Deliver the Vote!

Micromotives and Macrobehavior in Electoral Fraud

Ashlea Rundlett* and Milan W. Svolik†

Abstract

Most electoral fraud is not conducted centrally by incumbents but rather locally by a multitude of political operatives. How does an incumbent ensure that his agents deliver fraud when needed and as much as is needed? We address this and related puzzles in the political organization of electoral fraud by studying the perverse consequences of incentive conflicts between incumbents and their local agents. These incentive conflicts result in a herd dynamic among the agents that tends to either oversupply or undersupply fraud, rarely delivering the amount of fraud that would be optimal from the incumbent’s point of view. Our analysis of the political organization of electoral fraud explains why even popular incumbents often preside over seemingly unnecessary fraud, why fraud sometimes fails to deliver victories, and it predicts that the extent of fraud should be increasing in both the incumbent’s genuine support and reported results across precincts. A statistical analysis of anomalies in precinct-level results from the 2011-12 Russian legislative and presidential elections supports our key claims.

*Graduate Student, Department of Political Science, University of Illinois at Urbana-Champaign, 420 David Kinley Hall, 1407 West Gregory Drive, Urbana, IL 61801, rundlet2@illinois.edu.
†Contact author, Associate Professor, Department of Political Science, University of Illinois at Urbana-Champaign, 420 David Kinley Hall, 1407 West Gregory Drive, Urbana, IL 61801, msvolik@illinois.edu.
[Election] data needed doctoring to avoid embarrassing situations, as occurred in Oaxaca when a zealous, prospective candidate for office wanted to impress the government officials and mobilized 140 percent of those registered.

A “high government source” inside the PRI Presidency

And I admitted...that we rigged the election...I gave the order to change it from 93% to around 80%...Because more than 90, just psychologically, that is not well received.

Alexander Lukashenko about the 2006 presidential election in Belarus

1 Introduction

“You may have won the election, but I won the count!” was Anastasio Somoza’s rebuke to an opponent who accused him of rigging an election. A burgeoning literature depicts the many ways in which incumbents attempt to “win the count” and conducts increasingly sophisticated analyses of their detection and deterrence. Yet while “winning the count” may be directed and facilitated from above, its execution is primarily local. The most frequent forms of electoral fraud – ballot box stuffing, multiple voting, voter intimidation, or the falsification of counts – are ultimately executed at the level of individual polling stations, not by the incumbent but rather a machinery that typically consists of hundreds of political operatives, party members, and state employees.

In spite of rich descriptive accounts of such local-level fraud in qualitative and historical literature, most formal and analytical research on electoral fraud treats its political

---

1See Bezdek (1973, 41).
4On the conduct and forms of electoral fraud, see Birch (2011), Lehoucq (2003), Schedler (2013), and Simpser (2013).
5On the local execution of fraud, see Cantú (Forthcoming), Benton (2013), Larreguy, Olea, and Querubin (2014), Lehoucq (2003), Martinez-Bravo (2014), and Simpser (2013).
organization and execution as unproblematic. The incumbent’s machinery of manipulation is assumed to operate as a unified political actor, under the incumbent’s perfect political control. This approach leaves us with a number of puzzles: How does the incumbent ensure that his local agents deliver fraud precisely when needed and exactly as much as is needed? What motivates local agents to engage in fraud when doing so may result in criminal prosecution? Why does locally-conducted electoral fraud succeed in delivering a victory in some elections but fail in others?

In this paper, we study these puzzles and demonstrate that incentive conflicts in the political organization and execution of electoral fraud have far-reaching implications for its conduct, success, and empirical fingerprints. Two related but distinct incentive conflicts critically shape the political organization of electoral fraud: the principal-agent problem between an incumbent and his local agents, and the collective action problem among the agents. At the heart of the principal-agent problem is a conflict of interest between the incumbent and his agents about when to engage in fraud and how much of it to conduct. The incumbent’s preferences were eloquently summarized in John F. Kennedy’s facetious response to questions about the role of his father’s wealth in his political success: “I have just received the following wire from my generous daddy. It says, ‘Dear Jack, don’t buy a single vote more than is necessary. I’ll be damned if I’m going to pay for a landslide!’ ”

That is, even those incumbents who are willing to engage in fraud if it is needed for a victory want to avoid unnecessary fraud that will only raise suspicions. Most agents, meanwhile, prefer to conduct fraud when it carries the least risk – when the incumbent’s victory is assured and the agents’ actions are unlikely to be investigated. Agents are most reluctant to engage in fraud when the incumbent’s victory is in doubt and they worry about being prosecuted if the challenger were to win the election. Thus agents are least willing to

---

engage in fraud precisely when the incumbent needs it most!

This principal-agent problem between the incumbent and his agents is compounded by a collective action problem among the agents. It is most pronounced when the incumbent narrowly trails the challenger. In these scenarios, the incumbent’s agents understand that, if only enough of them engaged in fraud, they could secure the incumbent’s victory. At the same time, however, each agent’s doubts about other agents’ actions lead her to question the prudence of her own engagement in fraud. Hence even when fraud could secure the incumbent’s victory, the agents’ fear of its ultimate failure may turn it into a self-fulfilling prophecy.

In order to rigorously examine the interplay between principal-agent and collective action problems in the political organization and execution of electoral fraud, we develop a formal model with two key, novel features. First, the incumbent does not engage in fraud directly but instead depends on the illicit collaboration of a large number of local agents who must be motivated by the promise of a reward. This is a departure from existing formal research, where the incumbent’s machinery of fraud is assumed to act as a unitary actor (Egorov and Sonin 2012; Fearon 2011; Gandhi and Przeworski 2011; Little 2012; Rozenas 2013; Simpser 2013; Svolik and Chernykh 2013). A key aspect of this departure concerns the contingency of the agents’ reward – as well as their punishment – on the incumbent’s political survival: Each agent understands that, if she engages in fraud, she will obtain the promised reward only if the incumbent is reelected and may face prosecution if the incumbent is ultimately defeated.7

The second key feature of our formalization concerns the limited information available to the incumbent and his local agents when they are deciding whether to engage in fraud.

---

7Such politically contingent rewards play a key role in Gehlbach and Simpser’s (2015) analysis of bureaucratic incentives under authoritarianism, Smith, de Mesquita, and LaGatta’s (2013) study of group-level incentives in partisan electoral competition, and Dragu and Polborn’s (2013) analysis of administrative constraints on the arbitrary exercise of government power.
The difficulties that incumbents in hybrid regimes face when gauging their genuine popularity have been highlighted in research on electoral manipulation and democratization (Gehlbach and Simpser 2015; Little 2013; Miller Forthcoming; Rozenas 2013) and parallel classic accounts of incentives for “preference falsification” under authoritarianism (Kuran 1991; Wintrobe 1998). The novel feature of our model is in how the structure of this information paucity is tailored to the context of electoral manipulation: While both the incumbent and his local agents have only imperfect information about the incumbent’s genuine popularity, each local agent’s information is much more precise than the incumbent’s but at the same time confined to her own precinct.

The chief macro-political consequence of these two novel features is a herd dynamic among the agents that tends to result in either overwhelming victories for the incumbent or, less often, his resounding defeats. We obtain this prediction by a natural application of the global game approach to the analysis of collective action problems (Carlsson and van Damme 1993; Morris and Shin 2003). Its key advantage is to transform a setting that would otherwise suffer from a multiplicity of equilibria with contradictory predictions into one with a unique, tractable, and politically intuitive equilibrium. In our setting, the agents’ incentives result in a unique tipping-point equilibrium according to which an agent engages in fraud only if her local, private perception of the incumbent’s popularity exceeds a threshold. The intuition is as follows: The higher the incumbent’s genuine popularity in an agent’s precinct, the more popular she infers the incumbent to be nationwide; consequently, she anticipates that fewer agents need to engage in fraud in order to secure the incumbent’s victory, which in turn lowers her own risk of engaging in fraud.

---

8See Boix and Svolik (2013), Bueno de Mesquita (2010), Edmond (2013), Little (2012), and Shadmehr and Bernhardt (2011) for applications of global games and related techniques to the analysis of collective action problems in protests, repression, regime change, and authoritarian power-sharing. A distinctive political feature of our framework is that the regime strives to motivate the agents to work on its behalf rather than against it.
Thus while never observed perfectly by either the incumbent or the agents, the incumbent’s genuine nationwide popularity ends up playing a central role by tacitly coordinating the agents’ attempts to resolve their collective action problem.

The perverse consequence of such individual-level incentives is a herd dynamic at the aggregate level. Jointly, agents will tend to either oversupply or undersupply fraud, rarely delivering the amount of fraud that would be optimal from the incumbent’s point of view. At one extreme, when the incumbent is unpopular and therefore needs fraud the most, agents will tend to underdeliver it; at the other extreme, agents will deliver excess fraud when it is not needed at all. Put simply, the incumbent cannot order 51% of the vote and expect to get precisely that. The aggregate amount of fraud will approximate the desired level only when he narrowly trails the challenger. It is only in such elections that fraud will be both politically decisive for the incumbent’s victory and successful in securing it.

Our analysis of microincentives in the political organization of electoral fraud improves our understanding of the resulting macrobehavior in a number of ways. First, the equilibrium dynamic that we just described helps us to account for the puzzling, often contradictory accounts of incumbents who enjoy genuine popularity and at the same time engage in seemingly unnecessary fraud. In a seminal analysis of the Institutional Revolutionary Party’s (PRI) demise in Mexico, Magaloni (2006) observes that, while certainly present, fraud only served to embellish the already impressive popularity that the PRI enjoyed before the 1980s (see also Greene 2007; Simpser 2013). Similarly, students of contemporary Russia are puzzled by the embarrassingly obvious fingerprints of fraud in elections that the United Russia party, and especially Vladimir Putin and Dmitry Medvedev, could have won cleanly.  

According to a leading explanation for these perplexing
outcomes, inflated margins of victory serve to signal the incumbent’s invincibility and thus deter potential challengers or defectors (Magaloni 2006; Simpser 2013).

Our framework suggests an alternative mechanism: Rather than an intentional strategy, overwhelming incumbent victories are the unintended byproduct of the principal-agent and collective action problems in the political organization of electoral fraud. Because individual agents are most willing to conduct fraud when it carries the least risk – when the incumbent is genuinely popular – we should not be surprised to observe genuine popularity go hand in hand with excessive fraud at the aggregate level. Even those incumbents that expect to prevail generally find it optimal to promise their agents positive (even if small) rewards, hedging against the odds that they are being too optimistic about their own popularity. An active infrastructure of fraud thus serves not only those incumbents who cannot win a clean election, but also those who can yet want to insure against an unlikely defeat.

The same logic helps us understand why local-level fraud, even when encouraged by the incumbent, sometimes fails to secure his re-election. Because fraud is by definition illegal, the incumbent’s capacity to motivate agents to engage in fraud on his behalf is limited to the promise of a reward upon his reelection. Such politically contingent inducements, however, are least effective precisely when the incumbent needs the agents’ collaboration most – when he lacks genuine popularity. Our analysis of the ensuing collective action problem highlights how individual agents’ worries about the incumbent’s eventual defeat reverberate among them and, if sufficiently pronounced, multiply into an avalanche of defections from the incumbent. Agents’ fears of the incumbent’s eventual defeat become a self-fulfilling prophecy.

This reasoning suggests a mechanism of fraud deterrence that has not been explored by existing theoretical research but is implicit in recent empirical work. Extant analytical treatments of electoral fraud focus on the threat of a post-election protest or violence as the
chief deterrent against fraud (Fearon 2011; Little 2012; Przeworski 2011; Svolik and Chernykh 2013; Tucker 2007). By contrast, our arguments highlight that a major reason for the failure of fraud may be the incumbent’s inability to muster the machinery of fraud in the face of his declining popularity. This focus on the incentives faced by local-level agents parallels empirical research on election monitoring, the deterrent effect of which is also hypothesized to occur at the level of individual polling stations (Asuka et al. 2014; Hyde 2008; Ichino and Schüdeln 2012). Our results, however, suggest that the direct effect of such local-level deterrents – whereby they raise the risk of engaging in fraud for individual agents who are being monitored – may not be the only or even the most consequential one. Rather, the primary consequence of election monitoring may be indirect: when monitoring occurs, all agents anticipate that much greater efforts must be exerted at non-monitored polling stations in order to secure the incumbent’s victory, which in turn heightens the collective action dilemma among all agents, including those who are not being monitored. To our knowledge, such systemic consequences of local fraud deterrents have not yet been examined either empirically or theoretically.

An improved understanding of the microincentives faced by the agents who ultimately execute fraud also helps us anticipate its empirical fingerprints. The prevailing approach to fraud detection focuses on the identification of statistical anomalies in voting or turnout but is often less explicit about the political process that generates them. Our model clarifies that anomalies indicative of fraud may be the unintended consequence of incentive conflicts

---

11After all, only a small fraction of polling stations is visited by election observers during any single election; see e.g. Hyde (2011).
in the political organization of fraud and predicts a specific pattern that such anomalies should follow: Their occurrence across precincts should not be uniform but rather increasing in both the incumbent’s genuine popularity and his vote share. Yet at the same time, such anomalies alone do not imply that the incumbent stole an election that would have otherwise been won by the challenger. In fact, the fingerprints of fraud may be most pronounced precisely when fraud is not politically decisive.

We find preliminary empirical support for our arguments when we examine the pattern of fraud in the 2011 legislative and 2012 presidential elections in Russia. We confirm a finding from earlier analyses of these elections (Gehlbach 2012; Klimek et al. 2012; Kobak, Shpilkin, and Pshenichnikov 2012), according which one form of electoral manipulation involved the rounding of the incumbent Vladimir Putin’s and United Russia Party’s vote shares to a higher multiple of five by the regime’s local operatives. Crucially, we also identify a previously unnoticed pattern that is anticipated by our model: The extent of such anomalies is increasing in the incumbent’s precinct-level vote share. In order to quantify the extent of fraud, we develop a measure of the ruggedness in the distribution of Putin’s and United Russia’s results based on kernel density estimation techniques. Using two different benchmarks, we find that the distribution of Putin’s and United Russia’s results is too rugged at percentages corresponding to multiples of five to occur by chance, and this ruggedness is indeed increasing in Putin’s and United Russia’s precinct-level vote share, as predicted by our theoretical arguments. Our approach additionally reveals that a significant amount of fraud most likely involved the stealing of votes from the leading opposition candidate Gennady Zyuganov and his Communist Party. Overall, this case illustrates that Putin’s regime used fraud as insurance against an (arguably) unlikely defeat – with its oversupply as the undesirable byproduct of the principal-agent and collective action problems in the political organization of electoral fraud.
2 The Model

Consider the following electoral manipulation game between an incumbent and his agents.\textsuperscript{13} Each agent $i$ operates in one among a continuum of precincts of equal size and decides whether to engage in fraud on behalf of the incumbent at the time of the election. We denote agent $i$'s engagement or not in fraud by $a_i = \{f, n\}$, respectively. The incumbent, however, does not observe whether any agent engaged in fraud; he only observes the \emph{precinct-level election result} $R_i$ – his share of the vote in agent $i$'s precinct. Before the election, therefore, the incumbent promises each agent a reward (a higher salary, promotion, perks) commensurate with the election result in her precinct. More precisely, each agent obtains the payoff $wR_i$ after the incumbent’s victory, where $w \geq 0$ and we refer to it as the \emph{reward factor}.

The precinct-level election result $R_i$ depends on the incumbent’s genuine \emph{precinct-level popularity} $S_i$ and whether the agent engaged in fraud on behalf of the incumbent, $a_i = \{f, n\}$, as follows:\textsuperscript{14}

\[
R_i = \begin{cases} 
S_i + F & \text{if } a_i = f; \\
S_i & \text{if } a_i = n.
\end{cases}
\]

Above, the parameter $F$, $0 < F < \overline{F}$, denotes the share of the precinct-level election result due to the agent’s fraud and we interpret it as a measure of the precinct agents’ fraud capacity.\textsuperscript{15}

\textsuperscript{13}Proofs of all technical results can be found in the supplementary appendix.
\textsuperscript{14}This binary-action model has the same implications as a more general one, in which each agent chooses the amount of fraud from the interval $f \in (0, F)$. Given the payoffs in Figure 1, an agent who optimally engages in a positive amount of fraud does so at its maximum feasible level, effectively choosing either $f = 0$ or $f = F$. We intentionally leave unspecified the exact form that this fraud takes as long as it is ultimately executed locally by the incumbent’s agents.
\textsuperscript{15}This additive assumption about fraud production is only one – and an intentionally simple one – among several plausible ways of formalizing fraud production. Letting $R_i = (1 + F)S_i$ or $R_i = \frac{(1 + F)S_i}{(1 + F)S_i + (1 - S_i)}$ results in qualitatively identical insights but less transparent algebra. In the supplementary appendix, we derive $\overline{F}$, the maximum admissible value of $F$ that is implied by our informational assumptions and the global game.
Figure 1: Agent $i$’s payoffs as a function of her fraud decision $a_i$ and the election result $R$

Crucially, the agents obtain the reward $wR_i$ only if the incumbent is re-elected. By contrast, if the incumbent loses, each agent’s payoff depends on whether she engaged in fraud at the time of the election. If she did not engage in fraud, she obtains the payoff 0. If she did engage in fraud, the agent obtains the payoff $-cF$, where $c > 0$ stands for the political cost of fraud. It reflects the potential investigation of allegations of fraud in the agent’s precinct after the challenger takes office, possibly resulting in the agent’s criminal prosecution and conviction.\(^\text{16}\) Thus we are effectively assuming that the likelihood of a conviction or the severity of punishment is increasing in $F$, the share of the precinct-level election result due to the agent’s fraud.\(^\text{17}\)

The agents’ payoffs are summarized in Figure 1. We see that each agent’s incentive to engage in fraud depends on her expectation about the national-level election result $R$. If the agent expects the incumbent to win, $R \geq \frac{1}{2}$, then she prefers to conduct fraud since $w(S_i + F) > wS_i$. If, on the other hand, the agent expects the incumbent to lose, $R < \frac{1}{2}$, then she prefers to play fair since $-cF < 0$.

\(^{16}\)For instance, in the aftermath of the Orange Revolution and the re-run of the fraudulent second round of the 2004 presidential election, Ukrainian newspapers lamented that only “little people” had been prosecuted for fraud on behalf of the incumbent-endorsed Viktor Yanukovych, while the organizers of that fraud avoided prosecution by securing political “cover.” See e.g. “Two convicted of electoral fraud in central Ukraine,” \textit{BBC Monitoring Former Soviet Union}, 22 March, 2005; “Ukrainian court hands down two electoral fraud sentences,” \textit{BBC Monitoring Former Soviet Union}, 23 March, 2005; and “Weekly says organizers of Ukraine presidential election fraud go unpunished,” \textit{BBC Monitoring Former Soviet Union}, 27 April, 2006. The immunity from prosecution afforded by the incumbent’s victory is nicely illustrated by a wisecrack from machine-era Chicago politics, according to which “when a Cook County election is stolen, it stays stolen.” See Grossman, Guy, “It’s No News Here,” \textit{Chicago Tribune}, 23 November 1968, p. 16.

\(^{17}\)We may also interpret $c$ less politically, as the agent’s costly effort. In that case, $-cF$ should also be included in the top-left cell of the payoff table in Figure 1.

\begin{tabular}{|c|c|c|}
\hline
Agent $i$’s action $a_i$ & $R \geq \frac{1}{2}$ & $R < \frac{1}{2}$ \\
\hline
$f$ & $w(S_i + F)$ & $-cF$ \\
\hline
$n$ & $wS_i$ & 0 \\
\hline
\end{tabular}
The overall election result $R$ depends on both the incumbent’s genuine popularity and the actions of the agents. More specifically, the incumbent’s *popularity* at the time of the election $\theta$, $0 < \theta < 1$, corresponds to the fraction of the electorate that actually voted for the incumbent. We assume that $\theta$ is not perfectly known by any of the players but is commonly believed to be uniformly distributed on the unit interval $(0, 1)$. Instead, each agent privately observes the fraction $S_i$ of her precinct that voted for the incumbent, which is correlated with $\theta$ in the following way: $S_i$ is uniformly distributed on the interval $(\theta - \epsilon, \theta + \epsilon)$, $0 < \epsilon < \Phi$. We think of $\epsilon$ as “small” and interpret it as a measure of heterogeneity in the incumbent’s support across precincts. Thus when each agent decides whether to engage in fraud on behalf of the incumbent, she has only imperfect information about the incumbent’s genuine, national-level popularity.

Because we model the incumbent’s agents as atomless players on a continuum of precincts, any single agent’s decision to engage in fraud on behalf of the incumbent will be inconsequential at the national level. Jointly, however, the agents’ actions affect the outcome as the overall election result $R$ amounts to

$$E[R] = \int_{\theta-\epsilon}^{\theta+\epsilon} \frac{1}{2\epsilon} \left( S_i + 1_{\{a_i=f\}} \Phi \right) dS_i = \theta + \phi \Phi. \tag{1}$$

Above, $E[R]$ denotes the expected value of $R$, $1_{\{a_i=f\}}$ is an indicator function that equals 1 if agent $i$ engaged in fraud and 0 otherwise, and $\phi$ is the fraction of agents that engaged in fraud. We think of our assumption of atomless agents along a continuum of equally-sized precincts as capturing a country with a large number of precincts that are small relative to

---

18 The possibility of $S_i < 0$ and $S_i > 1$ when $\theta$ is within an $\epsilon$ distance of the boundaries 0 and 1, respectively, is irrelevant for the analysis that follows (and could be avoided by letting $S_i$ be uniformly distributed on the intervals $(0, \theta + \epsilon)$ and $(\theta - \epsilon, 1)$ when $\theta$ is within an $\epsilon$ distance of 0 and 1, respectively.)

19 In the supplementary appendix, we derive $\Phi$, the maximum admissible value of $\epsilon$ that is implied by our informational assumptions, $\Phi = \frac{1}{4}(1 - 2\Phi)$ for $\Phi < \frac{1}{2}$. 

20 Note that while $S_i$ is informative about $\theta$, the agents lack a common knowledge of $\theta$ for an arbitrarily small $\epsilon$. For a discussion of this feature of global games, see Morris and Shin (2003).
the country as a whole.\footnote{This approximation works well since the national-level election result in such a country is effectively the mean of precinct-level results, \( R = \frac{1}{N} \sum_{i=1}^{N} R_i \). By the law of large numbers, this mean converges in probability to \( (1) \), \( R = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (S_i + 1_{(a_i = f)} F) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} S_i + F \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} 1_{(a_i = f)} = \theta + \phi F. \) Our results extend to a setting with a finite number of agents; c.f. Morris and Shin (2003, Appendix B).}

The national-level election outcome \( R \) thus depends on the precinct agents’ fraud capacity \( F \), the fraction of agents engaging in fraud \( \phi \), and the incumbent’s election-day popularity \( \theta \). If \( \theta \geq \frac{1}{2} \), then \( R \geq \frac{1}{2} \) and the incumbent wins the election regardless of the agents’ actions. If \( \theta < \frac{1}{2} - F \), on the other hand, then the incumbent will be defeated even if all agents conducted fraud on his behalf, \( R < \frac{1}{2} \). Only when \( \frac{1}{2} - F \leq \theta < \frac{1}{2} \) does the election outcome depend on the fraction of agents \( \phi \) engaging in fraud. In turn, if the agents were able to observe \( \theta \) perfectly, they would all refrain from fraud when \( \theta < \frac{1}{2} - F \), engage in fraud if \( \theta \geq \frac{1}{2} \), and condition their actions on the actions of others when \( \frac{1}{2} - F \leq \theta < \frac{1}{2} \). In the latter, most politically interesting case, all agents engaging in fraud and refraining from fraud both constitute a Nash equilibrium. This indeterminacy disappears in our setting, where agents do not directly observe the incumbent’s national-level popularity \( \theta \).

We examine the perfect Bayesian Nash equilibria of this game. To recapitulate, the timing of actions is as follows: first, the incumbent sets the reward factor \( w \); next, the incumbent’s national- and precinct-level popularities \( \theta \) and \( S_i \) are drawn; each agent \( i \) privately observes \( S_i \) but not \( \theta \) and decides whether to engage in fraud; finally, depending on whether they engaged in fraud and on whether the incumbent is re-elected, agents are rewarded or suffer the cost of fraud. We proceed by backward induction.

Consider first the agents’ decision whether to engage in fraud in light of the incumbent’s popularity in her precinct \( S_i \). While each agent’s precinct-level result \( S_i \) is only an imperfect signal of \( \theta \), some values of \( S_i \) allow her to perfectly infer the outcome of the election. More specifically, our assumptions about the distribution of \( S_i \) imply that if
$S_i < \frac{1}{2} - F - \epsilon$, then $\theta < \frac{1}{2} - F$ and thus $R < \frac{1}{2}$. On the other hand, if $S_i \geq \frac{1}{2} + \epsilon$, then $\theta \geq \frac{1}{2}$ and thus $R \geq \frac{1}{2}$. For these values of $S_i$, therefore, each agent optimally refrains from and engages in fraud, respectively. But when $\frac{1}{2} - F - \epsilon \leq S_i < \frac{1}{2} + \epsilon$, agent $i$’s optimal action depends on her inference about other agents’ precinct-level results and actions.

Suppose therefore that agents with $S_i$ on the interval $[\frac{1}{2} - F - \epsilon, \frac{1}{2} + \epsilon)$ follow the threshold strategy

\[
\sigma(S_i) = \begin{cases} 
\text{engage fraud}, & a_i = f, \quad \text{if } S_i \geq S^*; \\
\text{do nothing}, & a_i = n, \quad \text{if } S_i < S^*.
\end{cases}
\]

According to $\sigma(S_i)$, agent $i$ engages in fraud if and only if the incumbent’s popularity in her precinct reaches at least some threshold value $S^*$. We will refer to $S^*$ as the agents’ fraud threshold.

When an agent who observes precinct-level popularity $S_i$ engages in fraud, she expects the payoff

\[
\Pr \left[ R \geq \frac{1}{2} \mid S_i \right] w(S_i + F) - \Pr \left[ R < \frac{1}{2} \mid S_i \right] cF .
\]

(2)

Meanwhile, the agent’s expected payoff from doing nothing is

\[
\Pr \left[ R \geq \frac{1}{2} \mid S_i \right] wS_i .
\]

(3)

According to the threshold strategy $\sigma(S_i)$, the threshold agent in whose precinct the incumbent’s popularity is $S_i = S^*$ must be indifferent between engaging in fraud and doing nothing. Letting $S_i = S^*$ and equating (2) to (3), we see that the following indifference condition holds for the threshold agent:

\[
\Pr \left[ R < \frac{1}{2} \mid S_i = S^* \right] = \frac{w}{c + w} .
\]

(4)
The indifference condition in (4) highlights the central role that each agent’s expectation about the outcome of the election plays in her decision to engage in fraud. The smaller the reward factor $w$, the stronger must be the threshold agent’s expectation that the incumbent will win. This will occur when the fraction of agents that engage in fraud $\phi$ satisfies the 

majority condition \[ R \geq \frac{1}{2} \text{ or equivalently } \theta + \phi F \geq \frac{1}{2}. \]

We may therefore refer to the value of $\phi$ at which the incumbent wins by a bare majority as the 

majority threshold \[ \phi^* = \frac{1}{2} - \theta. \]

According to our assumptions about the distribution of $S_i$ and the threshold strategy $\sigma(S_i)$, the fraction of agents that engage in fraud in equilibrium is \[ \phi = \frac{(\theta + \epsilon) - S^*}{2\epsilon}. \]

In turn, the majority threshold implies that the threshold agent’s belief that the incumbent will lose the election is \[ \Pr \left[ R < \frac{1}{2} \mid S_i = S^* \right] = \Pr [\phi < \phi^* \mid S_i = S^*] = \Pr \left[ \frac{(\theta + \epsilon) - S^*}{2\epsilon} < \phi^* \right] = \Pr [\theta < S^* + 2\epsilon\phi^* - \epsilon]. \] (5)

Substituting the majority threshold $\phi^*$ into (5), we obtain \[ \Pr \left[ R < \frac{1}{2} \mid S_i = S^* \right] = \Pr \left[ \theta < \frac{FS^* - \epsilon F + \epsilon}{F + 2\epsilon} \right]. \]
Given that $\theta$ and $S_i$ are uniformly distributed on the intervals $(0, 1)$ and $(\theta - \epsilon, \theta + \epsilon)$, respectively, the threshold agent believes that the incumbent’s popularity $\theta$ is uniformly distributed on the interval $(S^* - \epsilon, S^* + \epsilon)$.\textsuperscript{22} In turn,

$$
\Pr \left[ R < \frac{1}{2} \mid S_i = S^* \right] = \frac{FS^* - \epsilon F + \epsilon}{F + 2\epsilon} - (S^* - \epsilon) \frac{2\epsilon}{2\epsilon} \\
= \frac{1}{2} - S^* + \epsilon \\
= \frac{1}{2} - \frac{S^* + \epsilon}{F + 2\epsilon}.
$$

Substituting (6) into the indifference condition in (4), we see that the agents’ fraud threshold must be

$$
S^* = \frac{1}{2} - \frac{F}{c + w} - \frac{\epsilon}{c + w}.
$$

Jointly, the fraud threshold $S^*$ and majority threshold $\phi^*$ imply the existence of a popularity threshold $\theta^*$ such that, in equilibrium, the incumbent loses the election if $\theta < \theta^*$ and wins the election if $\theta \geq \theta^*$. That is, when the incumbent’s genuine popularity is exactly at the popularity threshold $\theta^*$, he wins by a bare majority,

$$
\theta^* = \frac{1}{2} - \phi^* F \quad \text{and} \quad \phi^* = \frac{(\theta^* + \epsilon) - S^*}{2\epsilon}.
$$

Substituting $S^*$ above and solving for $\theta^*$ and $\phi^*$, we see that

$$
\theta^* = \frac{1}{2} - F \frac{w}{c + w} \quad \text{and} \quad \phi^* = \frac{w}{c + w}.
$$

Finally, consider the incumbent’s optimal choice of the reward factor $w$ in light of the thresholds $S^*$, $\theta^*$, and $\phi^*$. Letting $b > 0$ be the incumbent’s payoff from winning the

\textsuperscript{22}This holds for the posterior density of $\theta|S_i$ for the range of $S_i$ under consideration, $\frac{1}{2} - F - \epsilon \leq S_i < \frac{1}{2} + \epsilon$. The posterior density of $\theta|S_i$ within a $2\epsilon$ distance of the boundaries 0 and 1 is different; see the supplementary appendix for details.
election and normalizing his payoff from losing to 0, his expected payoff becomes

\[
\Pr[\theta \geq \theta^*] b - w E[R] = (1 - \theta^*) b - w (E[\theta] + \phi F) = b(1 - \theta^*) - w \left( \frac{1}{2} + \phi F \right). \tag{7}
\]

In (7), we use that fact that \(E[\theta] = \frac{1}{2}\) and the equilibrium probability of the incumbent’s victory is \(1 - \theta^*\), given our assumption that, before the election, \(\theta\) is commonly believed to be uniformly distributed on the unit interval \((0, 1)\). Treating \(\theta^*\) and \(\phi\) as functions of \(w\) and maximizing (7) with respect to \(w\), we obtain

\[
w^* = \sqrt{\frac{cF[cF + 2\epsilon(b + c)]}{F^2 + 2\epsilon F + \epsilon}} - c,
\]

which is positive as long as \(b > \frac{c}{2F}\). Intuitively, the optimal reward factor \(w\) is increasing in the incumbent’s payoff from winning the election \(b\). When \(b \leq \frac{c}{2F}\), meanwhile, the incumbent does not value victory enough to be willing to pay for any fraud and optimally chooses \(w^* = 0\).

Proposition 1 summarizes our results so far.

**Proposition 1 (Collective Action and Electoral Fraud).** In the unique perfect Bayesian Nash equilibrium,

i. the incumbent chooses the reward factor \(w^*\);

ii. agent \(i\) engages in fraud if \(S_i \geq S^*\) and does nothing otherwise;

iii. the incumbent wins the election if \(\theta \geq \theta^*\) and is defeated otherwise;

iv. the fraction of agents that engage in fraud when the incumbent wins the election is \(\phi \geq \phi^*\).

**Proof.** Follows from the text. See the supplementary appendix for the derivation of \(w^*\) and the upper bounds on \(F\) and \(\epsilon\). \(\square\)
2.1 Comparative Statics and Political Implications

In order to highlight the political implications of our results so far, consider an illustration based on the parameters $c = 1$, $F = \frac{2}{10}$, $\epsilon = \frac{1}{10}$, and $b = 70$, which yield $S^* = 0.3$, $\theta^* = 0.35$, $\phi^* = 0.75$, and $w^* = 3$. That is, in equilibrium, agents engage in fraud only if the incumbent’s popularity in their precinct is greater than 30% and fraud secures the incumbent’s victory only if his national-level popularity is greater than 35%, or equivalently, when at least three-fourths of agents participate in fraud. Figure 2 employs these values to plot the equilibrium election result $R^*$ as a function of the incumbent’s genuine popularity $\theta$.

We see in Figure 2 that the agents’ equilibrium behavior can be partitioned into four qualitatively distinct intervals over $\theta$. At very low levels of the incumbent’s genuine
popularity, \( 0 < \theta < S^* - \epsilon \), no agent observes a precinct-level popularity high enough to warrant engaging in fraud and all agents correctly anticipate the incumbent’s defeat. When \( S^* - \epsilon \leq \theta < \theta^* \), fraud occurs but fails: if enough agents engaged in fraud, they could ensure the incumbent’s victory for some values of \( \theta \) on this interval, but because the incumbent’s popularity is too low, an insufficient number of agents ends up engaging in fraud. By contrast, when \( \theta^* \leq \theta < \frac{1}{2} \), enough agents engage in fraud to secure an undeserved victory for the incumbent. If we take the share of the election result that is due to fraud as a measure of such undeservedness, then the incumbent’s victory is most undeserved when \( \theta = S^* + \epsilon \) and all agents engage in fraud. Finally, when \( \frac{1}{2} < \theta < 1 \), fraud occurs but it is unnecessary: the incumbent is popular enough to win without fraud. In fact, when \( \theta \geq \frac{1}{2} + \epsilon \), all agents are aware that the incumbent will prevail without their complicity; they nonetheless engage in fraud because it boosts the election result in their precincts and thus leads to a higher reward. A perverse consequence of these incentives are national-level election results exceeding 100% at high values of \( \theta \).\(^{23}\)

This conflict between the incumbent’s needs and the agents’ equilibrium behavior is illustrated in Figure 3. The bottom axis denotes the incumbent’s national-level popularity \( \theta \); the top axis denotes the equilibrium election result \( R^* \); the dashed gray line plots the level of fraud \( \hat{F} \) that the incumbent needs for a victory; and the solid black line plots the level of fraud \( F^* \) that occurs in equilibrium. When \( \theta < \theta^* \), the incumbent needs more fraud for a victory than the agents deliver, \( F^* < \hat{F} \). When \( \theta \geq \theta^* \), on the other hand, the agents engage in a level of fraud that is unnecessary from the incumbent’s point of view, \( F^* > \hat{F} \), collectively delivering up to 20% in excess of what the incumbent needs. Only at exactly \( \theta^* \) is the incumbent’s demand for fraud and its supply by the agents in balance: This is when

\(^{23}\)This effect would disappear, i.e. the plot of \( R^* \) would flatten and converge to the 45 degree line at high levels of \( \theta \) if we assumed that, in addition to the strategic cost of fraud, agents face a direct cost of fraud that is increasing in the amount of fraud that they engage in.
the incumbent needs the national level of fraud to add up to 15% ($\phi^* F = 0.15$) and just the right fraction of agents – three-fourths ($\phi^* = 0.75$) – delivers it.

Realizations of the incumbent’s popularity across individual precincts thus play a central role in forming agents’ beliefs about the incumbent’s national-level popularity and, in turn, the degree of coordination needed for the incumbent’s victory. But the precise values of the thresholds $S^*$, $\theta^*$, and $\phi^*$ are also shaped by the parameters $F$, $c$, $\epsilon$, $b$, and the equilibrium reward factor $w^*$.

Consider the popularity threshold $\theta^*$, which is central not only strategically but also normatively: As $\theta^*$ declines, the interval $\frac{1}{2} - \theta^*$ along which the incumbent secures an undeserved victory becomes larger. An increase in the fraud capacity $F$ and the reward...
factor \( w^* \) lowers \( \theta^* \) and thus expands the range of \( \theta \)s along which the incumbent prevails in spite of being opposed by a majority of the electorate. That is, the greater the amount of fraud that each agent can produce within her precinct, the lower the demands on the agents’ coordination. Meanwhile, the greater the agents’ compensation for precinct-level results, the greater the risk that each agent is willing to take when engaging in fraud. The opposite holds for the cost parameter \( c \). At borderline values of \( F, w^*, \) and \( c \) (\( F \to 0, \)
\( w^* \to 0, \) or \( c \to \infty \)), no fraud occurs in equilibrium when it is actually needed by the incumbent as \( \theta^* = \frac{1}{2}, S^* = \frac{1}{2} + \epsilon, \) and \( \phi^* = 0. \) Intuitively, no agent is willing to risk prosecution when there is no way to inflate the incumbent’s vote share, when there is no personal benefit from doing so, or when the cost of failure is extreme.\(^{24}\)

Crucially, while a greater reward factor \( w^* \) lowers the fraud and popularity thresholds \( S^* \) and \( \theta^* \), it does not eliminate the collective action problem among the agents.\(^{25}\) Observe that as \( w^* \) tends to infinity, \( \lim_{w \to \infty} \phi^* = 1, \) and in turn, \( \lim_{w \to \infty} S^* = \frac{1}{2} - F - \epsilon \) and \( \lim_{w \to \infty} \theta^* = \frac{1}{2} - F, \) but

\[
S^* > \frac{1}{2} - F - \epsilon \quad \text{and} \quad \theta^* > \frac{1}{2} - F \quad \text{for any} \quad w^* > 0.
\]

That is, even when the agents’ compensation is arbitrarily large, there will be values of the incumbent’s popularity at which the agents could deliver the incumbent’s victory by conducting fraud but will fail to do so out of the fear that an insufficient fraction among them will engage in fraud.

\(^{24}\)These comparative statics for \( F \) and \( c \) (as well as for \( \epsilon \) and \( b \)), which we discuss in the supplementary appendix, continue to hold after accounting for their indirect influence on \( \theta^* \) via their effect on the incumbent’s choice of \( w^* \).

\(^{25}\)The converse holds for the cost parameter \( c \): while a greater cost of engaging in fraud raises the fraud and popularity thresholds \( S^* \) and \( \theta^* \), it cannot entirely prevent fraud from succeeding in equilibrium: \( S^* < \frac{1}{2} + \epsilon \) and \( \theta^* < \frac{1}{2} \) for any \( c > 0. \)
2.2 Fraud as Insurance against Defeat: Pre-election Expectations and the Equilibrium Supply of Fraud

How do pre-election expectations about the incumbent’s popularity affect his choice of the agents’ reward \( w \), and in turn, the equilibrium supply of fraud? In order to address this question, we must relax our assumption that \( \theta \) is uniformly distributed on the entire interval \((0, 1)\). This assumption in effect implied that the incumbent has no information about his likely national-level popularity on election day. In order to vary prior beliefs about \( \theta \), we now let \( \theta \) be uniformly distributed on a smaller interval, \((\hat{\theta} - \sigma, \hat{\theta} + \sigma)\), with \( \frac{F}{2} < \sigma < \frac{1}{2} \).\(^{26}\) We can therefore think of \( \hat{\theta} \) and \( \sigma \) as the mean and degree of uncertainty in the incumbent’s and the agents’ pre-election expectations about \( \theta \).

In this extended setting, pre-election expectations about \( \theta \) may vary from the pessimistic belief that unless he engages in fraud, the incumbent will lose the election for sure (when \( \hat{\theta} + \sigma < \frac{1}{2} - F \)) to the optimistic belief that incumbent will win the election for sure even if he does not engage in any fraud (when \( \hat{\theta} - \sigma > \frac{1}{2} \)). In turn, whenever \( \hat{\theta} < \frac{1}{2} - F - \sigma \) or \( \hat{\theta} > \frac{1}{2} + \sigma \), the incumbent optimally sets \( w^* = 0 \) since the agents’ actions cannot affect the election outcome. Meanwhile, when \( \frac{1}{2} - F - \sigma \leq \hat{\theta} \leq \frac{1}{2} + \sigma \), the equivalent of the incumbent’s expected payoff in (7) is

\[
\left( \frac{\hat{\theta} + \sigma - \theta^*}{2\sigma} \right) b - w \left( \hat{\theta} + \phi F \right).
\] (8)

Above, we used that fact that \( E[\theta] = \hat{\theta} \) and the equilibrium probability of the incumbent’s victory is now \( \frac{\hat{\theta} + \sigma - \theta^*}{2\sigma} \). Treating \( \theta^* \) and \( \phi \) as functions of \( w \) and maximizing (8) with respect

\(^{26}\)These prior assumptions allow for enough uncertainty about \( \theta \) so that at least for some values of \( \hat{\theta} \), the prior covers the interval \([\frac{1}{2} - F, \frac{1}{2}]\), which is sufficient for the existence of a unique equilibrium. For values of \( \hat{\theta} \) close to 0 or 1, however, there may be multiple equilibria. In our discussion, we restrict attention to the threshold equilibria from our basic model.
Figure 4: The effect of pre-election expectations about the incumbent’s popularity $\hat{\theta}$ on the equilibrium reward factor $w^*$ (left) and the expected equilibrium election result $R^*$ (right)

to $w$, we obtain

$$w^* = \sqrt{\frac{cF[eb + \sigma cF(2\epsilon + F)]}{\sigma[\hat{\theta}(2\epsilon + F) + F(2\epsilon + F - \frac{1}{2})]}} - c,$$

which is decreasing in $\hat{\theta}$.

This result is illustrated in the left panel of Figure 4, which is based on $\sigma = \frac{1}{4}$ (keeping the remaining parameter values the same as in the basic model.) Intuitively, as his expected popularity improves, the incumbent reduces the reward factor in order to avoid wasting resources on unnecessary fraud. Given the uncertainty in prior information about his eventual popular support $\theta$, however, the incumbent is willing to pay his agents a positive reward factor even when he expects to prevail – hedging against the odds that he is being too optimistic. We may therefore view the incumbent’s choice of $w$ as insurance against electoral defeat.

The right panel in Figure 4 plots the effect of the incumbent’s expected popularity on
the expected equilibrium outcome of the election (this is the analogue of Figure 2 from our earlier discussion.) We see that a key conclusion from our earlier analysis remains unchanged: in general, fraud is still either under- or over-supplied due to the interplay of the collective action and principal-agent problems in the execution of fraud. The key difference between our basic model and the present extension is that fraud no longer occurs when pre-election expectations about the incumbent’s popularity indicate that he can win the election without engaging in fraud. Lack of fraud can therefore arise out of two very different scenarios: in the first, the incumbent gives up on fraud because he desperately lacks popularity; in the second, the incumbent no longer bothers with fraud because he enjoys overwhelming popularity.

2.3 Differences in Competitiveness across Precincts: Persistent versus Variable Electoral Support

The previous extension introduced differences in pre-election expectations across elections but not across precincts. While the realizations of $\theta$ do eventually differ across precincts in our basic model, all precincts are ex-ante identical. In order to examine the consequences of differences in pre-election expectations across precincts, we now separate the incumbent’s support into two components, persistent and variable support. Variation across precincts in persistent support $P_i$ captures the commonly known, ex-ante differences between incumbent and opposition strongholds that exist in any real-world election. By contrast, we now refer to the privately observed and ex-ante identical realizations $S_i$ as the incumbent’s variable support. The precinct-level result thus amounts to

$$R_i = \alpha P_i + (1 - \alpha) S_i + 1_{\{a_i = f\}} F,$$
where \( \alpha \) denotes the weight of persistent relative to variable support. This modification implies that the incumbent wins the election as long as

\[
\alpha \pi + (1 - \alpha) \theta + \phi F \geq \frac{1}{2},
\]

with \( \pi \) denoting the national-level average of the persistent component of the incumbent’s genuine support.

This distinction between the persistent and variable components in the incumbent’s support suggests a change in the basis for agents’ rewards: rather than paying an agent by the overall result in her precinct, the incumbent now bases her reward on the difference between the overall precinct-level result \( R_i \) and the known level of the incumbent’s persistent precinct-level support \( P_i \), since the latter is guaranteed and commonly known. Put differently, an agent who delivers 70% where the incumbent was expected to receive only 50% of the vote is rewarded more than one who delivers 90% where no less was expected in the first place. We derive the incumbent’s optimal choice of the reward factor as well as the equilibrium fraud, popularity, and majority thresholds in the supplementary appendix.

This extension yields two intuitive insights. First, the equilibrium reward factor \( w^* \) is decreasing in \( \pi \), the national-level average of the persistent component. This result parallels that from the previous section as an increase in \( \pi \) amounts to a more optimistic expectation about the incumbent’s genuine popularity. Second, while the equilibrium supply of fraud is increasing in \( \theta \) just as in the basic model, precincts with the largest realizations of variable support \( S_i \) may not coincide with those where the overall result \( R_i \) is largest. Rather, fraud will be most abundant in precincts where the election outcome \( R_i \) exceeds pre-election expectations based on the persistent component \( P_i \) alone.
2.4 An Alternative Information Structure: The Normal Model

The key advantage of the uniform information structure that we have employed so far is the availability of closed form solutions for the fraud, popularity, and majority thresholds, and in turn, the ease with which their political implications can be studied. In order to establish the robustness of our results, we examine our model under an alternative information structure – the Normal model. Specifically, we denote by $\theta'$ the probit-transformed version of the incumbent’s popularity $\theta$ and assume that it follows the Normal distribution with mean $\theta'_0$ and variance $\sigma^2_0$, $\theta' \sim N(\theta'_0, \sigma^2_0)$. Analogously, we let $S'_i$ denote the probit-transformed version of the agents’ signals $S_i$ and assume that it follows the Normal density with the mean $\theta'$ and variance $\sigma^2$, $S'_i \sim N(\theta', \sigma^2)$.

We transform $\theta'$ and $S'_i$, whose support is on $(-\infty, \infty)$ onto $(0, 1)$, the natural interval $\theta$ and $S_i$, via the probit link, $\theta' = \Phi^{-1}(\theta)$ and $S'_i = \Phi^{-1}(S_i)$.

An appealing feature of the Normal information structure is that it allows for prior beliefs about $\theta$ of an arbitrary mean and precision while maintaining a positive amount of uncertainty about $\theta$ and $S_i$ along the entire support $(0, 1)$. Unlike in the case of the uniform parameterization however, the threshold agent’s indifference condition for the Normal model does not have a closed form solution. Equilibrium fraud, popularity, and majority thresholds must therefore be obtained numerically. Figure 5 illustrates the Normal model by plotting the effect of the pre-election expectation $\theta_0$ about the incumbent’s popularity on his equilibrium choice of the reward factor $w^*$ and the expected equilibrium election result $R^*$ at $\sigma^2_0 = 100, 1, \frac{1}{25}, \frac{1}{100}$ (clockwise starting at top-left.)

We see that key conclusions from our earlier analysis remain unchanged. As the right

---

27 Paralleling our earlier interpretation of $\epsilon$, we think of the variance $\sigma^2$ of $S'_i$ as a metric of heterogeneity in the incumbent’s support across precincts.

28 Uniqueness in the Normal model obtains as long as the signal $S_i$ is sufficiently precise relative to the prior belief about $\theta$; see the supplementary appendix.
Figure 5: The effect of pre-election expectations about the incumbent’s popularity $\theta_0$ on the equilibrium reward factor $w^*$ (left) and the expected equilibrium election result $R^*$ (right) for $\sigma_0^2 = 100, 1, \frac{1}{25}, \frac{1}{100}$ (clockwise starting at top-left.)

panel shows, fraud is either over- or under-supplied in equilibrium due to the interplay of the collective action and principal-agent problems. The key difference between the Uniform and the Normal models is that positive levels of fraud may occur at all values of $\theta$ in the latter. This is because even when $\theta$ is close to 0 (or 1), there is a small measure of agents who observe precinct-level signals implying that the incumbent is overwhelmingly popular (or unpopular) when the opposite is the case.

Figure 5 also indicates that the incumbent indeed views his choice $w$ as insurance against electoral defeat. Given only vague prior information about his eventual popular support $\theta$, the incumbent is willing to reward the agents for conducting fraud even when he expects to prevail, hedging against the odds that he is being too optimistic. While equilibrium comparative statics for the effect of $\sigma_0^2$ on $w^*$ are mathematically too complex to be tractable, simulations suggest that as the precision of prior information about $\theta$ improves, the incumbent reduces the agent’s rewards at high and low levels of his expected
popularity. This is consistent with our discussion in Section 2.2 of $w$ as insurance against defeat: when the incumbent expects an overwhelming victory, compensating the agents would only result in unnecessary fraud and thus be a waste of resources; when he expects an overwhelming defeat, any consequential reward factor would have to be so high as to render fraud too expensive.

### 3 Empirical Analysis

Our theoretical analysis leads to a number of empirical predictions. First, incentive conflicts between the incumbent and his agents result in either the undersupply or oversupply of fraud from the incumbent’s point of view. Second, a key feature of the herd dynamic that occurs among the incumbent’s agents is that small shifts in their perception of the incumbent’s viability may trigger large aggregate shifts in the amount of fraud conducted; this herd dynamic is strongest when the incumbent’s genuine popularity is close to the threshold $\theta^*$. Jointly, these predictions anticipate a pattern of elections that are won or lost by large margins correlated with the incumbent’s popularity. In these elections, the incumbent’s defeat may be instigated by only a minor decline in his genuine popularity, and while it may be widespread, electoral fraud is politically decisive in only a fraction of the elections in which it occurs. Finally, our analysis predicts that the extent of fraud – and thus its fingerprints – should be increasing in both the incumbent’s genuine support as well as his vote share across precincts. Hence we should expect a positive association between the incumbent’s popularity and the extent of fraud across both multiple elections and individual precincts in a single election.

As a first step toward the empirical assessment of our arguments, we focus on the last of these predictions: That the extent of fraud across individual precincts should be increasing
Figure 6: The distribution of Vladimir Putin’s precinct-level vote share in the 2012 presidential election

in the incumbent’s share of the vote. While fraud is difficult to identify in most elections, we take advantage of a particular form of fraud that arguably took place during the 2011 legislative and 2012 presidential elections in Russia: the rounding of the incumbent Vladimir Putin’s/United Russia Party’s precinct-level vote share to some higher multiple of five by the regime’s local operatives (Gehlbach 2012; Klimek et al. 2012; Kobak, Shpilkin, and Pshenichnikov 2012). Although this form of fraud was most likely only one among several forms of electoral manipulation during these elections, its execution at the level of individual precincts provides an opportunity to assess whether the extent of fraud was indeed increasing in the incumbent’s precinct-level vote share – as our theoretical model anticipates.

Several features of the 2011 legislative and 2012 presidential elections in Russia
correspond to the key elements of our model. Extant empirical research as well as journalistic accounts indicate that fraud did indeed occur in these elections (Enikolopov et al. 2013); that fraud was executed locally by operatives within the state bureaucracy, the public sector, and the United Russia party (Frye, Reuter, and Szakonyi 2012); that these operatives where motivated by the promise of a political or bureaucratic advancement or monetary rewards – and often also by the complementary threat demotion or dismissal (Reuter and Robertson 2012); that the regime had only rough information about its genuine support due to a pro-regime bias in public opinion surveys (Kalinin 2013); and that the regime anticipated that unnecessary fraud might occur and wanted to avoid it unless it was needed for a first-round victory. Due to space constraints, we focus below on the 2012 presidential election; our analysis of the 2011 parliamentary election – which provides even stronger support for our arguments – can be found in the supplementary appendix.

As a preliminary step, we establish that multiples of five are indeed over-represented in Vladimir Putin’s precinct-level vote shares. Figure 6 plots the distribution of Putin’s vote share in the 2012 presidential election across more than 90,000 precincts. In spite of the large number precincts, we see a suspicious lack of smoothness due to spikes that mostly coincide with multiples of five, especially in the range from 55% to 100%. In order to examine the distribution of digits in precinct-level vote shares more formally, we round each candidate’s vote share to the nearest multiple of 0.5, extract the unit and the first decimal place digits (e.g. both 76.481 and 46.532 become 6.5), and pool them into the twenty resulting digit pairs. The distribution of these pooled digit pairs in Vladimir Putin’s

---

29 For crowdsourcing reports about fraud during these elections, see www.kartanarusheniy.org.
30 For anecdotal accounts, see Judah (2013, 231-237).
31 According to the journalist Julia Ioffe, “the puzzling command from Moscow” to local party leaders was “victory for Putin in the first round – that is, over 51 percent – but no violations.” See Ioffe, Julia, “The Last Waltz,” Foreign Policy, 12 March 2012.
32 Our results do not depend on the extent of rounding; rounding to units or one decimal place leads to
Figure 7: The distribution of the pooled unit and the first decimal place digits in Putin’s precinct-level vote share (after rounding to the nearest multiple of 0.5)

Consistent with our discussion above, precinct-level vote shares that end in either 0.0 or 5.0 are over-represented for Putin, and crucially, only for Putin. This is confirmed by a series of likelihood ratio independence tests. Assuming that neighboring digits should be distributed approximately uniformly, we compute the $G^2$ statistic (Agresti 1996, 36) for the frequencies of 0.0 and 5.0 and the two digit pairs to their left and right. These are the digit pairs \{9.0, 9.5, 0.0, 0.5, 1.0\} and \{4.0, 4.5, 5.0, 5.5, 6.0\}, respectively. The $G^2$ statistics (76.6 and 40.0 with $df = 4$) strongly suggest that these digit frequencies are not uniform (both identical conclusions. We dropped all precincts with fewer than 50 voters in order to exclude special-category precincts (hospitals, military units) and to eliminate small-N effects on precinct-level vote shares.

Vote shares that end in 0.0 tend to be over-represented for the three minor candidates (Prokhorov, Zhirinovsky, and Mironov) but only because there are many precincts in which these candidates receive close to 0% of the vote. After excluding precincts where any candidate received less than 1% of the vote, precinct-level vote shares that end in either 0.0 or 5.0 remain over-represented for Putin but not for any other candidate.
$p$-values = 0). Once we exclude the digit pairs 0.0 and 5.0, however, the remaining digit frequencies are consistent with uniformity ($G^2 = 3.2$ and 1.7 with $df = 3$ implying $p$-values of 0.37 and 0.65, respectively). Thus the lack of uniformity in Putin’s digit distribution is overwhelmingly due to the over-representation of the multiples of 5 among his precinct-level vote shares.\footnote{This is implied by the partitioning property of chi-squared statistics (Agresti 1996, 39-40); an analysis based on standardized residuals implies the same conclusion. In the supplementary appendix, we also examine these results using Benford’s Law techniques.}

We now turn to our main empirical test and examine whether the over-representation of the multiples of 5 for Putin is indeed increasing in his precinct-level vote share as predicted by our model. In order to do so, we first develop a measure of the ruggedness in the distribution of Putin’s precinct-level results: We take the difference between the empirical distribution of a candidate’s precinct-level results and its optimal kernel density estimate.\footnote{A kernel density estimate is a smooth, nonparametric estimate of a density function. The extent of smoothing is determined by the choice of a bandwidth, which sets the weight that the estimator assigns to each data point’s neighboring observations (see e.g. Cameron and Trivedi 2005, Chapter 9).}

The former is based on a histogram with 0.5 bin width; the latter employs the (optimal) Epanechnikov kernel and the optimal bandwidth estimate (Cameron and Trivedi 2005, Chapter 9).\footnote{The optimal bandwidth estimate minimizes the mean integrated squared error based on a Gaussian kernel. In the supplementary appendix, we confirm the robustness of these results by using both twice and half the optimal bandwidth estimate (as recommended in Cameron and Trivedi 2005, 304).} Figure 8 plots the kernel density estimate of each candidate’s precinct-level results with a black dashed line along with their actual empirical distribution (gray solid line). We see a nearly perfect overlap between the distribution of precinct-level vote shares and the corresponding kernel density estimates for the three minor candidates (Prokhorov, Zhirinovsky, and Mironov), some ruggedness unrelated to multiples of 5 for Zyuganov, and significant ruggedness correlated with multiples of 5 for Putin.

Figure 8 further highlights that the ruggedness in the distribution of Putin’s precinct-level results is not only substantial but also increasing in his precinct-level vote share.
share, as our theoretical framework anticipated. In order to quantify how anomalous this ruggedness is and to evaluate the strength of its association with Putin’s precinct-level vote share, we employ two standards. The first is empirical: We judge the ruggedness of Putin’s precinct-level results by the standard of his four competitors. Specifically, we calculate the difference between the empirical distribution of Putin’s competitors’ precinct-level results and their kernel density estimate, pool these residuals, and use their 95th and 99th percentiles as our first benchmark for judging how anomalous the ruggedness of Putin’s precinct-level results is.\footnote{In order for these residuals to be comparable across candidates, we divide them by the height of the corresponding candidate’s KDE. Each residual then measures the difference between a candidate’s empirical distribution of precinct-level results and its KDE relative to its own height.} Figure 9 plots these residuals separately for Putin (diamonds), Zyuganov (squares), and the remaining three minor candidates (Prokhorov, Zhirinovsky, and Mironov). We see that with the exception of a few poorly fitting corner values for the minor candidates, all residuals above the 95th and 99th percentile benchmarks belong to either Putin or Zyuganov. Crucially, only Putin’s residuals coincide with the multiples of five and are significantly increasing in his vote share.\footnote{When we regress Putin’s residuals on his vote share, the vote share coefficient is positive and statistically significant at the 0.01 significance level; the regression coefficient on the other candidates’ vote share is statistically significant but negative.}
We arrive at identical conclusions when we employ an alternative, theoretical benchmark for evaluating the ruggedness of Putin’s precinct-level results. We compute the 95% and 99% asymptotic confidence intervals for the kernel density estimate of Putin’s results and treat the observations that lie outside these confidence intervals as anomalously rugged. Just as before, Putin’s and Zyuganov’s residuals are significantly larger than those of the remaining candidates and only Putin’s residuals are increasing in his precinct-level vote share.\footnote{Due to space constraints, these plots are presented in the supplementary appendix.} Hence judging by two different standards – that of Putin’s competitors and asymptotic confidence intervals – the distribution of Putin’s results is indeed suspiciously rugged at multiples of five and this ruggedness is increasing in his precinct-level vote share as anticipated by our theoretical arguments.

Our analysis of the ruggedness in the distribution of precinct-level results in the 2012
Russian presidential election also sheds light on how fraud was conducted. Throughout our discussion above, Zyuganov was the only candidate other than Putin with a significant amount of ruggedness in his precinct-level results. Yet crucially, this ruggedness did not coincide with multiples of five and was not increasing in his vote share. This observation suggests that rather than simply stuffing ballots, Putin’s local operatives may have been stealing votes from Zyuganov and falsifying the final protocols accordingly. This makes logistical sense: Zyuganov was the only major opposition candidate in this election and hence the only candidate in most precincts with a large enough number of votes that could be transferred to Putin in order to round up Putin’s vote share to some higher multiple of five. In order to evaluate this hypothesis, we add Putin’s and Zyuganov’s precinct-level votes and examine the ruggedness in the resulting distribution of vote shares. As Figure 10 reveals, the significant ruggedness in the two candidates’ individual vote-share distributions now disappears – supporting our hypothesis of vote-stealing from Zyuganov.40

To summarize, our analysis of the 2011-2012 Russian legislative and presidential elections (see the supplementary appendix for the former) supports a key prediction from our theoretical model: The extent of fraud in these elections is indeed increasing in the incumbent Vladimir Putin’s and United Russia’s precinct-level vote share and, crucially, in only their vote share. We took advantage of a particular type of fraud that occurred during these elections – the rounding of the regime candidate’s and party’s vote share to some higher multiple of five. Consistent with the findings from earlier analyses of these elections, we found that precinct-level vote shares corresponding to multiples of five are indeed over-represented in Putin’s and United Russia’s results but not those of other candidates or parties. By analyzing the ruggedness in the distribution of precinct-level results of all

---

40This is confirmed by a formal analysis of the KDE residuals in the supplementary appendix. Zyuganov and his Communist party have a history as the main opposition target for manipulation, see e.g. Myagkov, Ordeshook, and Shakin (2009, Chapter 3).
competing candidates and parties, we found that the votes necessary for this fraud were most likely stolen from the leading opposition candidate Gennady Zyuganov and his Communist Party.

Nonetheless, there are some aspects of these elections that our data and analysis do not allow us to address. Chief among them is why fraud took the particular form of the rounding of the incumbent’s precinct-level vote share to some higher multiple of five. Plausible explanations include regional targets that themselves were multiples of five, attempts by local operatives to signal complicity without leaving direct evidence (c.f. Kalinin and Mebane 2011), or the human tendency to fabricate numbers that are multiples of five (c.f. Kobak, Shpilkin, and Pshenichnikov 2014). Unfortunately, we cannot discriminate among these conjectures with the available data. Our results – just like many
qualitative accounts of fraud during these elections – also indicate that a different method of manipulation was employed in some ethnic republics and “special” precincts (e.g. hospitals and military units). Results from these places show close to 100% turnout and vote-share for the regime, suggesting that they were delivered wholesale with fraud that was executed at the top rather than the bottom of the administrative hierarchy.\textsuperscript{41} Finally, we are unable to eliminate the alternative hypothesis that anomalies indicative of fraud may be increasing in Putin’s and United Russia’s precinct-level vote share because observers, local electoral commission members, or even ordinary citizens are less likely to deter fraud in precincts where the regime is highly popular.\textsuperscript{42}

4 Conclusion

Why do incumbents who could arguably win a clean election engage in fraud? Our analysis of the principal-agent and collective action problems in the political organization of electoral fraud suggests an answer. Because most fraud is ultimately executed by a large number of local operatives with an incentive to conceal their actions, incumbents’ control over whether and how much fraud will be conducted is limited. Each agent understands that the difference between her promotion and prosecution rests on whether her involvement in fraud will result in the incumbent’s eventual victory or defeat. In turn, the agents’ perception of the incumbent’s popularity and the ensuing burden on their collective complicity plays a crucial coordinating role. The aggregate result is too much fraud for incumbents who need it the least and too little fraud for those who need it the most. The seeming invincibility of some incumbents and the surprising fragility of others are thus two

\textsuperscript{41}The right tail in Figure 10 corresponds to such precincts. 
\textsuperscript{42}Enikolopov et al.’s (2013) experimental evidence indicates that a significant amount of fraud occurred even in Moscow, where Putin’s regime is arguably least popular.
sides of the same political logic.

Our analysis of the 2011-2012 Russian legislative and presidential elections finds support for one prediction based on these intuitions: That the extent of fraud should be increasing in the incumbent Vladimir Putin’s and United Russia’s precinct-level vote share. Yet this prediction is only one among several implied by our theory. The Russian case, along with pre-1980s Mexico and Daniel Ortega’s Nicaragua (especially the 2008 local and 2011 presidential elections), illustrates the scenario in which an incumbent employs fraud as insurance against defeat, with its oversupply becoming the undesirable byproduct when that insurance proves unnecessary. Meanwhile, Mexico in 1988 and 2000, Senegal in 2012, and Sri Lanka in 2015 illustrate the undersupply of fraud that obtains when the machinery of fraud is reluctant to follow an incumbent with a declining popularity.

Our arguments apply most directly to political systems that have been alternately referred to as hybrid, electoral authoritarian, or competitive authoritarian regimes (Bunce and Wolchik 2011; Levitsky and Way 2010; Schedler 2013). A distinguishing feature of these regimes is the presence of elections that are competitive enough to allow for an active opposition yet at the same time systematically favor the incumbent. Our arguments explain why these regimes may be significantly more fragile than existing research admits. Even if the infrastructure of fraud favors the incumbent, his genuine popularity still plays a crucial role in shaping the agents’ perception of the risk of engaging in fraud. Electoral authoritarianism should therefore be characterized by a punctuated dynamic, with an oversupply of fraud that lasts as long as the incumbent enjoys genuine popularity and is interrupted by an undersupply of fraud as soon as that popularity dwindles. Even manipulated elections thus hold the potential for a democratic opening.

Our analysis of the political organization of electoral fraud suggests a number of

\[ F \]

Our model captures this pro-incumbent bias via the fraud capacity parameter $F$.  

38
directions for future research. Our model intentionally focuses on forms of manipulation that may be classified as locally-executed, election-day fraud. This category captures some of the most frequent forms of fraud: ballot box stuffing, voter intimidation, count falsification, vote buying, multiple voting, etc. Nonetheless, these forms of fraud are frequently preceded and sometimes followed by their centrally-executed counterparts. Our results point to several trade-offs inherent in a regime’s decision about the timing and the level of manipulation. As we have shown, incumbents who rely on locally-executed election-day fraud end up with only limited control over how much of it will be conducted. By contrast, top-level election-day fraud provides such control – since it amounts to the re-writing of the election result by the incumbent – but it is brazen and therefore best used as a weapon of last resort. Manipulation before the election, meanwhile, allows for more subtle ways of stacking the deck against the opposition, but because of its timing, it remains vulnerable to a public exposure and condemnation by the opposition. Future research would ideally account for these trade-offs explicitly, by incorporating the choice of the timing and level of manipulation within a single model.

Our analysis also highlights a novel, distinctively political feature of the principal-agent problem in the execution of electoral fraud. Just like in many other political and economic agency problems, the incumbent – the principal – observes the agents’ actions only imperfectly: the incumbent cannot condition the agents’ reward on the actual amount of fraud since the latter is intentionally concealed. Yet unlike in most settings, the principal’s problem is further complicated by the politically conditional nature of the agents’ rewards: the agents know that they will obtain the promised rewards only if the incumbent eventually wins the election. To facilitate our analysis of this principal-agent problem, we worked with a simple reward structure, according to which the agents’ rewards linearly
increase in the precinct-level election outcome – but only if the incumbent wins. Yet real-world fraud reward structures are never written or even explicitly stated, often include a mix of positive and negative incentives, and are typically embedded within the institutional infrastructure of regime parties and government bureaucracies. Future research should explain these qualitative features of the political organization of electoral fraud as an optimal choice within a more general model of political agency.

Finally, our intuitions naturally extend to other settings, most directly to patronage politics, election campaigns, and repression. Just as in the present setting, the rewards and punishment of patronage brokers, campaign operatives, and repressive agents are contingent upon the success or survival of their principals. In patronage politics, for example, a broker’s effort on behalf of a candidate should be driven by her expectation of the candidate’s ultimate victory and hence the likelihood that her effort will not be wasted. Meanwhile in repression, the willingness of a repressive agent to engage in legally questionably suppression of opposition should be driven by his expectation of the regime’s survival and hence his immunity from prosecution. Just as in the case of fraud, candidates and leaders who want to motivate their operatives to engage in electioneering or repression have only limited control over whether and how much of it will eventually be conducted. This is because patronage brokers, campaign operatives, and repressive agents have one thing in common: they all want to work for a winner.

44The ubiquity of linear contracts in real-world principal-agent relationships is a long-standing puzzle (see e.g. Holmstrom and Milgrom 1987). Carroll (2015) shows that a linear contract is optimal when the principal is uncertain about the actions available to the agent and evaluates alternative contracts by their worst-case performance – conditions that likely hold in the present setting.

45For an analysis of principal-agent problems in clientelism and repression, see Stokes et al. (2013) and Svolik (2012), respectively.
References


Cameron, A. Colin, and Pravin K. Trivedi. 2005. *Microeconometrics: Methods and Applica-


Columbia University and University of Wisconsin.


cation: Case of Russia.” Unpublished Manuscript, University of Michigan.


Little, Andrew T. 2013. “An Informational Theory of Noncompetitive Elections.” Unpub-


Miller, Michael K. Forthcoming. “Elections, information, and policy responsiveness in authoritarian regimes.” *Comparative Political Studies*.


