\widehat{m}_2}\left(\frac{\widehat{m}_2}{1-\underline{p}} - \frac{\widehat{m}_1}{\underline{p}}\right)\underline{p}^{(c)} \qquad (B26)$$

The first-order condition of the optimal c is

$$h\left(\epsilon(U,c)\right)\left[(1-\beta)S^{0\,(c)} + D^{0\,(c)}\right] + h^{(c)}\left(\epsilon(U,c)\right)[(1-\beta)S^0 + D^0] = 0 \qquad (B27)$$

where the $(1-\beta)S^{0\,(c)} + D^{0\,(c)}$ is defined in (B26), and $(1-\beta)S^0 + D^0$ is defined in (B25), and

$$h^{(c)}\left(\epsilon(U,c)\right) = h^{(g)}g^{(c)} < 0 \; where \; g^{(c)} = -g\frac{S^{0\,(c)}}{S^0} > 0 \qquad (B28)$$

||

B2 Expected job tenure

Extending the baseline model to allow for Bayesian learning about the matching quality does not compromise the closed-form representation of the expected job tenure, i.e., the expected match duration.

Specifically, let $\hat{T}(p)$ be the expected remaining duration of a match when current posterior about matching quality is p. $\hat{T}(p)$ solves the following boundary value problem:

$$\Sigma(p)\hat{T}''(p) - s\hat{T}(p) = -1 \qquad (B29)$$

The boundary conditions are:

$$\hat{T}\left(\underline{p}\right) = 0 \; and \; \lim_{p\to 1}\hat{T}(p) = \frac{1}{s} \qquad (B30)$$

Intuitively, the remaining tenure is zero if p hits the separation threshold, $\underline{p}$. Meanwhile, as both parties are certain that the match quality is high, only event that could end the match is the exogenous matching destruction event, with arrival intensity s.

Again, the homogenous part of (B29) admits a general form of solution:

$$\gamma_1 p^{\tau_1}(1-p)^{\tau_2} + \gamma_2 p^{\tau_2}(1-p)^{\tau_1} \qquad (B31)$$

where τ_1 and τ_2 is negative and positive root of the equation $\tau(\tau-1) - \frac{2s}{\phi^2} = 0$, respectively.

$$\tau_1 = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2s}{\phi^2}} \; and \; \tau_2 = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2s}{\phi^2}} \qquad (B32)$$

Therefore, the general solution of the boundary problem (B29) and (B30) takes the form

$$\hat{T}(p) = \gamma_1 p^{\tau_1}(1-p)^{\tau_2} + \gamma_2 p^{\tau_2}(1-p)^{\tau_1} + \frac{1}{s} \qquad (B33)$$

Since $\lim_{X\to 1}\hat{T}(p) = \frac{1}{s} < \infty$, I have $\gamma_2 = 0$. γ_1 can be determined by another boundary condition $\hat{T}(\underline{p}) = 0$.

$$\gamma_1 \underline{p}^{\tau_1}\left(1-\underline{p}\right)^{\tau_2}+\frac{1}{s}=0 \Rightarrow \gamma_1=-\frac{1}{s\underline{p}^{\tau_1}\left(1-\underline{p}\right)^{\tau_2}} \qquad (B34)$$

Bringing (B34) to (B33) leads to

$$\hat{T}(p)=\frac{1}{s}\left[1-\left(\frac{p}{\underline{p}}\right)^{\tau_1}\left(\frac{1-p}{1-\underline{p}}\right)^{\tau_2}\right] \qquad (B35)$$

where τ_1 and τ_2 are given in (B32). It is straightforward that the expect tenure of a job decreases in coupon rate c and increases in the current posterior p. Recall that wage also increases in the current cash flow state p. Again I obtain a positive relationship between job tenure and wage.

B3 Derive the stationary cross-sectional probability density function of p

To close the equilibrium, in this section, I characterize the stationary cross-sectional distribution of the posterior on match-specific quality p in the economy. The stochastic process governing the evolution of the posterior p, is a Brownian motion with respect to the filtration $\{\mathcal{F}_t^X\}$. Obviously, p is a Kolmogorov-Feller diffusion process. Let $\tilde{\hat{f}}(p)$ be the transition probability density function for p in the economy. Similar to Section 3.5.5, the Fokker-Planck equation, also known as Kolmogorov forward equation of the posterior p, governs the dynamics of $\tilde{\hat{f}}(p)$.

$$\frac{d\tilde{\hat{f}}(p)}{dt}=\frac{d^2}{dp^2}\left[\Sigma(p)\tilde{\hat{f}}(p)\right]-s\tilde{\hat{f}}(p) \qquad (B36)$$

Let $\hat{f}(p)$ denote the stationary $\tilde{\hat{f}}(p)$, I have the following boundary value problems governing the dynamics of $\hat{f}(p)$:

$$\frac{d^2}{dp^2}\left[\Sigma(p)\hat{f}(p)\right]-s\hat{f}(p)=0 \qquad (B37)$$

with boundary conditions[59]:

$$\hat{f}\left(\underline{p}+\right)=0 \qquad (B38a)$$

[59] $p+:=\lim_{p'\downarrow p} p'$ and $p-:=\lim_{p'\uparrow p} p'$

$$\Sigma(p_0)\left[\hat{f}'(p_0 -) - \hat{f}'(p_0 +)\right] = s\int_{\underline{p}}^{1}\hat{f}(p)dp + \Sigma\left(\underline{p}\right)\hat{f}'\left(\underline{p}+\right) \qquad (B38b)$$

$$g\left[1 - \int_{\underline{p}}^{1}\hat{f}(p)dp\right] = s\int_{\underline{p}}^{1}\hat{f}(p)dp + \Sigma\left(\underline{p}\right)\hat{f}'\left(\underline{p}+\right) \qquad (B38c)$$

As in the appendix A5, define $\hat{f}^i(p),\ i \in \{0,1\}$, in which $\hat{f}^0(p)$ represents the probability density function for $p \in [\underline{p}, p_0)$ and $\hat{f}^1(p)$ represents the probability density function for $p \in (p_0, 1]$. The solution procedure of the boundary problem (B37) and (B38a) — (B38c) is more complicated than the geometric Brownian motion case, due to the fact that a general solution of the homogeneous part of (B37) is difficult to obtain directly. I resort to the following change-of-variable technique.

Specifically, define $\eta^i(p) \coloneqq p^2(1-p)^2\hat{f}^i(p)$, multiplying both sides of (B37) by $p^2(1-p)^2$, and dividing both sides using $\frac{\phi^2}{2}$. (B37) becomes:

$$p^2(1-p)^2\eta''(p) - \frac{2s}{\phi^2}\eta(p) = 0 \qquad (B39)$$

Again from Polyanin and Zaitsev (2003, 2.1.7 — 6, equation 216), a general solution of (B39) takes the form of

$$\eta^i(p) = \xi_1^i p^{\widetilde{m}_1}(1-p)^{\widetilde{m}_2} + \xi_2^i p^{\widetilde{m}_2}(1-p)^{\widetilde{m}_1} \quad p \neq p_0 \qquad (B40)$$

where $\widetilde{m}_1$ and $\widetilde{m}_2$ is negative and positive root for the equation $\frac{\phi^2}{2}\widetilde{m}(\widetilde{m}-1) - s = 0$.

$$\widetilde{m}_1 = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2s}{\phi^2}}\ and\ \widetilde{m}_2 = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2s}{\phi^2}} \qquad (B41)$$

Then, inverting $\hat{f}^i(p)$ from $\eta^i(p)$,

$$\begin{aligned}\hat{f}^i(p) &= \eta^i(p)p^{-2}(1-p)^{-2}\\ &= \xi_1^i p^{\widetilde{m}_1-2}(1-p)^{\widetilde{m}_2-2} + \xi_2^i p^{\widetilde{m}_2-2}(1-p)^{\widetilde{m}_1-2}\\ &= \xi_1^i p^{\widetilde{m}_1-2}(1-p)^{-\widetilde{m}_1-1} + \xi_2^i p^{-\widetilde{m}_1-1}(1-p)^{\widetilde{m}_1-2} \qquad (B42)\end{aligned}$$

The last equality comes from the fact that $\widetilde{m}_1 + \widetilde{m}_2 = 1$. The expression $\int_{\underline{p}}^{1}\hat{f}(p)dp$ must be integrable, i.e.,

$$\int_{\underline{p}}^{1} \hat{f}(p)dp = \hat{f}(p_0) + \int_{\underline{p}}^{p_0} \hat{f}^0(p)dp + \int_{p_0}^{1} \hat{f}^1(p)dp < \infty \qquad (B43)$$

Notice that

$$\int_{p_0}^{1} \hat{f}^1(p)dp = \int_{p_0}^{1} \xi_1^1 p^{\widetilde{m}_1-2}(1-p)^{-\widetilde{m}_1-1}dp + \int_{p_0}^{1} \xi_2^1 p^{-\widetilde{m}_1-1}(1-p)^{\widetilde{m}_1-2}dp$$

Let $\hat{\iota} = min(1, p_0^{-\widetilde{m}_1-1})$, then I have

$$\int_{p_0}^{1} \xi_2^1 p^{-\widetilde{m}_1-1}(1-p)^{\widetilde{m}_1-2}dp > \xi_2^1 \hat{\iota} \int_{p_0}^{1} (1-p)^{\widetilde{m}_1-2}dp = \xi_2^1 \hat{\iota} \frac{(1-p)^{\widetilde{m}_1-1}}{\widetilde{m}_1-1}|_{p_0}^{1} \rightarrow \infty$$

as $p \rightarrow 1$, since $\widetilde{m}_1 < 0$ thus $\widetilde{m}_1 - 1 < -1$. $\int_{p_0}^{1} \hat{f}^1(p)dp < \infty$, therefore $\xi_2^1 = 0$, otherwise $\int_{p_0}^{1} \hat{f}^1(p)dp$ explodes as $p \rightarrow 1$.

On the other hand,

$$\int_{p_0}^{1} \xi_1^1 p^{\widetilde{m}_1-2}(1-p)^{-\widetilde{m}_1-1}dp > \xi_1^1 \int_{p_0}^{1} (1-p)^{-\widetilde{m}_1-1}dp = \xi_1^1 \frac{(1-p)^{-\widetilde{m}_1}}{-\widetilde{m}_1}|_{p_0}^{1} < \infty$$

because $\widetilde{m}_1 < 0$. Therefore ξ_1^1 can be nonzero.

At $p = \underline{p}$, by boundary condition $\hat{f}\left(\underline{p}+\right) = 0$, I have

$$\xi_1^0 \underline{p}^{\widetilde{m}_1-2}\left(1-\underline{p}\right)^{-\widetilde{m}_1-1} + \xi_2^0 \underline{p}^{-\widetilde{m}_1-1}\left(1-\underline{p}\right)^{\widetilde{m}_1-2} = 0$$

$$\xi_2^0 = -\xi_1^0 \left(\frac{\underline{p}}{1-\underline{p}}\right)^{2\widetilde{m}_1-1}$$

The two remaining unknown coefficients ξ_1^0 and ξ_1^1 are determined by (B38b) and (B38c).

Now I present the stationary cross-sectional distribution density function for $p \in \left[\underline{p}, 1\right] \setminus \{p_0\}$ in equilibrium:

$$\hat{f}(p) = \begin{cases} \xi_1^1 p^{\widetilde{m}_1-2}(1-p)^{-\widetilde{m}_1-1}, & p_0 < p \leq 1 \\ \xi_1^0 p^{\widetilde{m}_1-2}(1-p)^{-\widetilde{m}_1-1}\left[1-\left(\frac{\underline{p}}{1-\underline{p}}\frac{1-p}{p}\right)^{2\widetilde{m}_1-1}\right], & \underline{p} \leq p < p_0 \end{cases} \qquad (B44)$$

where $\widetilde{m}_1 = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2s}{\phi^2}}$.

The expression of the stationary cross-sectional density function $\hat{f}(p)$ of posterior p takes the Double-Pareto distribution form, as shown in the stochastic growth literature (e.g., Gabaix, 2009; Achdou, Han, Lasry, Lions and Moll, 2015).

Appendix C Asymmetric information about firm productivity θ

C1 Separating equilibrium

C1.1 Proof of Proposition 6

Let $IV \coloneqq (1-\beta)S^0(c;\theta_k) + D^0(c;\theta_k)$ be the intrinsic value of financial claims. Along any iso-value curve, I have:

$$\omega \frac{\partial M}{\partial c} + (1-\omega)\frac{\partial IV}{\partial c} = 0$$

$$\Rightarrow \frac{\partial M}{\partial c} = -\frac{1-\omega}{\omega}\frac{\partial IV}{\partial c} = -\frac{1-\omega}{\omega}\left[\frac{\beta}{\delta} - \frac{\beta-\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_k}}\right)^{\nu}\right] \quad (C1)$$

where the last equality follows from the fact that Nash bargaining outcome holds for separating equilibrium. Taking the derivative with respect to θ_k on both sides of (C1), I have:

$$\frac{\partial}{\partial\theta_k}\left(\frac{\partial M}{\partial c}\right) = -\frac{1-\omega}{\omega}\left[-\frac{\beta-\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_k}}\right)^{\nu}(-\nu)\frac{1-\nu}{-\nu}\frac{\theta_k}{\delta-\mu}\frac{\delta}{\delta F + c + rU}\frac{-\nu}{1-\nu}\frac{\delta-\mu}{-\theta_k^2}\frac{\delta F + c + rU}{\delta}\right]$$

$$= \frac{1-\omega}{\omega}\frac{\beta-\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_k}}\right)^{\nu}\frac{\nu}{\theta_k} < 0 \quad (C2)$$

since $\nu < 0$. ||

C1.2 Proof of Lemma 2

Notice that the difference between left hand side and right hand side of (IC.L) (59) is continuous in c^s. Consider $c^s = c_L$, i.e., the separating coupon rate is equal to the full-information first best coupon choice of the low-type firms. Under this case, it is obvious that left hand side of (IC.L) (59) is greater than the right-hand side, since the first term on the left hand side is greater than the second term, which is equal to a proportion $1-\omega$ of the right hand side of (IC.L) (59).

At $\bar{c}$ specified in footnote 48, under the parameter restriction, left hand side of (IC.L) (59) is strictly smaller than the right hand side. By continuity, there exists a $c^{s*} \in (c_L, \bar{c})$ such that the left hand side of (IC.L) (59) is equal to the right hand side.

Notice that at c_L, the left hand side of (IC.L) (59) is strictly increasing in c, because the derivative of the first term on the left hand side with respect to c at $c = c_L$ is greater than zero, whereas the derivative of the second term with respect to c at $c = c_L$ is equal to zero, by the optimality of c_L and optimal $c_H > c_L$. Moreover, the left hand side of (IC.L) (59) is concave since it is a linear combination of two concave shareholder value functions. Thus the left hand side of (IC.L) (59) admits a unique maximizing coupon rate, denoted as c^m. Since at c^{s*}, the left hand side of (IC.L) (59) is equal to the right hand side, I must have $c^{s*} > c^m$. Using the fact that the left hand side of (IC.L) (59) decreases in c for $c > c^m$, I have that (IC.L) (59) holds for $\forall c^s \in [c^{s*}, \bar{c}]$. ||

C1.3 Proof of Lemma 3

I focus on case in which $c^{s*} > c_H$, since in this case, the separating equilibrium creates under-employment problem. Notice that the difference between left hand side and right hand side of (IC.H) (60) is continuous in c^s. Consider $c^s = c_H$, i.e., the separating coupon rate is equal to the full-information first best coupon choice of the high-type firm. Under this circumstance, it is obvious that left hand side of (IC.H) (60) is greater than the right-hand side, since the high-type firm cannot fare better than its full-information first best scenario. At $\bar{c}$ specified in footnote 48, left hand side of (IC.H) (60) is strictly smaller than the right hand side. By continuity, there exists a $c^{s**} \in (c_H, \bar{c})$ such that the left hand side of (IC.H) (60) is equal to the right hand side. Notice that the left hand side of (IC.H) (60) is strictly decreasing in c^s for $c^s \in (c_H, \bar{c})$. Therefore, I have that (IC.H) (60) holds for $\forall c^s \in [c_H, c^{s**}]$.

In order to show that the separating equilibrium exits, I must show that $c^{s*} < c^{s**}$. I prove this inequality by demonstrating that c^{s**} satisfies the (IC.L) (59). Notice that at c^{s**}, (IC.H) (60) holds with equality by definition. Therefore (IC.L) (59) at c^{s**} is equivalent to:

$$\omega[(1-\beta)S^0(c_L;\theta_L)+D^0(c_L;\theta_L)]+(1-\omega)[(1-\beta)S^0(c_L;\theta_H)+D^0(c_L;\theta_H)]$$
$$-(1-\omega)[(1-\beta)S^0(c^{s**};\theta_H)+D^0(c^{s**};\theta_H)]+$$
$$(1-\omega)[(1-\beta)S^0(c^{s**};\theta_L)+D^0(c^{s**};\theta_L)]<$$
$$(1-\beta)S^0(c_L;\theta_L)+D^0(c_L;\theta_L) \quad (C3)$$

$\Leftrightarrow$

$$(1-\omega)[(1-\beta)S^0(c^{s**};\theta_H)+D^0(c^{s**};\theta_H)]-$$
$$(1-\omega)[(1-\beta)S^0(c^{s**};\theta_L)+D^0(c^{s**};\theta_L)]>$$
$$(1-\omega)[(1-\beta)S^0(c_L;\theta_H)+D^0(c_L;\theta_H)]-(1-\omega)[(1-\beta)S^0(c_L;\theta_L)+D^0(c_L;\theta_L)] \quad (C4)$$

I prove (C4) by showing that the function

$$[(1-\beta)S^0(c;\theta_H)+D^0(c;\theta_H)]-[(1-\beta)S^0(c;\theta_L)+D^0(c;\theta_L)] \quad (C5)$$

increases in c. (C5) is equal to[60]

$$(1-\beta)(\theta_H-\theta_L)\Pi(X_0)+\frac{\alpha\nu-\beta}{1-\nu}\frac{\delta F+c+rU}{\delta}\left[\left(\frac{X_0}{\underline{X_H^c}}\right)^{\nu}-\left(\frac{X_0}{\underline{X_L^c}}\right)^{\nu}\right] \quad (C6)$$

Taking the derivative of (C6) with respect to c gives:

$$\frac{\alpha\nu-\beta}{1-\nu}\frac{1}{\delta}\left[\left(\frac{X_0}{\underline{X_H^c}}\right)^{\nu}-\left(\frac{X_0}{\underline{X_L^c}}\right)^{\nu}\right]+\frac{\alpha\nu-\beta}{1-\nu}(-\nu)\frac{1}{\delta}\left[\left(\frac{X_0}{\underline{X_H^c}}\right)^{\nu}-\left(\frac{X_0}{\underline{X_L^c}}\right)^{\nu}\right]>0 \quad (C7)$$

I have proved that $c^{s*}<c^{s**}$. Therefore, there exists a c^s that enforces a separating equilibrium and $c^s\in[c^{s*},c^{s**}]$. ||

C2. Pooling equilibrium

C2.1 “Screening demand”

As mentioned in Section 5.3.2, I compare the value accrued to the firm from pooling in the capital market, to that from the least-cost separating equilibrium outcome, characterized by coupon choices by the high and low productivity firms, $(c^{s,LC},c_L)$. The incentive compatibility constraint for the low-type firms is:

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[60] $\underline{X_k^c}$ is the optimal separation threshold of the firm with productivity type $k\in\{L,H\}$ for a particular coupon rate c, as in (28).

$$(ICpool.L)\ \omega\begin{Bmatrix} p[(1-\beta)S^0(c^p;\theta_H)+D^0(c^p;\theta_H)]+ \\ (1-p)(1-\beta)[S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]\end{Bmatrix}+$$
$$(1-\omega)(1-\beta)[S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]\geq$$
$$(1-\beta)S^0(c_L;\theta_L)+D^0(c_L;\theta_L) \qquad (C8)$$

The second term of the left hand side of (C8) reflects the fact that if the wage demand is rejected by the firm, with probability β, the match dissolves and all financial claims against future cash flows from the match are worthless.

Next, I consider the incentive constraint for the high-productivity firm, which is:

$$(ICpool.H)\ \omega\begin{Bmatrix} p[(1-\beta)S^0(c^p;\theta_H)+D^0(c^p;\theta_H)]+ \\ (1-p)(1-\beta)[S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]\end{Bmatrix}+$$
$$(1-\omega)[(1-\beta)S^0(c^p;\theta_H)+D^0(c^p;\theta_H)]\geq$$
$$(1-\beta)S^0(c^{s*};\theta_H)+D^0(c^{s*};\theta_H) \qquad (C9)$$

First, notice that whenever $c^{s*}<c_H$, no pooling equilibrium could survive the equilibrium refinement in Maskin and Tirole (1992). Under this case, the least-cost separating equilibrium is characterized by (c_H, c_L). In other words, the high-productivity firm separates itself from low-productivity firm at its full-information first best coupon rate. It cannot not do any better than the full-information first best scenario. Therefore, in order for the existence of a pooling equilibrium in "screening offer" case, I must have $c^{s*}>c_H$. Moreover, if ω is too small, the (IC_{pool}.L) (C8) cannot hold.

Let me compare the two incentive constraints, (IC_{pool}.L) (C8) and (IC_{pool}.H) (C9), notice that the first terms in the braces are the same for (IC_{pool}.L) (C8) and (IC_{pool}.H) (C9). Take the difference in differences between the second term of the left hand side and the term on the right hand side, for both (IC_{pool}.L) (C8) and (IC_{pool}.H) (C9).

$$(1-\omega)[(1-\beta)S^0(c^p;\theta_H)+D^0(c^p;\theta_H)]-[(1-\beta)S^0(c^{s*};\theta_H)+D^0(c^{s*};\theta_H)]-$$
$$(1-\omega)(1-\beta)[S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]+(1-\beta)S^0(c_L;\theta_L)+D^0(c_L;\theta_L) \qquad (C10)$$

Taking derivative of (C10) with respect to ω, I get

$$-[(1-\beta)S^0(c^p;\theta_H)+D^0(c^p;\theta_H)]+(1-\beta)[S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]<0 \qquad (C11)$$

At $\omega=1$, (C10) is smaller than zero. Therefore, I must have a $\underline{\omega}$ such that whenever $\omega>\underline{\omega}$, ($IC_{pool}$.H) (C9) implies ($IC_{pool}$.L) (C8), which is the case I focus on.

Notice that the left hand side of (IC_{pool}.H) (C9) admits a unique maximizing c^p. This is because the left hand side of (IC_{pool}.H) (C9) is a linear combination of nonnegative concave functions, thus is concave, and the choice set of c^p is a compact set. The first order condition for the maximization problem is[61]:

$$(1-\omega+\omega p)\left[\frac{\beta}{\delta}-\frac{\beta-\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_H^p}}\right)^{\nu}\right]+\omega(1-p)(1-\beta)\frac{\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_L^p}}\right)^{\nu} \qquad (C12)$$

Denote the maximizing coupon rate is c_1^{p*}, I need restrictions on model parameters so that (IC_{pool}.H) (C9) holds for c^{p*}. Notice that $c_1^{p*} < c_H$, because the second term inside the braces of the left hand side of (C9) decreases in c^p. ||

C2.2 "Pooling demand"

As mentioned in Section 5.3.2, I compare the value accrued to the firm from pooling in the capital market, to that from the least-cost separating equilibrium outcome, characterized by coupon choice, $(c^{s,LC}, c_L)$.[62] The incentive compatibility constraint for the low-type firms is:

$$(ICpool.L)\ \omega\begin{Bmatrix}p[S^0(c^p;\theta_H)-\beta S^0(c^p,\theta_L)+D^0(c^p;\theta_H)]+\\(1-p)[(1-\beta)S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]\end{Bmatrix}+$$
$$(1-\omega)[(1-\beta)S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]\geq(1-\beta)S^0(c_L;\theta_L)+D^0(c_L;\theta_L) \qquad (C13)$$

Next, I consider the incentive constraint for the high-productivity firm, which is:

$$(ICpool.H)\omega\begin{Bmatrix}p[S^0(c^p;\theta_H)-\beta S^0(c^p,\theta_L)+D^0(c^p;\theta_H)]+\\(1-p)[(1-\beta)S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]\end{Bmatrix}+$$
$$(1-\omega)[S^0(c^p;\theta_H)-\beta S^0(c^p,\theta_L)+D^0(c^p;\theta_H)]\geq(1-\beta)S^0(c^{s*};\theta_H)+D^0(c^{s*};\theta_H) \quad (C14)$$

The term $S^0(c^p;\theta_H)-\beta S^0(c^p,\theta_L)+D^0(c^p;\theta_H)$ reflects the fact that the high-productivity firm enjoys an information rent with the amount equal to the difference in expected matching surplus values between the high and low productivity firms, when the worker makes a

[61] $\underline{X_k^p}$ denotes the optimal separation threshold for firms with productivity type $k\in\{L,H\}$, when coupon rate is c^p

[62] One subtlety is that if the worker makes a pooling demand, he might leave the match when cash flow state X hits the separation threshold for the low-productivity firm. I assume that the high-productivity firm can always make a flow of side-payments equal to rU, to keep the worker until X hits the separation threshold for the high-productivity firm.

pooling wage demand. Similar to "screening demand" case, if ω is too small, the (IC_{pool}.L) (C13) does not hold.

Let me compare the two incentive constraints, (IC_{pool}.L) (C13) and (IC_{pool}.H) (C14), notice that the first term under the braces is the same for (IC_{pool}.L) (C13) and (IC_{pool}.H) (C14). Take the expression similar to (C10),

$$(1-\ \omega)[S^0(c^p;\theta_H)-\beta S^0(c^p;\theta_L)+D^0(c^p;\theta_H)]-$$
$$[(1-\beta)S^0(c^{s*};\theta_H)+D^0(c^{s*};\theta_H)]-$$
$$(1-\omega)[(1-\beta)S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]+(1-\beta)S^0(c_L;\theta_L)+D^0(c_L;\theta_L) \qquad (C15)$$

Taking derivative of (C15) with respect to ω, I get

$$-[S^0(c^p;\theta_H)-\beta S^0(c^p;\theta_L)+D^0(c^p;\theta_H)]+[(1-\beta)S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]<0 \qquad (C16)$$

At $\omega=1$, (C15) is smaller than zero. Therefore, I must have a $\underline{\omega}$ such that whenever $\omega>\underline{\omega}$, ($IC_{pool}$.H) (C14) implies ($IC_{pool}$.L) (C13), which is the case I focus on.

Notice that the left hand side of (IC_{pool}.H) (C14) admits a unique maximizing c^p. This is because the left hand side of (IC_{pool}.H) (C14) is a linear combination of nonnegative concave functions, thus is concave, and the choice set of c^p is a compact set. To further clarify this observation, rewrite the left hand side of (IC_{pool}.H) (C14) as

$$(1-\omega+\omega p)[(1-\beta)S^0(c^p;\theta_H)+D^0(c^p;\theta_H)]+\omega(1-p)[(1-\beta)S^0(c^p;\theta_L)+D^0(c^p;\theta_L)]$$
$$+(1-\omega+\omega p)\beta[S^0(c^p;\theta_H)-S^0(c^p,\theta_L)]$$

The first two terms are obviously concave. The sign of the second order derivative of the last term with respect to c^p is determined by that of the term in the brackets, whose second order derivative with respect to c^p is $\frac{-\nu}{\delta}\frac{1}{\delta F+c+rU}\left[\left(\frac{X_0}{\underline{X_H^p}}\right)^\nu-\left(\frac{X_0}{\underline{X_L^p}}\right)^\nu\right]<0$.

The first order condition for the maximization problem is:

$$(1-\omega+\omega p)\left[\frac{\beta}{\delta}-\frac{\beta-\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_H^p}}\right)^\nu\right]+\omega(1-p)\left[\frac{\beta}{\delta}-\frac{\beta-\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_L^p}}\right)^\nu\right]+$$
$$(1-\omega+\omega p)\frac{\beta}{\delta}\left[\left(\frac{X_0}{\underline{X_H^p}}\right)^\nu-\left(\frac{X_0}{\underline{X_L^p}}\right)^\nu\right]=0 \qquad (C17)$$

Simplifying,

$$\frac{\beta}{\delta} + (1-\omega+\omega p)\frac{\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_H^p}}\right)^{\nu} - \frac{\beta-\omega(1-p)\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_L^p}}\right)^{\nu} = 0 \qquad (C18)$$

Recall that at c_L, the full-information optimal coupon rate for the low-productivity firm, the first order condition of the asset value for low-productivity firm is such that $\frac{\beta}{\delta} - \frac{\beta-\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_L^p}}\right)^{\nu} = 0$. Therefore, at $c^p = c_L$, the second term of (C17) is equal to zero, and (C17) becomes:

$$(1-\omega+\omega p)\left[\frac{\beta}{\delta} + \frac{\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_H^p}}\right)^{\nu} - \frac{\beta}{\delta}\left(\frac{X_0}{\underline{X_L^p}}\right)^{\nu}\right] > (1-\omega+\omega p)\left[\frac{\beta}{\delta} - \frac{\beta-\alpha\nu}{\delta}\left(\frac{X_0}{\underline{X_L^p}}\right)^{\nu}\right] \qquad (C19)$$

The inequality comes from the fact that $\underline{X_L^P} > \underline{X_H^P}$ and $\nu < 0$. Therefore, at c_L, the left hand side of (C17) is greater than zero. Therefore, I have $c_2^{p*} > c_L$, by the concavity of (IC_{pool}.H) (C14). It is obvious that $c_2^{p*} < c_H$. If the parameter restrictions are such that (IC_{pool}.H) (C14) holds at c_2^{p*}, then $c_2^{p*} \in (c_L, c_H)$, between the full-information first best coupon choices of the low-productivity and high-productivity firms. ||

Figure 1 Comparative statics of optimal coupon rate c

Figure 1A c and bargaining power β

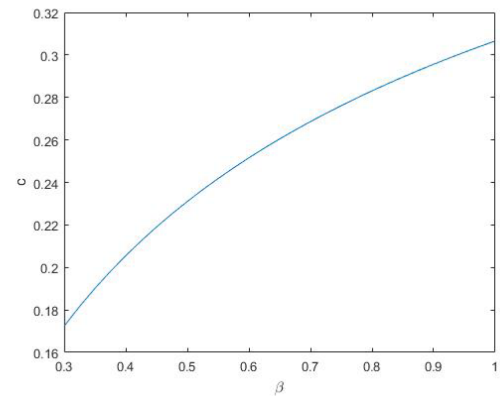

Figure 1B c and search efficiency A

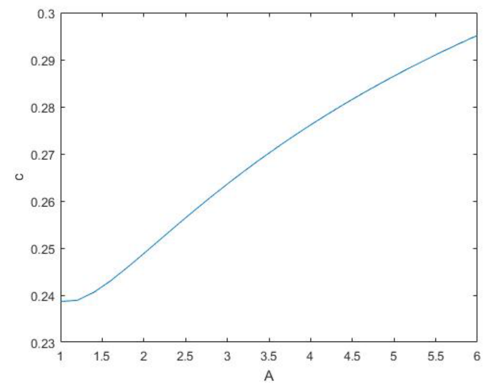

Figure 1C c and volatility σ

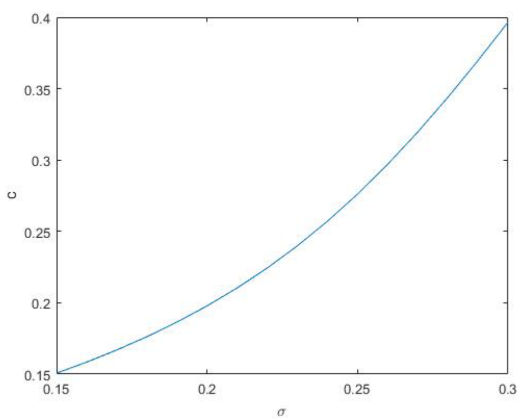

Figure 1D c and bankruptcy cost α

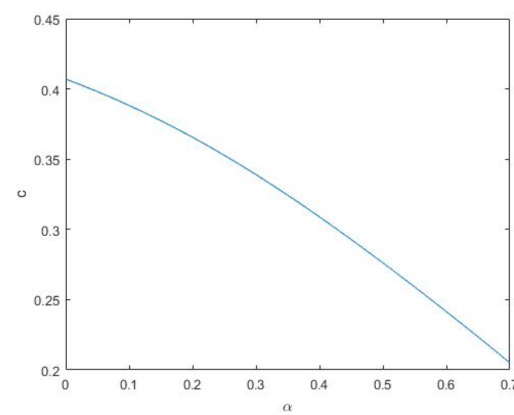

Figure 2 Comparative statics of expected tenure

Figure 2A Expected tenure and bargaining power β

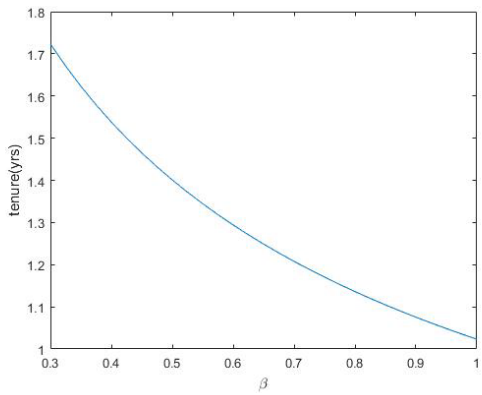

Figure 2B Expected tenure and search efficiency A

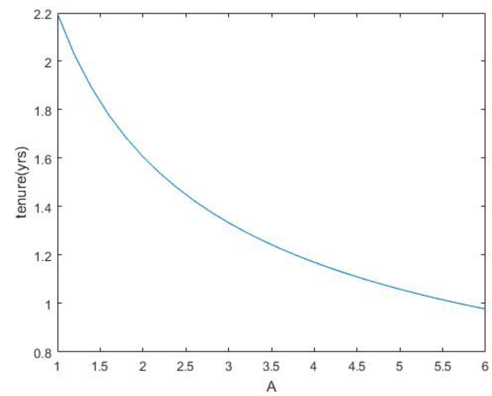

Figure 2C Expected tenure and volatility σ

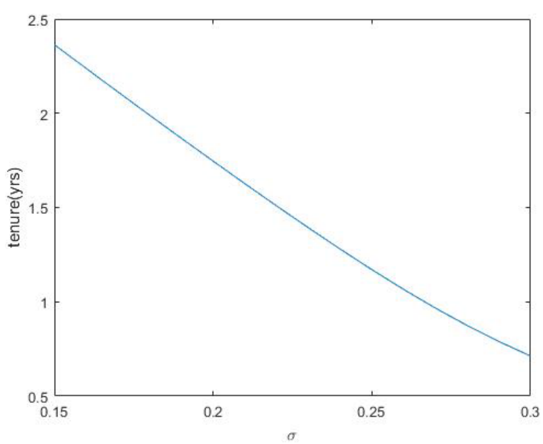

Figure 2D Expected tenure and bankruptcy cost α

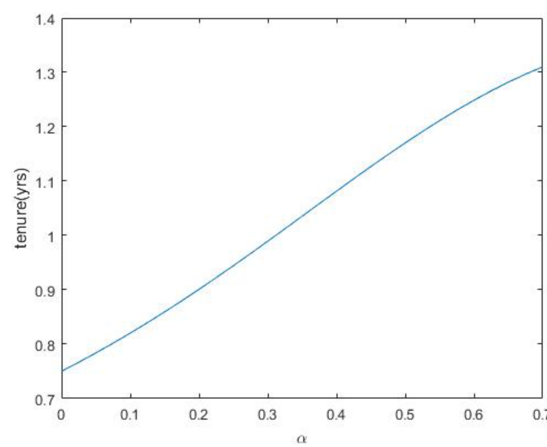

Figure 3 Comparative statics of the stationary cross-sectional density function of X

Figure 3A $f(X)$ and bargaining power β

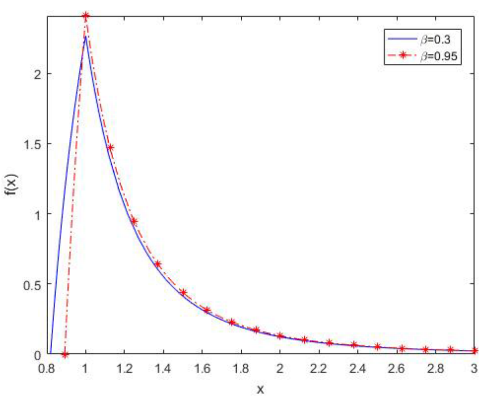

Figure 3B $f(X)$ and search efficiency A

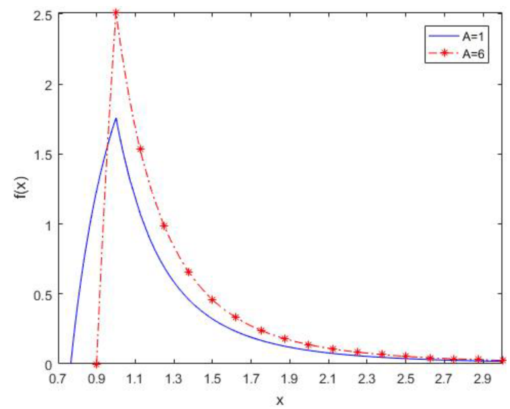

Figure 4 Comparative statics of unemployment rate u

Figure 4A u and bargaining power β

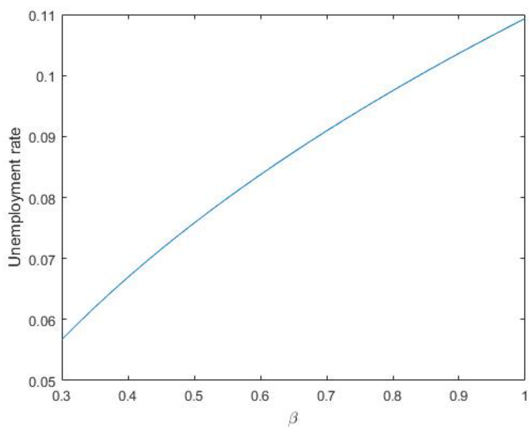

Figure 4B u and search efficiency A

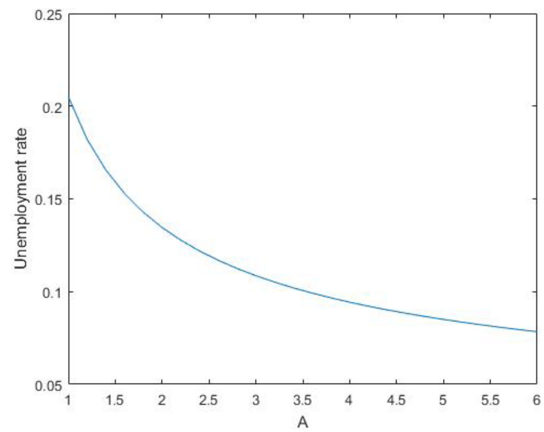

Figure 4C u and volatility σ

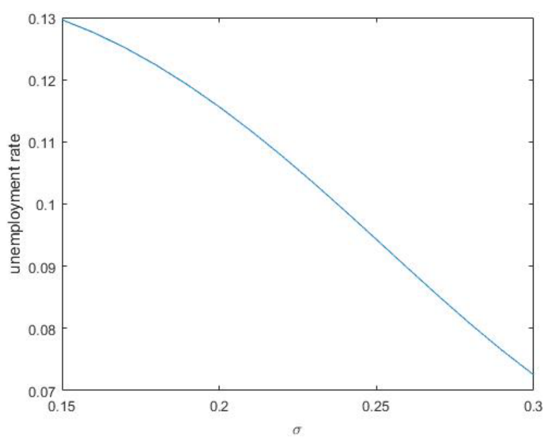

Figure 4D u and bankruptcy cost α

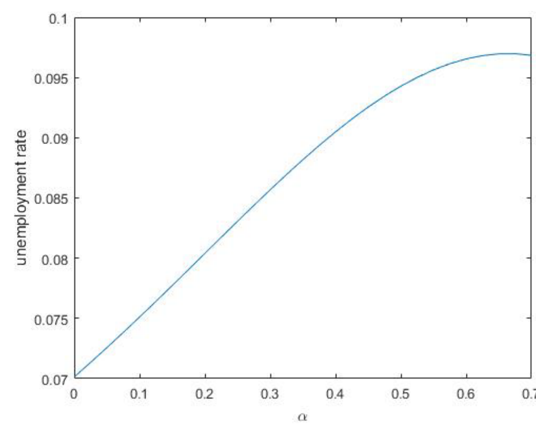

Figure 5 Comparative statics of initial wage w_0

Figure 5A w_0 and bargaining power β

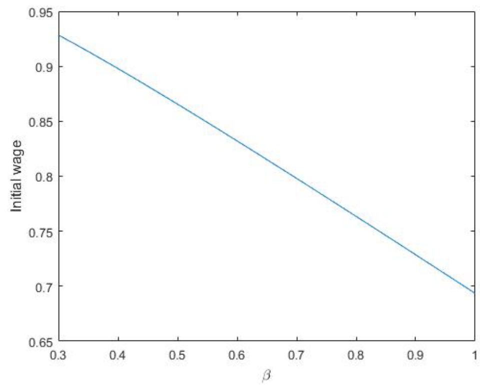

Figure 5B w_0 and search efficiency A

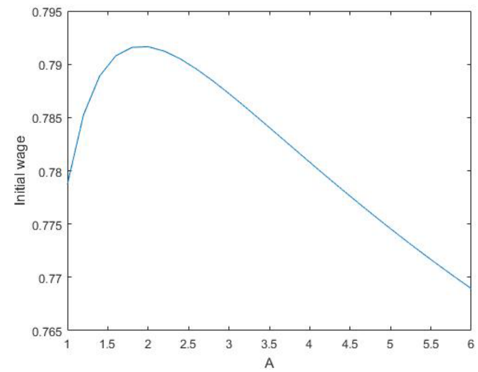

Figure 5C w_0 and volatility σ

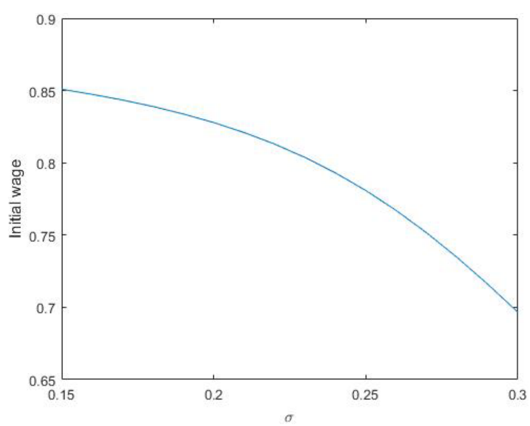

Figure 5D w_0 and bankruptcy cost α

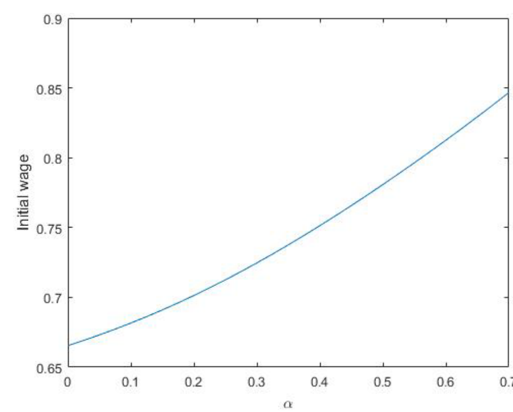

Figure 6 Comparative statics of labor force participation e

Figure 6A e and search efficiency A

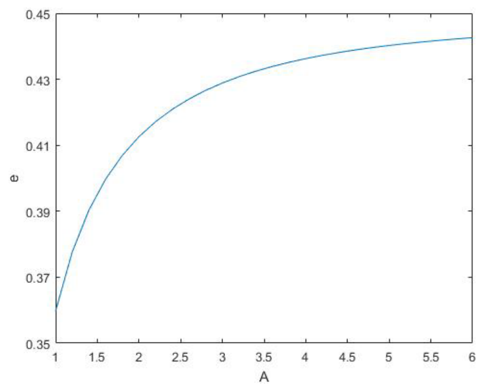

Figure 6B e and volatility σ