Use of Chaining Strategies in the Presence of Disruption Risks

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In this paper, we study the use of the chaining strategy introduced by Jordan and Graves (1995) for designing a robust production/supply chain network in which the links and nodes are susceptible to disruptions. We introduce the concept of fragility to quantify the change in system performance resulting from a disruption. Although one may anticipate that networks with longer chains will be more robust (will have smaller expected fragility) than shorter ones, our study reveals that this is not always true. We show that the expected fragility with respect to a single link failure decreases as the size of the chain decreases; however, the expected fragility with respect to a single node failure increases as the size of the chain decreases. We also show that multiple failures in a network can be decomposed into a set of multiple subnetworks each with a single failure; hence, we can analyze the impact of large-scale disruptions by studying each single failure subnetwork. Simulation experiments are used to extend insights from single link or node failures to multiple failure cases.
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1. Introduction

When designing production/supply chain systems, dealing with demand and supply uncertainty has been an important topic of concern in the literature. While carrying redundancy certainly helps hedging against such uncertainties, Sheffi and Rice (2005) emphasize the value of flexibility. Flexibility increases the resiliency and responsiveness without incurring the costs of high inventory or excess capacity. Honda's recent experience also confirms flexibility as an essential weapon in facing demand uncertainties – with the flexibility of producing SUVs (CR-V) and compact cars (Civic) on the same production line, Honda was able to match consumers' demand faster than any of their rivals when fuel prices were fluctuating (Wall Street Journal, 2008).

One of the foundational contributions in the flexibility literature is by Jordan and Graves (1995) (hereafter J-G). In a single stage, multiple product, multiple plant setting, it was shown that *chaining* is the most effective network configuration to hedge against demand uncertainty. Using a limited number of links, a chaining configuration maximizes the degree of flexibility that results in the minimum possible demand shortfall; i.e., minimum *lost sales cost*. They further show that longer chains outperform shorter chains and thus having one long chain that connects all supply and demand nodes is the the ideal network configuration. While this is a very inspiring finding, it implicitly assumes that the cost of establishing each link is identical regardless of its location. In this paper, we explicitly capture the *fixed cost* of flexibility investment. Assuming that the cost of flexibility increases with product dissimilarity, we show that establishing longer chains (a chain that covers broader range of product types) are more expensive than multiple shorter ones. Imagine the cost of building two auto plants: one that is only capable of producing two similar products within the same product family; the other one that produces two extremely different products, a compact car and a pick-up truck, for example. A plant designed to produce models based on the same platform will have a significantly lower fixed cost than a plant designed to produce two models based on completely different platforms. Considering these two costs, we can obtain an optimal size of chains that minimizes the total cost. To illustrate this point we conducted a numerical study with 24 supply and demand nodes. Figure 1 plots total costs of all possible chaining configurations.[1] For this example, the optimal average chain size is 4.833 with four chains of size 5 chains and one size 4 chain.

While J-G only considers demand uncertainty, we also consider supply uncertainty. In particular, we consider two types of disruptions: link and node failures. A link failure represents the case in which a plant is no longer able to produce a specific product as a result of a product-specific supplier or machine failure. Link failures can be commonly observed in the high-tech industry where each type of product goes through many product-specific processes. For example, in the semiconductor industry, each wafer goes through many product-specific stages and a contamination in one of the workstations results in a disruption for that specific product line. In such cases, a plant typically produces an alternative product to maintain a targeted utilization level. A node failure, on the other hand, represents the case in which the entire supply node fails such that it cannot serve any demand. A strike or a fire at a facility are examples of incidents that lead to node failures.

In this paper, we analyze how chaining flexibility performs (and should be modified) when link

[1] We set the capacity of each supply node equal to the mean of each demand node. The coefficient of variation on each demand was 20%. Details of the simulations will be elaborated more in §3 and 4.

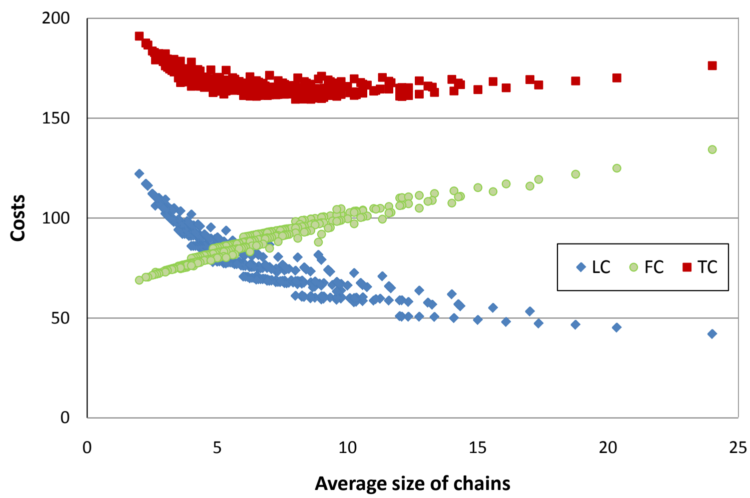

Figure 1: Total cost for various chaining configurations on a size 24 network

or/and node failures are taken into account. In particular, we focus on the J-G insight that one long chain connecting all supply and demand nodes is superior to multiple shorter chains. To study the impact of disruptions on network designs, we introduce two flexibility strategies: (a) a *coverage strategy* that grows the size of chains following J-G's flexibility guideline; and (b) a *containment strategy* that reduces the size of chains, contrary to the prevalent belief in the flexibility literature. The coverage strategy increases the ability of one particular supplier to (indirectly) assist another supplier in meeting larger than expected demand. The containment strategy, in contrast, contains the impact of a disruption to a smaller region of the network with shorter chains. Our goal is to understand if there are circumstances that favor the containment strategy over the coverage strategy in the presence of random network disruptions.

The main contributions of our study are as follows:

(1) We show that the common intuition on chaining flexibility guidelines may not always hold in the presence of network disruptions. If a network is more susceptible to a link failure, the containment strategy is preferable, whereas the coverage strategy is preferable when the network is more susceptible to a node failure [See §4].

(2) We introduce an approximation scheme for deriving the expected fragility for single component failures. We further show that multiple failures can be decomposed into single failures; hence, the expected fragility of a network subject to multiple failures can also be calculated approximately. [See §5].

(3) We provide managerial insights under a more generalized setting using a simulation study. We

show that when the failure probability is high (i.e., multiple network components fail concurrently), the containment strategy becomes very effective in mitigating risks [See §6].

The remainder of this paper is structured as follows. In Section 2, we review the related literature. In Section 3, we provide the detailed description of the model. In Section 4, we perform a simulation on a basic network setting and derive some observations. In Section 5, we analytically support the observations made from the network failures starting with single failure cases and progressing to multiple failure cases. A simulation study of a more generalized random failure case is performed in Section 6, and we conclude the paper by suggesting managerial insights and addressing future research in Section 7. All proofs and detailed derivations are provided in appendices.

2. Literature Review

Initiated by Jordan and Graves (1995), chaining configurations have been extensively studied on various applications. These include: cross-trained workforces (Jordan et al. 2004, Hopp et al. 2004, Iravani et al. 2005), service systems (Gurumurthi and Benjaafar 2004, Iravani et al. 2007, Tekin et al. 2009), and flexible transshipments (Lien et al. 2005, Herer et al. 2006). For supply chain network design, Garavelli (2003) observes that, given some extent of flexibility, a network with a chaining configuration provides exceptional performance in various measures (such as WIP and production lead time). Graves and Tomlin (2003) provide similar flexibility guidelines for reducing operational inefficiencies in multistage supply chains. Hopp et al. (2010) show that, in a multi-echelon supply chain, flexibility (including chaining) investment should be made in the stage in which the demand variability is the highest.

While the above papers provide a good foundation in identifying the benefit of chaining configurations and/or provide tactics on how to implement chaining configurations, they do not consider the risk of disruptions. In the last few years, there has been increased focus on the design and analysis of strategies to mitigate supply disruptions (e.g., Kleindorfer and Saad 2005). One interesting point from these studies is that the mitigation strategies for supply disruptions often differ from those used for demand uncertainties. Tomlin and Wang (2005) show that the common intuition that a flexible strategy is always preferred to a dedicated strategy can be wrong when the underlying suppliers are not perfectly reliable and the firm is not risk-neutral. Snyder and Shen (2006) and Schmitt et al. (2008) show that mitigation strategies for supply uncertainty not only differ from those of demand uncertainty, but often tend to be the exact opposite (e.g., order frequency, inventory placement).

Moreover, some recent studies point out the constraining aspect of flexibility. Bish et al. (2005) caution that flexibility may not always be cost-effective since it may cause greater swings in production, consequently increasing the order variability and inventory cost. Muriel et al. (2006) extend the idea to larger systems and show that limited flexibility (such as chaining) results in a considerable increase in production variability, resulting in higher operational costs. Pinker et al. (2010) point out that some form of flexibility leads a firm to staff with too little slack to respond to demand shocks, thus resulting in higher costs in the long run. It is in this context that our paper attempts to revisit the insights of J-G in the presence of disruption risks. It is not certain whether the prevalent wisdom on chaining literature will remain the same in the presence of disruption risk and the goal of this paper to provide insights on chaining strategies under these circumstances.

Unfortunately, analytically evaluating the performance of chaining is challenging (as discussed in Jordan and Graves (1995)) due to its combinatorial nature of the problem. For this reason, papers that provide concrete analytical results on chaining are limited. Aksin and Karaesmen (2007) provide some structural properties on the interaction between flexibility and capacity by using a network flow model. In another study, Chou et al. (2010) analyze the asymptotic performance of long chains by using a random walk process. Along with an extensive numerical study, we also provide an approximation scheme as an alternative method to understand (at least partially) chaining in the presence of disruptions.

3. The Model

3.1 Basic Model

We focus on a single-stage supply chain with multiple products and plants. Consider a bipartite network $G = (V, E)$ where V is the set of nodes and E is the set of links. The set V consists of the set of suppliers S_i (left hand nodes) and the set of product demands D_j (right hand nodes); i.e., $V = \{S_i \mid i \in I\} \cup \{D_j \mid j \in J\}$. Each link (i, j), that is denoted by e_{ij}, represents a supplier's ability to produce the corresponding product. We first focus on a basic setting: a *symmetric* network of size $|I| = |J| = N$ (a network containing N identical supply and demand nodes) with *non-overlapping* chaining configurations (each chain does not share common nodes). This simplifies the analysis and allows us to clearly understand factors that support containment or coverage. We later study asymmetric networks and modified chaining configurations in Section 6. We denote a configuration of a network G that consists of N demand (and supply) nodes with chains of size n_i by $G(n_1, n_2, ..., n_\ell)$ where $\sum_{i=1}^{\ell} n_i = N$ for all $n_i > 1$. We do not consider size 1 chains in our analysis as such chains do not exhibit any flexibility. Figure 2 illustrates examples of four different chaining configurations, $G(8)$, $G(3,5)$, $G(3,2,3)$, and $G(2,2,2,2)$, on a size 8 network. If a size N network consists of identically sized chains of size n, we compress the notation to $G(n|N)$; for example, $G(2,2,2,2) = G(2|8)$. We denote the set of all possible chaining configurations with network size N by $\mathcal{G}_N$; i.e., $\mathcal{G}_N = \left\{G(n_1, n_2, ..., n_\ell) : \sum_{i=1}^{\ell} n_i = N \text{ where } \ell \geq 1, n_i > 1 \; \forall i\right\}$.

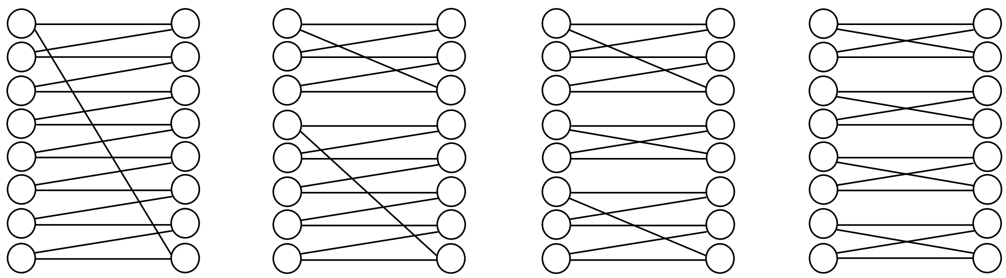

Figure 2: Examples of various chaining configuration on a size 8 network

The network designer makes a decision on the flexibility investment by choosing the network configuration G that meets the customers' demand most effectively. This decision implies which links to establish. We denote this by using $y_{ij} = 1$ if e_{ij} is used; and 0 otherwise. We denote the stochasticity in the model by ξ. Without considering disruptions, only demand is random; hence, $\xi = d$. We later augment ξ with a network component condition to capture disruptions.

We formulate the model in two stages. The first stage problem is referred to as a *network design problem* in which the optimal configuration G is found to minimize the total cost. The total cost consists of the fixed cost (FC) associated with establishing links, l_{ij}, and the lost sales cost (LC) that represents the penalty associated with expected shortfalls (unmet demands), SF. In our analysis, we assume that the penalty cost p per unit that is common across all products. Therefore, the total cost of a network G, $TC(G)$ is formulated as follows:

$$\begin{aligned} \text{Minimize } \; TC(G) \; &= \sum_{(i,j)\in E} l_{ij} y_{ij} + p\,\mathbb{E}_\xi[SF(G)] \qquad [P1] \\ \text{subject to } \; G \; &\in \; \mathcal{G}_N \end{aligned}$$

where the shortfall, $SF(G)$, is obtained by the second stage problem.

The second stage problem is referred to as a *capacity allocation problem* in which the shortfall is minimized by properly allocating the capacity of each production plant after the demand realization. We denote the capacity of supply node i by c_i and the demand for product j by d_j. Then, the resulting formulation of the second stage problem is as follows:

$$
\begin{aligned}
SF(G) = \quad & \text{Minimize} \quad && \sum_{j \in J} s_j && [P2] \\
& \text{subject to} && \sum_{i \in I} x_{ij} + s_j \geq d_j && \forall j \in J \\
& && \sum_{j \in J} x_{ij} \leq c_i && \forall i \in I \\
& && x_{ij} \leq M y_{ij} && \forall i \in I, \forall j \in J \\
& && \boldsymbol{x} \in \mathbb{R}_+^{|E|},\ \boldsymbol{s} \in \mathbb{R}_+^{|J|}
\end{aligned}
$$

where M represents a large number (e.g., $M = \sum_i^{|I|} c_i$). With the restriction on the number of links to be used ($|E| = 2|I| = 2|J|$) and assuming l_{ij} to be a constant, the model is equivalent to the basic model of J-G.

Without loss of generality, let us index the product demands based on product similarity. We let the cost of a link, l_{ij}, as a strictly increasing and strictly convex function of product similarity. For ease of notation, we use $FC(n)$ to denote $FC(G(n))$. Similarly, we omit G in other cost terms as well. Based on the cost structures, we derive the following two propositions.

Proposition 1. *The fixed cost for network $G(n)$, $FC(n)$, is a strictly increasing and strictly convex function of n.*

Proposition 2. *The lost sales cost for network $G(n)$, $LC(n)$, is increasing with n. Further, $LC(n)$ asymptotically becomes linear as n grows; i.e., $\lim_{n \to \infty} \frac{LC(n)}{n} = \alpha$ where α is a constant.*

In light of Propositions 1 and 2, $TC(n)$ will also be a strictly convex function for large n. While it is challenging to analytically show that $LC(n)$ is a strict convex function, one can show numerically that this is true for small n as well, thus $TC(n)$ is also a strictly convex function of n. When this condition holds, we show that it is always better to "equalize" the size of the chains as much as possible. We summarize this in the following theorem.

Theorem 1. *If $TC(n) = FC(n) + LC(n)$ is strictly convex in n, then there exists an n^* such that any optimal configuration of the network will consist of chains of size n^* or $n^* + 1$, only.*

From Theorem 1, the configuration set $\mathcal{G}_N$ reduces to configurations with equalized chain sizes only (that can differ at most by one). We denote this set by $\hat{\mathcal{G}}_N \subseteq \mathcal{G}_N$. For example, for $N = 12$, the set of interest will be $\hat{\mathcal{G}}_N = \{G(2|12), G(2,2,2,3,3), G(3|12), G(4|12),\ G(6|12), G(12)\}$. Finally, a firm chooses an optimal configuration G_N^* from among the set $\hat{\mathcal{G}}_N$. Note that the optimal flexibility configuration G_N^* may not be a singleton.

3.2 Network Fragility

Currently, $[P1]$ and $[P2]$ do not consider disruptions. Now we consider a set of disruption events which we denote by Δ. We denote a set of scenarios with disruptions along with a demand realization by Ω_Δ and a set of scenarios without disruptions by $\Omega_\circ$; thus $\Omega = \Omega_\Delta \cup \Omega_\circ$. We reflect the impact of random disruptions by making the network G stochastic along with demand d; i.e., $\boldsymbol{\xi} = (\boldsymbol{d}, \boldsymbol{G})$ is random. An explicit expression in a functional form will be $\boldsymbol{\xi}(\omega)$ where ω is a possible scenario $\omega \in \Omega$. We denote the first stage problem including disruptions by $[P3]$ and the corresponding second stage problem by $[P4]$. Note that $[P4]$ now makes a capacity allocation decision over the network after disruptions. Further, we define the expected impact of a disruption as the expected fragility giving us the following definition:

Definition 1. *The expected fragility of a network G with respect to disruption Δ is the difference in expected shortfalls after and before the disruption. That is,*

$$\mathbb{E}[\mathcal{F}_\Delta(G)] \equiv \mathbb{E}_\xi[SF(G)|\Omega_\Delta] - \mathbb{E}_\xi[SF(G)|\Omega_\circ].$$

Note that the fixed cost is a sunk cost and disruption only affects the lost sales cost. By using this definition, the total cost of network G can be expressed as:

$$TC(G) = \mathbb{E}[TC(G)|\Omega_\circ] + p\,\mathbb{P}(\Omega_\Delta)\mathbb{E}[\mathcal{F}_\Delta(G)]. \tag{1}$$

The first term in (1) is a network design problem which minimizes the total cost *without* considering the impact of disruptions (equivalent to the objective function of $[P1]$). The second term represents the expected fragility of the network G with respect to disruption Δ. Hence, the total cost of a network configuration G is decomposed into two parts: one that corresponds to the total cost without disruptions and a second that corresponds to the expected fragility of the network. For the derivation of equation (1), please refer to the appendix.

Problem $[P3]$ can be solved optimally when the probabilities of the disruption events, $\mathbb{P}(\Omega_\Delta)$, are precisely given. Unfortunately, these values are hard to estimate in most of the cases. In this paper, we solve problem $[P3]$ in two steps: we first solve for and compute the network configurations which are optimal for $[P1]$; that is, we find G_N^*. We then assess the fragilities of other configurations in $\hat{\mathcal{G}}_N$ to identify the *directionality* (coverage or containment) that reduces the expected fragility. This directionality results in smaller expected total cost when disruptions occur. Using this, we derive the following proposition:

Proposition 3. *Let there be two network configurations, $G(n_1)$ and $G(n_2)$, that have identical expected total cost conditional on there being no disruption. If the expected fragility of G_1 is smaller than that of G_2 $\big($i.e., $\mathbb{E}\big[\mathcal{F}_\Delta(G_1)\big] \leq \mathbb{E}\big[\mathcal{F}_\Delta(G_2)\big]\big)$, then G_1 has a smaller expected total cost than G_2 accounting for disruptions $\big($i.e., $\mathbb{E}\big[TC(G_1)\big] \leq \mathbb{E}\big[TC(G_2)\big]\big)$ irrespective of $\mathbb{P}(\Omega_\Delta)$.*

To be more precise with the directionality of fragility, we define a function $\eta(\cdot)$ which maps a configuration G of a size N network to a real number as follows:

$$\eta(G(n_1, n_2, ..., n_\ell)) = \frac{\sum_{i=1}^{\ell} n_i^2}{N}.$$

The function η measures the *weighted average chain size* (number of demand nodes in a chain) for a given network. Hence, the coverage strategy corresponds to increasing values of $\eta(G)$, while the containment strategy results in smaller values of $\eta(G)$. A network designer will prefer the coverage strategy if the expected fragility of a network decreases as $\eta(G)$ increases. Similarly, the containment strategy will be preferred if the expected fragility decreases as $\eta(G)$ decreases. Our approach in the rest of the paper is to understand how fragility decreases as the average chain size $\eta(G)$ changes. By contrasting the average chain size of G_1 with G_2, $\eta(G_1)$ and $\eta(G_2)$, we obtain the directionality of fragility for a given set of disruptions.

Proposition 4. *If for a single failure δ, the expected fragility is monotone decreasing (increasing) on the integers, i.e., $\mathbb{E}[\mathcal{F}_\delta(2)] \geq \mathbb{E}[\mathcal{F}_\delta(3)] \geq \mathbb{E}[\mathcal{F}_\delta(4)] \geq \dots$ $\big(\mathbb{E}[\mathcal{F}_\delta(2)] \leq \mathbb{E}[\mathcal{F}_\delta(3)] \leq \mathbb{E}[\mathcal{F}_\delta(4)] \leq \dots\big)$, then for any two network configurations $G_1, G_2 \in \hat{\mathcal{G}}_N$ where $\eta(G_1) < \eta(G_2)$, we have $\mathbb{E}[\mathcal{F}_\delta(G_1)] \geq \mathbb{E}[\mathcal{F}_\delta(G_2)]$ $\big(\mathbb{E}[\mathcal{F}_\delta(G_1)] \leq \mathbb{E}[\mathcal{F}_\delta(G_2)]\big)$.*

By using Proposition 4, the directionality of fragility among configurations in $\hat{\mathcal{G}}_N$ with respect to a single failure δ can be obtained by studying the expected fragility of single-chained networks; i.e., $\mathbb{E}[\mathcal{F}_\delta(2)]$, $\mathbb{E}[\mathcal{F}_\delta(3)]$, $\mathbb{E}[\mathcal{F}_\delta(4)], \dots$. Next, we perform simulation experiments to study the directionality of the expected fragility for such networks.

4. Directionality of Fragility

4.1 Single Component Failure on a Symmetric Network

In this section, we evaluate the expected fragility for a single component failure on a symmetric network. Analyses of multiple component failures and generalized networks are provided later in Section 6. The insights that we gain from single component failures can be useful in drawing flexibility guidelines when the failure probability is *very small*. That is, when the probability of disruption is sufficiently small that a disruption event only consists of either a single link or node failure, the insights are valid. The results of single component failure will also be used later to analyze the case of multiple failures.

When a single link or node failure occurs on a network, it only affects one chain in the network while the remaining chains do not exhibit any failures. Using this fact, we can show that it is sufficient to consider *only* the fragility of single-chained networks to establish whether one prefers longer versus shorter chains.

We consider the following two systems throughout the study: a *balanced* system in which a supplier's capacity equals the expected product demand ($\mu = 100, c = 100$) and a system with 20% excess capacity ($\mu = 100, c = 120$). We only present the results of these two systems as these sufficiently describe the characteristics of the network for other settings as well. Demand for each product was generated as $d_j \overset{iid}{\sim} N(\mu, \sigma^2)$ with different levels of demand variability as measured by the coefficient of variation ($CV = \frac{\sigma}{\mu}$) ranging from 10% to 50% in increments of 10%. The size of the network was varied from 2 to 50 in increments of 2 each with a single chaining configuration. Each setting was replicated 50,000 times. Note that smaller size networks correspond to smaller $\eta(G)$ values and vice versa. The simulation follows a two-stage procedure: first, the product demand and disruption (single failure) are realized; second, the supply is allocated such that the shortfalls are minimized.

Single Link Failure. The simulation results for the single link failure case on a balanced system and a system with 20% excess capacity are shown in Figure 3(a) and (b), respectively, and the findings are summarized as follows:

Observation 1. *The expected fragility with respect to a single link failure decreases as the size of the network decreases.*

Observation 2. *The expected fragility with respect to a single link failure decreases as the demand variability decreases and as excess capacity increases.*

Interestingly, Observation 1 suggests that the containment strategy is preferable to the coverage strategy when dealing with single link failures. Figure 3 also illustrates that, the expected fragility is bounded as the size of the chain increases (levels off at an asymptote), implying that the damage of a single link failure can be bounded with the chaining configuration and can be reduced by introducing excess capacity and reducing the demand variability.

Single Node Failure. The expected fragility with respect to a single node failure is shown in

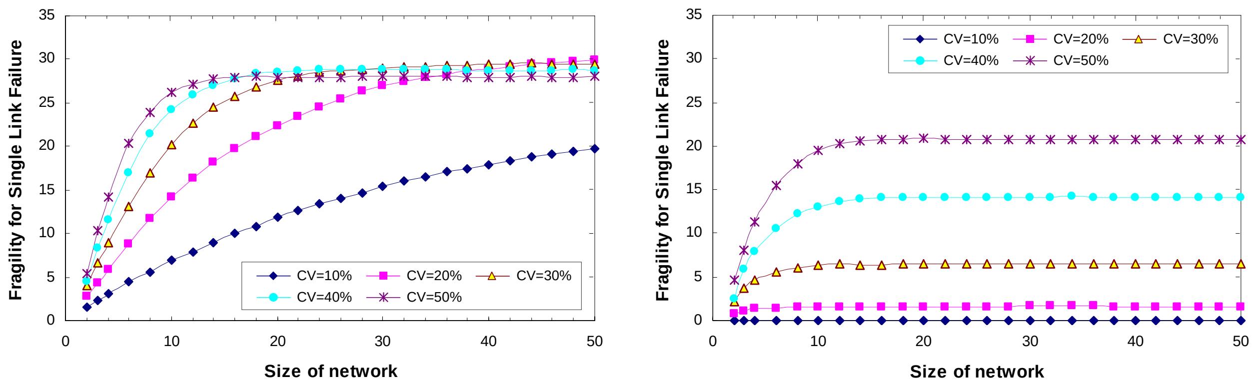

Figure 3: The expected fragility of single link failure on a symmetric network for: (a) balanced system and (b) system with 20% excess capacity

Figure 4 for identical settings as the previous single link failure case and the findings are summarized as follows:

Observation 3. *The expected fragility with respect to a single node failure decreases as the size of the network increases (except for the high demand variability case on a balanced system).*

Observation 4. *(a) The expected fragility with respect to a single node failure decreases as excess capacity increases.*
(b) With some level of excess capacity, the expected fragility with respect to a single node failure decreases as demand variability decreases. However, for a balanced system, the expected fragility with respect to a single node failure decreases as demand variability increases.

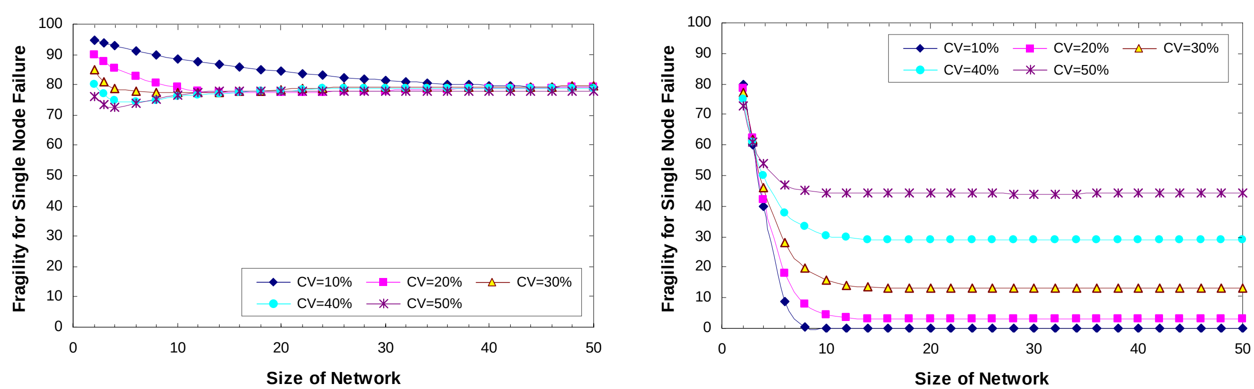

Figure 4: The expected fragility of single node failure on a symmetric network for: (a) balanced system and (b) system with 20% excess capacity

Observation 2 suggests that the coverage strategy is preferable to the containment strategy when dealing with single node failures. This is exactly the opposite to the single link failure case. For the balanced system, interestingly, higher demand variability results in a lower level of expected fragility. However, as excess capacity is introduced, the expected fragility decreases as demand variability decreases and as excess capacity increases. As in the link failure case, the expected fragility also approaches an asymptote at a certain level depending on the demand variability and excess capacity. We note that, for a balanced system, the expected fragility with respect to a node failure is much higher than that of a link failure. Further, unlike the link failure case, the expected fragility is bounded below by the asymptote and does not approach 0 regardless of the size of the chain. This reveals that the impact of a disruption for a node failure cannot be fully mitigated by only implementing flexibility unless some level of excess capacity is available.

4.2 Discussion

Based on the observations from Section 4.1, we derive the following guidelines for incorporating flexibility in supply chain network design.

First, if a network is more susceptible to link failures, a containment strategy is preferred over a coverage strategy. The containment strategy localizes the shock of a link failure so that the remaining chains still benefit from maintaining the chaining structure after the disruption. Second, if a network is more susceptible to node failures, a coverage strategy is preferred over a containment strategy in general. This is because the impact of losing a node fundamentally differs from that of losing a link – the loss of a node leads to a reduction in supply capacity whereas a link failure simply reduces the flexibility without reducing any supply capacity. In the case of a single node failure, increasing the coverage by fully utilizing the remaining capacity becomes more valuable than containing the shock locally. These results are interesting since the likelihood of link and node failures may differ significantly depending on the industry, hence the preferred flexibility configuration may differ significantly for each firm. Lastly, reducing the demand variability and adding excess capacity reduce the expected fragility for both types of failures.

We wish to prove the properties of fragility as shown in these observations. However, analytically proving these results is difficult. Next, we provide two additional supporting arguments on the directionality of fragility: First, we show that the expected fragility can be approximated by a random walk in §5. Using this approximation, we show that Observations 1 and 3 hold. Second, we extend the observations numerically by conducting more extensive simulations in §6.

5. Analytical Approach

To better understand the characteristics and to enhance the computation of chaining performances, we propose a random walk approximation scheme. These approximations are also useful for analyzing large-scale networks with multiple failures. The derivation of this approximation scheme is in Appendix B.1 and we only present the final result in this section.

5.1 Single Component Failures

Single Link Failure. We start with a balanced system on a size N network. We assume each demand is an independent and identically distributed normal random variable with mean μ and variance σ^2 and each supply node has a capacity of $c = \mu$. Hence $c = c_1 = \cdots = c_N = \mu_1 = \cdots = \mu_N = \mu$ and $\sigma_1 = \cdots = \sigma_N = \sigma$. Now, define a random walk process $\{R_n, n \geq 0\}$ where $R_0 = 0$ and $R_n = \sum_{i=0}^{n} z_i$ with $z_i \sim N(0, \sigma^2)$. Then, we can approximate the expected fragility of the network with respect to a single link failure as:

$$\mathbb{E}\big[\widetilde{\mathcal{F}}_{1LF}(N)\big] = \mathbb{E}\big[\max_{0 \leq n \leq N} R_n\big] - \mathbb{E}\big[\max\big\{0, R_N\big\}\big]. \tag{2}$$

In other words, the expected fragility of a single link failure can be measured as the expected value of the maximal (the maximum position that the random walk has reached during the entire walk) minus the maximum between the starting point and the value of the random walk at the N^{th} step.

Now consider the case in which the capacity of each supplier exceeds the mean demand, $\mu < c$. Let the excess capacity be e; i.e., $z_j \sim N(-e, \sigma^2)$ where $c - \mu = e > 0$. The mean of each step in the random walk, z_j, now has a negative value and the entire random walk is negatively drifted. In the presence of excess capacity, the accuracy of the approximation improves as explained in the appendix. Although (2) is not an exact expression of the expected fragility of the single link failure, we analytically confirm the directionality of fragility as suggested in Observation 1.

Theorem 2 (Approximate expected fragility for $1LF$)**.** *For a random single link failure on both balanced and excess capacity systems, the approximate expected fragility of a large network is greater than that of a smaller network; i.e.,* $\mathbb{E}\big[\widetilde{\mathcal{F}}_{1LF}(N)\big] > \mathbb{E}\big[\widetilde{\mathcal{F}}_{1LF}(N-1)\big]$.

Single Node Failure. Again, by using the random walk, $\{R_n, n \geq 0\}$ where $R_0 = 0$ and $R_n = \sum_{i=0}^{n} z_i$, we approximate the expected fragility of a single node failure in a network of size N as:

$$\mathbb{E}\big[\widetilde{\mathcal{F}}_{1NF}(N)\big] = \mathbb{E}\big[\max\big\{0, R_N + c\big\}\big] - \mathbb{E}\big[\max\big\{0, R_N\big\}\big]. \tag{3}$$

By using this approximation, we confirm Observation 3.

Theorem 3 (Approximate expected fragility for $1NF$)**.** *For a random single node failure on both balanced and excess capacity systems, the expected fragility of a large network is less than that of a smaller network; i.e.,* $\mathbb{E}\big[\widetilde{\mathcal{F}}_{1NF}(N)\big] < \mathbb{E}\big[\widetilde{\mathcal{F}}_{1NF}(N-1)\big]$.

In general, the approximation works very well with some excess capacity. For balanced systems, the accuracy decreases as the size of the network increases unless demand variability is small (e.g., the trend of the expected fragility being reversed for the high demand variability case for the single link failure is not captured from the approximation). In reality, however, most firms maintain some excess capacity and thus we expect the proposed approximation to work reasonably well. While one can clearly enhance the performance (by dropping less terms in the approximation), the proposed scheme provides a good proxy for determining the directionality of the fragility while still remaining analytically tractable. The performance of this approximation is demonstrated in Appendix B.2.

5.2 Multiple Component Failures

We now study the fragility of systems subject to multiple random failures. It is important to notice that multiple component failures in the network – failures of either links or nodes or their combinations – disconnect the network into multiple subnetworks. Further, each resulting subnetwork has an equivalent network topology to that of a network with a single component (link or node) failure. Figure 5 shows an example of multiple failures (one node failure and two link failures) on a size 8 network along with an equivalent set of subnetworks with single link or node failure.

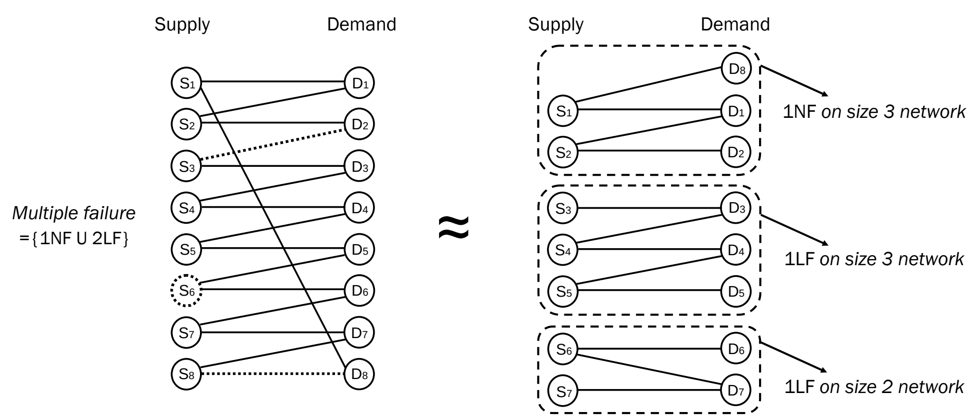

Figure 5: An example of multiple failures and equivalent subnetworks with single failure

By using this decomposition property, any combination of multiple failures can be interpreted as a collection of single link or node failures. This highlights the value of the proposed fragility approximation scheme for single component failures as the fragility of the entire system with multiple failures can be approximated by the sum of the fragilities of the associated single element failures.

6. Extensions

Our observations and analyses so far are based on a symmetric network with a single chained network configuration. In this section, we perform a series of simulations on stylized, but more general, networks to test the robustness on directionality of fragility by relaxing the above assumptions. Except for the network topology, the simulation settings are identical to those in Section 4 unless specifically mentioned. For the simulation study, we focus on two systems: balanced and 20% excess capacity, both with demand variability of 20%. Similar trends hold for other levels of excess capacity and demand variability as well.

6.1 Single Component Failures

6.1.1 Asymmetric Network

We first investigate the fragility on asymmetric networks with modified chaining configurations. In general, firms are likely to have different numbers of plants and products. On an asymmetric network, the conventional chaining configuration is no longer applicable. We employ four asymmetric configurations with stylized chaining configurations as shown in Figure 6. Conforming to the spirit

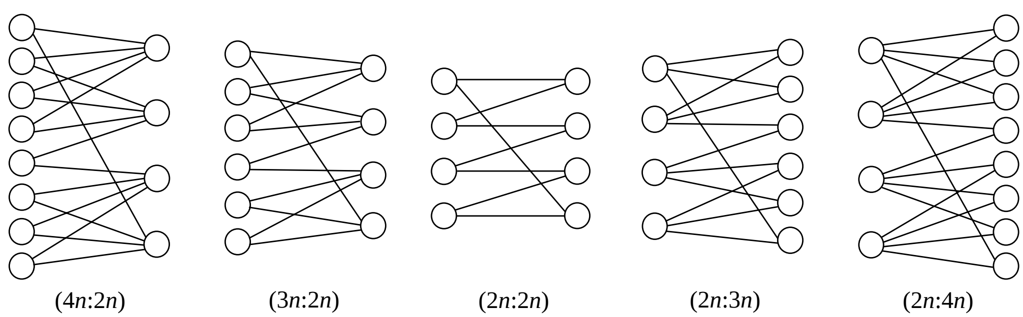

Figure 6: Configurations tested for asymmetric networks where $n = 2$

of chaining, each node has at least two connected links; however, the average number of links per node is increased. The notation below each configuration in Figure 6 indicates the ratio between the number of supply and demand nodes. For example, a configuration $(4n:2n)$ has four supply nodes for every two demand nodes. For $n = 3$, we would thus result in a configuration with 12 supply nodes and 6 demand nodes. For the balanced system, the total supply equals the total expected demand. In particular, we fix the mean of total demand for each unit group to $200n$; i.e., each demand node in configuration $(4n:2n)$ has a mean demand of 100 while in configuration $(2n:4n)$, each demand node has a mean of 50. For the 20% excess capacity case, we equally distribute the excess capacity to each supply node. We vary the number of unit groups n from 1 to 10. The expected fragility with respect to a single link and node failure on the balanced and excess capacity systems is shown in Table 1.

Table 1: The expected fragility of single link and node failures on an asymmetric balanced and a 20% excess capacity system

		Link Failure					Node Failure				
	n	$4n$:$2n$	$3n$:$2n$	$2n$:$2n$	$2n$:$3n$	$2n$:$4n$	$4n$:$2n$	$3n$:$2n$	$2n$:$2n$	$2n$:$3n$	$2n$:$4n$
Balanced System	1	0.00	0.00	2.70	0.00	0.00	39.68	56.15	89.85	92.39	92.89
	2	0.77	1.15	5.87	0.82	0.98	37.18	52.83	85.43	89.50	89.62
	3	1.39	2.07	8.83	1.53	1.42	36.26	50.26	82.59	87.43	87.95
	4	2.03	3.31	11.70	2.27	1.72	35.90	50.12	80.62	86.22	86.76
	5	2.32	3.96	14.20	3.79	1.95	35.78	49.76	79.05	85.41	86.40
	6	2.82	4.38	16.36	4.46	2.15	35.77	49.57	77.97	84.89	86.29
	7	3.04	5.09	18.21	5.01	2.41	35.79	49.56	77.62	84.77	86.22
	8	3.23	5.61	19.71	5.99	2.66	35.77	49.56	77.63	84.70	86.19
	9	3.38	6.54	21.11	7.13	2.84	35.76	49.56	77.61	84.66	86.31
	10	3.53	8.29	22.32	8.32	3.03	35.76	49.56	77.59	84.64	86.39
20% Excess Capacity	1	0.00	0.00	0.77	0.00	0.00	22.63	40.10	78.48	80.95	80.06
	2	0.10	0.15	1.33	0.04	0.01	6.35	14.18	42.04	43.91	44.13
	3	0.11	0.16	1.46	0.05	0.02	1.75	4.23	18.11	25.22	31.90
	4	0.12	0.16	1.51	0.06	0.02	1.21	1.68	7.76	17.60	30.40
	5	0.13	0.17	1.51	0.06	0.02	0.96	1.13	4.27	16.58	30.34
	6	0.14	0.17	1.50	0.06	0.03	0.88	0.97	3.39	16.30	30.39
	7	0.14	0.18	1.50	0.07	0.02	0.80	0.91	3.16	16.07	30.40
	8	0.14	0.20	1.52	0.07	0.03	0.76	0.86	3.05	15.99	30.28
	9	0.15	0.20	1.55	0.08	0.03	0.78	0.86	3.02	15.89	30.16
	10	0.16	0.21	1.52	0.09	0.03	0.77	0.85	3.01	15.78	30.09

The directionality of the expected fragility on asymmetric networks is consistent with that on symmetric networks. From both the balanced and excess capacity systems, we observe that the expected fragility with respect to a single link failure decreases as the size of the network decreases, while the expected fragility with respect to a single node failure decreases as the group size increases.

6.1.2 Higher Degree of Connectivity with Identical Chains

We next investigate the fragility with higher degrees of connectivity on a symmetric network. Instead of having one chaining configuration, a firm may be better off having a higher degree of connectivity. We first consider multiple chains with identical structures by increasing the degree of chaining from 2 to 4 as shown in Figure 7. For the simulation, we increased the size of the network, N, from 2 to 20. The simulation results are shown in Table 2 for both the balanced and 20% excess capacity systems with a demand CV of 20%.

We again observe that for configurations with higher degrees of chaining (degree 3 and 4), the expected fragility as a result of a single link failure decreases as the size of the chain decreases. For a single node failure, the expected fragility decreases as the size of the chain increases. Both

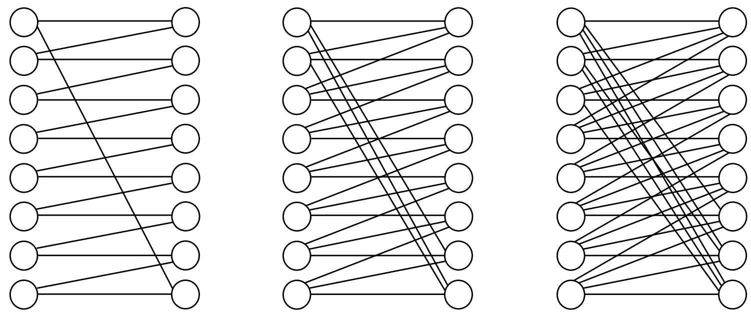

Figure 7: Configurations tested for multiple identical chain networks where $N = 8$

Table 2: The expected fragility of a single link and a single node failure with higher degrees of chain on balanced and 20% excess capacity systems

	Balanced System						20% Excess Capacity System					
	Link Failure			Node Failure			Link Failure			Node Failure		
N	deg.2	deg.3	deg.4	deg.2	deg.3	deg.4	deg.2	deg.3	deg.4	deg.2	deg.3	deg.4
2	2.20	0.00	0.00	89.46	N/A	N/A	0.77	0.00	0.00	78.48	N/A	N/A
4	5.87	0.00	0.00	85.83	85.82	85.79	1.33	0.00	0.00	42.24	42.23	42.22
6	8.83	0.00	0.00	82.51	82.53	82.51	1.46	0.00	0.00	18.11	17.68	17.66
8	11.09	0.00	0.00	80.42	80.23	80.21	1.51	0.00	0.00	7.76	5.96	5.95
10	13.60	0.00	0.00	79.73	78.63	78.62	1.51	0.00	0.00	4.27	1.84	1.84
12	15.87	0.00	0.00	79.41	77.07	77.06	1.50	0.00	0.00	3.39	0.54	0.54
14	16.87	0.01	0.00	79.45	75.61	75.61	1.51	0.00	0.00	3.16	0.13	0.13
16	18.46	0.01	0.00	79.25	74.26	74.26	1.52	0.00	0.00	3.05	0.03	0.03
18	19.86	0.02	0.00	79.23	72.95	72.95	1.55	0.00	0.00	3.02	0.00	0.00
20	22.14	0.04	0.00	79.21	72.38	72.38	1.52	0.00	0.00	3.00	0.00	0.00

findings are consistent with Observations 1 and 3 made on degree 2 chains. Also, from the node failure case on a balanced system from Table 2, we confirm that the risk of a node failure cannot be fully mitigated by having flexibility unless some level of excess capacity is introduced.

6.1.3 Higher Degree of Connectivity with Non-identical Chains

We next investigate the case in which the network has a higher degree of connectivity with non-identical chains. In this case, the current definition of the containment and coverage strategies becomes ambiguous as each network contains chains of different sizes. To study the directionality of fragility on a broader collection of network configurations, we study the relationship between some simple network measures and the expected fragility.

We limit our analysis to the $N = 12$ network; however, similar results hold for other settings as well. Along with the 6 configurations in $\hat{\mathcal{G}}_{12}$, we introduce 18 additional configurations as shown in Figure 12 in the Appendix C. We first created 9 new configurations that are products of two combinations of 6 basic configurations; that is, superposing one configuration on top of the other one to generate a network with richer connectivity. Then we generate another 9 configurations that are variants of the 6 basic configurations (by adding bridge links that join two separate groups). Based on simple linear regression, we find the result shown in Table 3 that consists of three critical network measures that are defined as follows:

(a) group size (GS) = average size of each disjoint group (not necessarily a chain)

$$= \frac{N}{\text{number of groups}}$$

(b) minimal chain size (MinCS) = the size of the smallest chain in the network

(c) maximal chain size (MaxCS) = the size of the largest chain in the network

Table 3: Linear regression result of expected fragility with network measures

Single Link Failure				
$\mathbb{E}[\mathcal{F}_{1LF}] = -0.101 + 0.292\,\text{GS} + 1.02\,\text{MinCS}$ $S = 0.7258,\ R^2 = 94.6\%,\ R^2_{adj} = 94.1\%$				
Predictor	Coef	SE Coef	t	p
Constant	-0.1007	0.3703	-0.27	0.788
GS	0.2925	0.0417	7.02	0.000
MinCS	1.0195	0.0706	14.45	0.000

Single Node Failure				
$\mathbb{E}[\mathcal{F}_{1NF}] = 90.0 - 0.546\,\text{GS} - 0.502\,\text{MaxCS}$ $S = 1.0444,\ R^2 = 93.0\%,\ R^2_{adj} = 92.3\%$				
Predictor	Coef	SE Coef	t	p
Constant	90.0485	0.5066	177.74	0.000
GS	-0.5456	0.0689	-7.91	0.000
MaxCS	-0.5020	0.0675	-7.44	0.000

Observe that the expected fragility of link and node failures are both described very well (R^2 and R^2_{adj} are all greater than 92% for both cases) by two simple network measures each. For a single link failure, the expected fragility decreases as the group size and minimal chain size decrease. This finding aligns with the containment strategy because more groups and smaller chains imply greater containment. Smaller values of the group size and minimal chain size reflect an enhanced ability of a network to contain the shock to a limited region. On the other hand, the expected fragility of a single node failure decreases as the group size and maximal chain size increase. This finding aligns with the coverage strategy because fewer groups and larger chains imply greater coverage. Larger values of the group size and maximal chain size reflect an enhanced range of coverage (how much unused capacities can be pooled) for each demand node. Detailed statistics of each configuration are provided in Table 5 in the Appendix C.

6.2 Multiple Component Failures

We now investigate the fragility for multiple random failures on symmetric networks. Since the main goal of the study is to suggest the directionality of fragility, we focus on *whether the configurations are ordered monotonically in expected fragility.* That way, the suggested directionality holds for all possible choices of configurations. We simulate on the network of size N= 12, thus $\hat{\mathcal{G}}_{12} = \{G(2|12), G(2,2,2,3,3), G(3|12), G(4|12), G(6|12), G(12)\}$. The average sizes of these chains, $\eta(G)$, are 2, 2.5, 3, 4, 6, and 12, respectively.

We start with the balanced system and then continue with the excess capacity case. Within each configuration, we varied the probability of a link failure and a node failure from 0 to 0.2 in increments of 0.01. For the regions in which finer detail was required, we decreased the increment interval to 0.001. For each fixed $\mathbb{P}$(link failure) and $\mathbb{P}$(node failure), we generated 200 scenarios of randomly disrupted networks, and within each disrupted network we generated 500 subscenarios of random demands. All supply nodes have equal capacity of $c = 100$ and each demand d_j is an i.i.d. normal random variable, $N(\mu, \sigma^2)$, truncated at $\mu \pm 2\sigma$, with $\mu = 100$ and $\sigma = 20$.

Based on this experiment, we categorize the region depending on which flexibility strategy is preferred. This result is depicted in Figure 8. To emphasize the point further, we take a slice of Figure 8 at $\mathbb{P}$(node failure) $= 0.05$ in Figure 9. Region 1 represents the region in which the containment strategy is always preferred. In this region, the monotonicity of the expected fragility held in ascending order with respect to $\eta(G)$; i.e., $\mathbb{E}[\mathcal{F}(2|12)] < \mathbb{E}[\mathcal{F}(2,2,2,3,3)] < \mathbb{E}[\mathcal{F}(3|12)] < \mathbb{E}[\mathcal{F}(4|12)] < \mathbb{E}[\mathcal{F}(6|12)] < \mathbb{E}[\mathcal{F}(12)]$. In contrast, region 3 represents the region in which the coverage strategy is always preferred. In this region, the order of monotonicity is reversed, $\mathbb{E}[\mathcal{F}(12)] < \cdots < \mathbb{E}[\mathcal{F}(2|12)]$. Region 2 is an ambiguous region in which monotonicity does not strictly hold. However, it is worth observing that in this region the range of expected fragilities is very small; there are no significant differences in expected fragilities.

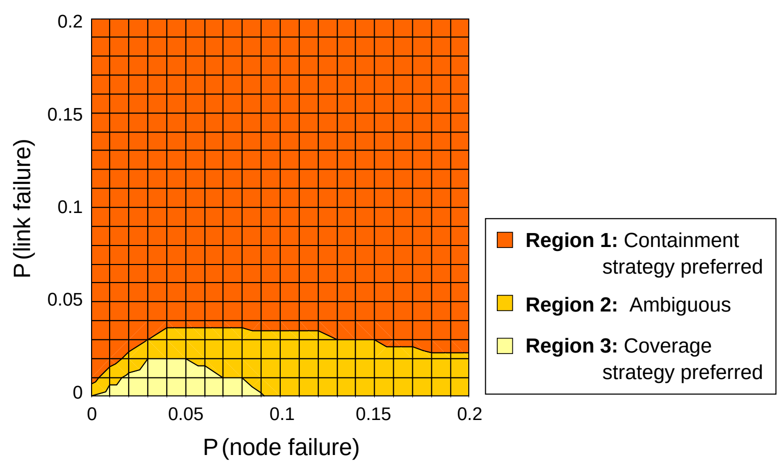

Figure 8: Regions where monotonicity holds for size 12 network

It is also interesting to observe that the coverage strategy is favored only for a small region and that the containment strategy becomes the dominant strategy in many instances. That is, containment is preferred when link failure probabilities exceed a certain level (around 0.04 for this case) regardless of the node failure probabilities. This is because any combination of link and/or node failures disconnect the network into multiple subnetworks as we mentioned in Section 5.2. This is equivalent to having started with shorter chains. Thus, for the multiple failure case, the value of a coverage is significantly reduced since long chains are likely to result in multiples of shorter incomplete chains anyway. On the other hand, a containment strategy is beneficial as it preserves the chances of maintaining chaining structures after disruptions.

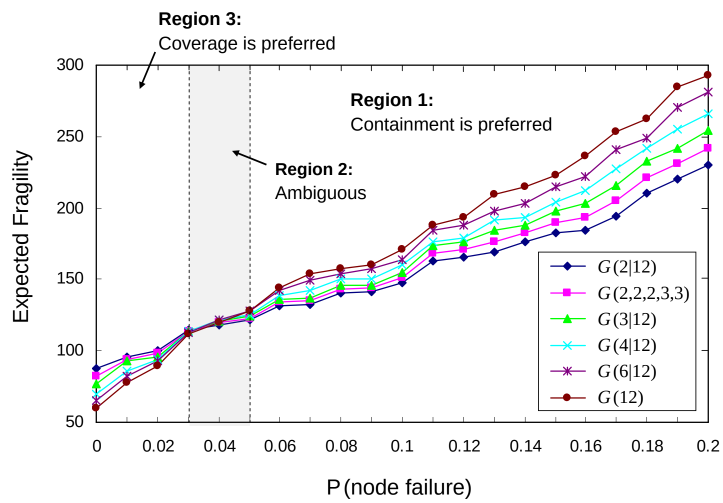

Figure 9: Expected fragilities of all possible configurations for size 12 network: with node failure probability fixed at 0.05

An identical simulation experiment was performed for systems with excess capacity. We observe that when excess capacity is introduced, the containment strategy becomes even more effective. Our computational results show that for a system with even 5% excess capacity (i.e., $c = 1.05\mu$), the containment strategy is preferred for almost any combination of link and node failures. This implies that if a firm has enough capacity to bounce back from node failures, they should focus on the containment strategy in general.

6.3 Guidelines for a Flexibility Policy

Our work provides the following guidelines for implementing flexibility:

(1) When the disruption probability is low enough that the chance of a disruption with multiple failures is sufficiently small, we can hedge the risk of disruptions by using the results from single component failures in §4.1. We observe that a firm should use the containment strategy if their

supply chain network is more susceptible to link failures; however, it is preferable to use the coverage strategy if the network is more susceptible to node failures.
(2) When the disruption probability is large and multiple failures are likely, the containment strategy becomes a very effective mitigation tool for both link and node failures. That is, the benefit of a coverage strategy becomes limited due to disconnection of the network following disruptions.
(3) Lastly, investing in some level of excess capacity is essential as doing so reduces the expected fragility significantly. Relying only on flexibility has a limited ability to mitigate the impact of disruptions, especially for node failures, and thus a mixture of appropriate flexibility along with excess capacity is recommended. Additionally, it is also desirable to reduce demand variability to hedge against supply chain disruptions.

7. Conclusions

Supply chains are subject to numerous disruptions ranging from natural disasters to terrorist attacks. Although most of these events have a very low probability of occurring, when they do happen, they often come with catastrophic consequences. Inspired by Jordan and Graves' flexibility guideline, we study the use of the chaining concept in designing a robust supply chain network by taking into account potential disruptions and the underlying cost structure. We introduced fragility to quantify the change in system performance resulting from a disruption. Based on this, we showed that by understanding the directionality of fragility a firm can design supply chain networks that reduce the impact of disruptions. We also showed that multiple failure cases can be decomposed into sets of single component failure subnetworks which can then be analyzed individually. A random walk approximation scheme was introduced to support the simulation results for the expected fragility of the network. Finally, using extensive simulation experiments, we identified the optimal directionality for the network configuration when failures may occur in a random manner.

We believe that flexibility and fragility are very important concepts, not only in the context of supply chain network design, but also in a broad range of network systems in practice. To conclude, we identify four important directions in which to extend this study: first, exploring the possibilities of partial and correlated failures of the network elements; second, guidelines with link- and/or node-specific failure probabilities; third, extending the results to the case of targeted disruptions such as those caused by terrorists; and lastly, extending the concept of containment and coverage to more generalized network configurations. Reinvestigating our findings under such conditions will provide richer insights into how to design and operate such network systems.

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Appendix

A. Proof of the Main Results

Proof of Proposition 1: Let $l(i-j+1)$ represents the link cost of l_{ij}. The fixed cost for network $G(n)$ is $FC(n) = nl(1)+(n-1)l(2)+l(n)$. Note, $nl(1)$ and $(n-1)l(2)$ are increasing linear functions since $l(1)$ and $l(2)$ are constant, and $l(n)$ is a strictly increasing and strictly convex function. Hence $FC(n)$ is a strictly increasing convex function of n. ■

Proof of Proposition 2: Note that $LC(n)$ is a linear function of $SF(n)$, thus it is equivalent to study the characteristics of $SF(n)$ instead of $LC(n)$. First, it is easy to see that $SF(n)$ is an increasing function. For the second part of the proof, consider an incomplete chain of size n in which the long diagonal link, e_{1n}, is omitted as depict in Figure 10. Denoting the shortfall resulting from such a configuration with size n by $SF^{IC}(n)$, we know that $SF^{IC}(n)-c \leq SF(n) \leq SF^{IC}(n)$. Thus showing $\frac{SF^{IC}(n)}{n} \to \alpha$ as $n \to \infty$ implies $\frac{SF(n)}{n} \to \alpha$ as $n \to \infty$.

Now we focus on $SF^{IC}(n)$. For the incomplete chaining configuration depicted in Figure 10, the greedy algorithm which sequentially allocates the capacity by numbered orders in the figure provides the optimal solution. For example, (a) we start the cycle $k=1$ by making S_1 first serve D_1. We denote the amount of unmet demand up to demand j by u_j, thus $u_1 = (d_1 - c_1)^+$. If $u_j = 0$, then the current cycle ends. Otherwise, (b) S_2 then serves unmet demand u_1. If $u_1 > c_2$, then this creates shortfall of $u_1 - c_2$. We denote the shortfall up to demand j by w_j, thus $w_1 = (u_1 - c_2)^+$. (c) If $c_2 > u_1$, the unused capacity from S_2, $(c_2 - u_1)$, serves the next demand d_2. By repeating this, we obtain $w_j = (u_j - c_{j+1})^+$ and $u_j = (d_j - (c_j - u_{j-1})^+)^+$. Whenever $u_j = 0$, we consider the cycle ended and we restart the algorithm for the next cycle $k \leftarrow k+1$. We define m^k as the length of the cycle. The total shortfall for each cycle is $sf^k = \sum_{j=1}^{m^k} w_j$. Then, a process $N(n) = \sum_{t=1}^{\infty} \mathbf{1}\{M_t \leq n\}$ where $M_t = \sum_{k=1}^{t} m^k$ is a renewal process since m^k is a

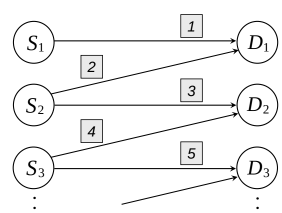

Figure 10: Sequential capacity allocation algorithm for incomplete chain

nonnegative independent and identically distributed random variable. Further, $SF^{IC} = \sum_{k=1}^{N(n)} sf^k$ is a renewal reward process since $\{(m^k, sf^k), k \geq 1\}$ is an independent and identically distributed random variable. Since $\mathbb{E}sf < \infty$ and $\mathbb{E}m < \infty$, we conclude that $\frac{SF^{IC}(n)}{n} \to \frac{\mathbb{E}sf}{\mathbb{E}m} = \alpha$ as $n \to \infty$ by the Renewal Reward Theorem (Kulkarni, 1995). Therefore, $LC(n)$ is an increasing function that asymptotically becomes linear with n. ■

Proof of Theorem 1: Suppose there exists an optimal configuration of network G_1 which includes two different chains with sizes of n_1 and n_2 where $n_1 > n_2$ and $|n_1 - n_2| > 1$. Now consider another configuration G_2 that modifies G_1 by changing the size of the two chains to $n_1 - 1$ and $n_2 + 1$ while the other configurations remain the same. Note that $G_2 \in \mathcal{G}_N$ as well. By convexity of $TC(n)$, we know $TC(n_1) + TC(n_2) > TC(n_1 - 1) + TC(n_2 + 1)$. This implies that $TC(G_1) > TC(G_2)$ and leads us to a contradiction. Therefore, the optimal network configuration consists of chains with sizes of only n^* or $n^* + 1$. $\blacksquare$

Derivation of equation (1): The objective function of $[P3]$ can be rewritten as follows:

$$\begin{aligned}
TC(G) &= \sum_{(i,j)\in E} l_{ij}y_{ij} + p\,\mathbb{E}_\xi[SF(G)] \\
&= \sum_{(i,j)\in E} l_{ij}y_{ij} + p\,\mathbb{P}(\Omega_\circ)\mathbb{E}_\xi[SF(G)|\Omega_\circ] + p\,\mathbb{P}(\Omega_\Delta)\mathbb{E}_\xi[SF(G)|\Omega_\Delta] \\
&= \sum_{(i,j)\in E} l_{ij}y_{ij} + p\,\mathbb{P}(\Omega_\circ)\mathbb{E}_\xi[SF(G)|\Omega_\circ] + p\,\mathbb{P}(\Omega_\Delta)\mathbb{E}_\xi[SF(G)|\Omega_\Delta] \\
&\qquad\qquad + p\,\mathbb{P}(\Omega_\Delta)\mathbb{E}_\xi[SF(G)|\Omega_\circ] - p\,\mathbb{P}(\Omega_\Delta)\mathbb{E}_\xi[SF(G)|\Omega_\circ] \\
&= \sum_{(i,j)\in E} l_{ij}y_{ij} + p\,\mathbb{E}_\xi[SF(G))|\Omega_\circ] + p\,\mathbb{P}(\Omega_\Delta)\big(\,\mathbb{E}_\xi[SF(G)|\Omega_\Delta] - \mathbb{E}_\xi[SF(G)|\Omega_\circ]\,\big) \\
&= \sum_{(i,j)\in E} l_{ij}y_{ij} + p\,\mathbb{E}[SF(G)|\Omega_\circ] + p\,\mathbb{P}(\Omega_\Delta)\mathbb{E}[\mathcal{F}_\Delta(G)] \\
&= \mathbb{E}[TC(G)|\Omega_\circ] + p\,\mathbb{P}(\Omega_\Delta)\mathbb{E}[\mathcal{F}_\Delta(G)]. \blacksquare
\end{aligned}$$

Proof of Proposition 3: Two configurations G_1 and G_2 belong to $\mathcal{G}_N^*$; i.e., $\mathbb{E}\big[TC(G_1)|\Omega_\circ\big] = \mathbb{E}\big[TC(G_2)|\Omega_\circ\big]$. The change in the expected total cost considering a disruption set Δ is $\mathbb{E}[TC(G_1)] - \mathbb{E}[TC(G_2)] = \mathbb{P}(\Omega_\Delta)\, p\, \big[\mathbb{E}[\mathcal{F}_\Delta(G_1)] - \mathbb{E}[\mathcal{F}_\Delta(G_2)]\big]$ by using (1). Hence, if the expected fragility of G_1 is smaller than that of G_2, $\mathcal{F}_\Delta(G_1) - \mathcal{F}_\Delta(G_2) < 0$, then $\mathbb{E}[TC(G_1)] - \mathbb{E}[TC(G_2)] < 0$, thus configuration G_1 is preferred over G_2. $\blacksquare$

Proof of Proposition 4: (a) Let $n_1 \le \eta(G_1) < n_1 + 1$ and $n_2 \le \eta(G_2) < n_2 + 1$ where $n_1 \le n_2$ and $n_1, n_2 \in \mathbb{N}$. Since $G_1, G_2 \in \hat{\mathcal{G}}_N$, we know that G_1 consists of $\beta_1 (> 0)$ size n_1 chains and γ_1 size $n_1 + 1$ chain where $n_1\beta_1 + (n_1 + 1)\gamma_1 = N$. Similarly, G_2 consists of $\beta_2 (> 0)$ size n_2 chains and γ_2 size $n_2 + 1$ chain where $n_2\beta_2 + (n_2 + 1)\gamma_2 = N$. Hence,

$$\mathbb{E}[\mathcal{F}(G_1)] = \beta_1 \frac{n_1}{N}\mathbb{E}[\mathcal{F}(n_1)] + \gamma_1 \frac{(n_1+1)}{N}\mathbb{E}[\mathcal{F}(n_1+1)] = \beta_1 \frac{n_1}{N}\mathbb{E}[\mathcal{F}(n_1)] + \big(1 - \beta_1 \frac{n_1}{N}\big)\mathbb{E}[\mathcal{F}(n_1+1)],$$
$$\mathbb{E}[\mathcal{F}(G_2)] = \beta_2 \frac{n_2}{N}\mathbb{E}[\mathcal{F}(n_2)] + \gamma_2 \frac{(n_2+1)}{N}\mathbb{E}[\mathcal{F}(n_2+1)] = \beta_2 \frac{n_2}{N}\mathbb{E}[\mathcal{F}(n_2)] + \big(1 - \beta_2 \frac{n_2}{N}\big)\mathbb{E}[\mathcal{F}(n_2+1)].$$

If $n_1 < n_2$, then $\mathbb{E}[\mathcal{F}(n_1)] \ge \mathbb{E}[\mathcal{F}_\delta(G_1)] \ge \mathbb{E}[\mathcal{F}(n_1+1)] \ge \mathbb{E}[\mathcal{F}(n_2)] \ge \mathbb{E}[\mathcal{F}_\delta(G_2)] \ge \mathbb{E}[\mathcal{F}(n_2+1)]$. If $n_1 = n_2$, we know that $\beta_1 > \beta_2$ since $\eta(G_1) < \eta(G_2)$. This implies that $\mathbb{E}[\mathcal{F}(n_1)] \ge \mathbb{E}[\mathcal{F}_\delta(G_1)] \ge$

$\mathbb{E}[\mathcal{F}_\delta(G_2)] \geq \mathbb{E}[\mathcal{F}(n_2)]$. Therefore, $\mathbb{E}[\mathcal{F}_\delta(G_1)] \geq \mathbb{E}[\mathcal{F}_\delta(G_2)]$.

(b) Proof follows as in part (a). ■

Proof of Theorem 2: Using the definition of $\widetilde{\mathcal{F}}_{1LF}(N)$, we have

$$\begin{aligned}
&\mathbb{E}\big[\widetilde{\mathcal{F}}_{1LF}(N)\big] = \mathbb{E}\big[\max_{0\le n\le N} R_n\big] - \mathbb{E}\big[\max\{0, R_N\}\big] \\
&= \int_0^\infty \mathbb{P}\big(\max_{0\le n\le N} R_n > y\big)dy - \int_0^\infty \mathbb{P}\big(R_N > y\big)dy \\
&= \int_0^\infty \Big[\mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N > y\big) + \mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N \le y\big)\Big]dy - \int_0^\infty \mathbb{P}\big(R_N > y\big)dy \\
&= \int_0^\infty \mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N \le y\big)dy.
\end{aligned}$$

Here we use the fact that $\mathbb{E}[X] = \int_0^\infty \mathbb{P}(X > x)dx$ for a non-negative random variable X. Now, showing $\mathbb{E}\big[\widetilde{\mathcal{F}}_{1LF}(N)\big] > \mathbb{E}\big[\widetilde{\mathcal{F}}_{1LF}(N-1)\big]$ is equivalent to showing

$$\int_0^\infty \mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N \le y\big)dy > \int_0^\infty \mathbb{P}\big(\max_{0\le n\le N-2} R_n > y, R_{N-1} \le y\big)dy.$$

Next we claim that for all $y > 0$, $\mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N \le y\big) > \mathbb{P}\big(\max_{0\le n\le N-2} R_n > y, R_{N-1} \le y\big)$. Define a passage time M such that $M = \min\{n | R_n > y\}$. Then it follows that

$$\begin{aligned}
\mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N \le y\big) &= \mathbb{P}\big(M \le N-1, R_N \le y\big) \\
&= \mathbb{E}\big[\mathbb{P}\big(M \le N-1, R_N \le y \big| M, R_M\big)\big] \\
&= \mathbb{E}\big[\mathbb{P}\big(R_N \le y \big| M, R_M\big) \cdot \mathbf{1}_{\{M\le N-1\}}\big] \\
&= \mathbb{E}\big[\mathbb{P}\big(R_M + z_{M+1} + \cdots + z_N \le y \big| M, R_M\big) \cdot \mathbf{1}_{\{M\le N-1\}}\big].
\end{aligned}$$

(a) For a balanced system,

$$\begin{aligned}
\mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N \le y\big) &= \mathbb{E}\big[\mathbb{P}\big(S'_{N-M} \le y - R_M \big| M, R_M\big) \cdot \mathbf{1}_{\{M\le N-1\}}\big] \\
&\quad \text{where } S' \text{ is a random walk starting from } S'_0 = R_M \\
&= \mathbb{E}\big[\Phi\big(\frac{y - R_M}{\sigma\sqrt{N-M}}\big) \cdot \mathbf{1}_{\{M\le N-1\}}\big] \text{ since } S'_{N-M} \sim N\big(0, \sigma\sqrt{N-M}\big).
\end{aligned}$$

Likewise, $\mathbb{P}\big(\max_{0\le n\le N-2} R_n > y, R_{N-1} \le y\big) = \mathbb{E}\big[\Phi\big(\frac{y-R_M}{\sigma\sqrt{N-M-1}}\big)\cdot\mathbf{1}_{\{M\le N-2\}}\big]$. Note that $\mathbf{1}_{\{M\le N-1\}} > \mathbf{1}_{\{M\le N-2\}}$. Further, $\Phi\big(\frac{y-R_M}{\sigma\sqrt{N-M}}\big) > \Phi\big(\frac{y-R_M}{\sigma\sqrt{N-M-1}}\big)$ since $y - R_M < 0$.

Hence, for all $y > 0$, $\mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N \le y\big) > \mathbb{P}\big(\max_{0\le n\le N-2} R_n > y, R_{N-1} \le y\big)$, and thus $\int_0^\infty \mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N \le y\big)dy > \int_0^\infty \mathbb{P}\big(\max_{0\le n\le N-2} R_n > y, R_{N-1} \le y\big)dy$.

(b) For the system with excess capacity,

$$\mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N \le y\big) = \mathbb{E}\big[\mathbb{P}\big(S'_{N-M} \le y - R_M \big| M, R_M\big) \cdot \mathbf{1}_{\{M\le N-1\}}\big]$$

where $S'_{N-M} \sim N\big(-e(N-M),(N-M)\sigma^2\big)$ due to the excess capacity. Thus, we now have

$$\mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N \le y\big) = \mathbb{E}\big[\Phi\big(\frac{y-R_M-e(N-M)}{\sigma\sqrt{N-M}}\big)\cdot \mathbf{1}_{\{M\le N-1\}}\big] \text{ and}$$
$$\mathbb{P}\big(\max_{0\le n\le N-2} R_n > y, R_{N-1} \le y\big) = \mathbb{E}\big[\Phi\big(\frac{y-R_M-e(N-M-1)}{\sigma\sqrt{N-M-1}}\big)\cdot \mathbf{1}_{\{M\le N-2\}}\big].$$

Note that $\mathbf{1}_{\{M\le N-1\}} > \mathbf{1}_{\{M\le N-2\}}$. Also, $\Phi\big(\frac{y-R_M}{\sigma\sqrt{N-M}}\big) > \Phi\big(\frac{y-R_M}{\sigma\sqrt{N-M-1}}\big)$ holds for $N-M\ge 1$. Hence, for all $y>0$, $\mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N\le y\big) > \mathbb{P}\big(\max_{0\le n\le N-2} R_n > y, R_{N-1}\le y\big)$, and thus $\int_0^\infty \mathbb{P}\big(\max_{0\le n\le N-1} R_n > y, R_N\le y\big)dy > \int_0^\infty \mathbb{P}\big(\max_{0\le n\le N-2} R_n > y, R_{N-1}\le y\big)dy$. Therefore, we conclude $\mathbb{E}\big[\widetilde{\mathcal{F}}_{\scriptscriptstyle 1LF}(N)\big] > \mathbb{E}\big[\widetilde{\mathcal{F}}_{\scriptscriptstyle 1LF}(N-1)\big]$. ■

Proof of Theorem 3: Using the definition of $\widetilde{\mathcal{F}}_{\scriptscriptstyle 1NF}(N)$, we have

$$\begin{aligned}
\mathbb{E}\big[\widetilde{\mathcal{F}}_{\scriptscriptstyle 1NF}(N)\big] &= \mathbb{E}\big[\max\{0, R_N + c\}\big] - \mathbb{E}\big[\max\{0, R_N\}\big]\\
&= \int_0^\infty \mathbb{P}\big(R_N + c > y\big)dy - \int_0^\infty \mathbb{P}\big(R_N > y\big)dy\\
&= \int_{-c}^\infty \mathbb{P}\big(R_N > y\big)dy - \int_0^\infty \mathbb{P}\big(R_N > y\big)dy\\
&= \int_{-c}^0 \mathbb{P}\big(R_N > y\big)dy.
\end{aligned}$$

(a) For a balanced system, we it follows that

$$\begin{aligned}
\mathbb{E}\big[\widetilde{\mathcal{F}}_{\scriptscriptstyle 1NF}(N)\big] - \mathbb{E}\big[\widetilde{\mathcal{F}}_{\scriptscriptstyle 1NF}(N-1)\big] &= \int_{-c}^0 \Big[\mathbb{P}\big(R_N > y\big) - \mathbb{P}\big(R_{N-1} > y\big)\Big]dy\\
&= \int_{-c}^0 \Big[\mathbb{P}\big(R_{N-1} < y\big) - \mathbb{P}\big(R_N < y\big)\Big]dy < 0. \qquad (4)
\end{aligned}$$

Since for all $y<0$, $\mathbb{P}\big(R_N < y\big) > \mathbb{P}\big(R_{N-1} < y\big)$.

(b) For the system with excess capacity,

$$\begin{aligned}
\mathbb{E}\big[\widetilde{\mathcal{F}}_{\scriptscriptstyle 1NF}(N)\big] - \mathbb{E}\big[\widetilde{\mathcal{F}}_{\scriptscriptstyle 1NF}(N-1)\big] &= \int_{-c}^0 \Big[\mathbb{P}\big(R_{N-1} < y\big) - \mathbb{P}\big(R_N < y\big)\Big]dy\\
&= \int_{-c}^0 \Big[\Phi\big(\frac{y+(N-1)e}{\sqrt{N-1}\sigma}\big) - \Phi\big(\frac{y+Ne}{\sqrt{N}\sigma}\big)\Big]dy.
\end{aligned}$$

For $-c<y<0$ and $N>1$, $\frac{y+Ne}{\sqrt{N}\sigma} > \frac{y+(N-1)e}{\sqrt{N-1}\sigma}$ holds. Therefore, $\mathbb{E}\big[\widetilde{\mathcal{F}}_{\scriptscriptstyle 1NF}(N)\big] < \mathbb{E}\big[\widetilde{\mathcal{F}}_{\scriptscriptstyle 1NF}(N-1)\big]$. ■

B.1. Derivation of the Random Walk Approximation

Recall the symmetric network of size N where the supply nodes S_i $(\forall i\in I)$ with capacity c_i serve the demand nodes D_j $(\forall j\in J)$ with demand d_j. Graves and Jordan (1991) show that at optimality, the dual for the second stage problem from $[P2]$ (or $[P4]$) is $SF = \max_{J' \subseteq J} \Big[\sum_{j \in J'} d_j - \sum_{i \in P(J')} c_i \Big]$ where supply node $i \in P(J')$ iff there exists at least one demand node $j \in J'$ such that there is a link in the supply chain network between S_i and D_j. That is, the solution of $[P3]$ is obtained by identifying the *cut* which generates the maximum shortfall where for a given subset of demand J', $P(J')$ is the subset of suppliers who are capable of supplying at least one of the demands in J'. We refer to the cut which generates the maximum shortfall as the *maximum cut* $C(J'^*)$ where $J'^* = \arg\max_{J'} \Big[\sum_{j \in J'} d_j - \sum_{i \in P(J')} c_i \Big]$. Then, for a balanced system network, the expected shortfall $\mathbb{E}\big[SF(N)\big]$ for this system is

$$\begin{aligned}\mathbb{E}\big[SF(N)\big] &= \mathbb{E}\Big[\max_{J'} \Big[\sum_{j \in J'} d_j - \sum_{i \in P(J')} c_i\Big]\Big] \\ &= \mathbb{E}\Big[\max\big\{0,\ (z_1-c), \cdots, (z_N-c),\ (z_1+z_2-c), (z_1+z_3-2c), \cdots, (z_{N-1}+z_N-c), \\ &\qquad \cdots,\ (z_1+z_2+\cdots+z_N)\big\}\Big] \text{ where } z_j = d_j - c. \qquad (5)\end{aligned}$$

In this expression, each term inside the max operator corresponds to the shortfall associated with each possible cut. For each cut, a random variable z_j $(= d_j - c)$ represents the unmet demand from node D_j after being served by node S_j. Also, $-c$ $\big($or $-2c, \dots\big)$ represents the extra capacity coming from other suppliers through the flexible configuration. Examples of possible cuts and their corresponding shortfalls are illustrated in Figure 11. For the balanced system, $\mathbb{E}\big[z_j\big] = 0$ and hence

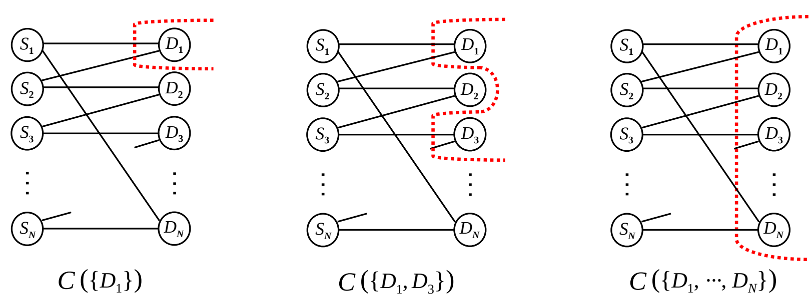

Figure 11: Examples of cuts: shortfalls for each of these cuts are $(z_1 – c)$, $(z_1 + z_3 – 2c)$ and $(z_1 + \cdot\cdot + z_N)$, respectively

each cut has a mean of either $0, -c, -2c$ and so on. We refer to the cuts with a mean of 0 as 0^{th} *order cuts* and the collection of these cuts for a size N network as set $\mathbb{Z}^0(N)$. These cuts represent the case in which the total capacity of the subset equals the total expected demand. Similarly, the remainder of the cuts with lower means are referred to as *lower order cuts* and their collection is termed set $\mathbb{Z}^{-0}(N)$. These cuts represent the case in which the total capacity of the subset exceeds the total expected demand with higher degrees of flexibility. Then (5) can be expressed as $\mathbb{E}\big[SF(N)\big] = \mathbb{E}\big[\max\big\{0, (z_1+z_2+\cdots+z_N), \mathbb{Z}^{-0}(N)\big\}\big] = \mathbb{E}\big[\max\big\{\mathbb{Z}^0(N),\, \mathbb{Z}^{-0}(N)\big\}\big]$. When demand variability is low, the probability of the maximum cut $C(J'^*)$ being achieved by the cuts from $\mathbb{Z}^{-0}(N)$ is small since these cuts have lower means than the cuts from set $\mathbb{Z}^0(N)$. Therefore, the expected shortfall of the system can be approximated by dropping the cuts with lower order means, $\mathbb{Z}^{-0}_{1LF}(N)$. Hence, we get

$$\mathbb{E}\big[SF\big] \simeq \mathbb{E}\big[\widetilde{SF}\big] = \mathbb{E}\big[\max\big\{\mathbb{Z}^0(N)\big\}\big] = \mathbb{E}\big[\max\big\{0, (z_1 + \cdots + z_N)\big\}\big]. \tag{6}$$

In other words, we only consider the cuts which have higher chances of achieving the maximum shortfall while ignoring the lower order cuts in which the subset of demands has enough suppliers (in expectation) to serve the subset.

Single Link Failure. Without loss of generality, we let link e_{11} fail. Conditioning on a single link failure, the expected shortfall $\mathbb{E}\big[SF_{1LF}(N)\big]$ is

$$\begin{aligned}\mathbb{E}\big[SF_{1LF}(N)\big] &= \mathbb{E}\Big[\max_{\bar{J}'}\Big[\sum_{j\in\bar{J}'} d_j - \sum_{i\in P_{1LF}(\bar{J}')} c_i\Big]\Big] \\ &= \mathbb{E}\big[\max\big\{0,\, z_1, (z_2-c), \cdots, (z_N-c),\, (z_1+z_2), (z_1+z_3-c), \cdots, (z_{N-1}+z_N-c), \\ &\qquad \cdots,\, (z_1+z_2+\cdots+z_N)\big\}\big] \quad \text{where } z_j = d_j - c.\end{aligned} \tag{7}$$

We use "bar" on the network elements to indicate the network configuration *after* the disruption. In the first equality, we use $P_{1LF}(\bar{J}')$ to denote the subset of suppliers for demands in $\bar{J}'$ after a link failure. After losing a link, the shortfalls from the cuts including D_1 have increased by c with the loss of service capability from supplier S_1. However, note that the cuts including D_1 along with D_N are not affected since the capacity of S_1 is utilized by serving the demand d_N. With the loss of a link, we observe that new cuts are introduced to the 0^{th} order set, $\mathbb{Z}^0_{1LF}(N)$, and they are systematically arranged in the form of random sums. Hence, the expected shortfall with respect to a single link failure is $\mathbb{E}\big[SF_{1LF}(N)\big] = \mathbb{E}\big[\max\big\{0,\, z_1, (z_1+z_2),\, (z_1+z_2+z_3),\, \cdots,\, (z_1+z_2+\cdots+z_N),\, \mathbb{Z}^{-0}_{1LF}(N)\big\}\big]$. By dropping $\mathbb{Z}^{-0}_{1LF}(N)$, we can also approximate the expected shortfall for the single link failure as

$$\mathbb{E}\big[\widetilde{SF}_{1LF}(N)\big] = \mathbb{E}\big[\max\big\{0, z_1, (z_1+z_2), \cdots, (z_1+\cdots+z_N)\big\}\big]. \tag{8}$$

By using (6) and (7), therefore, the expected approximated fragility for a single link failure is $\mathbb{E}\big[\widetilde{\mathcal{F}}_{1LF}(N)\big] = \mathbb{E}\big[\widetilde{SF}_{1LF}(N)\big] - \mathbb{E}\big[\widetilde{SF}(N)\big] = \mathbb{E}\big[\max_{0\le n\le N} R_n\big] - \mathbb{E}\big[\max\big\{0, R_N\big\}\big]$. For the case with excess capacity, the expression of the expected approximated fragility for a single link failure is identical to above while $z_j \sim N(-e, \sigma^2)$ with $c - \mu = e > 0$. In the presence of excess capacity the approximation becomes more accurate because the chance of a cut from $\mathbb{Z}^{-0}_{1LF}(N)$ achieving $C(J'^{*})$ becomes even lower than it is in the balanced system. In fact, the more excess capacity there is, the better the approximation performs.

Single Node Failure. For a single node failure, we similarly let the first supply node S_1 fail without loss of generality. Conditioning on a single node failure, the expected shortfall $\mathbb{E}\big[SF_{1NF}(N)\big]$ can be expressed as

$$\begin{aligned}
\mathbb{E}\big[SF_{1NF}(N)\big] &= \mathbb{E}\Big[\max_{\bar{J}'}\Big[\sum_{j\in\bar{J}'} d_j - \sum_{i\in P_{1NF}(\bar{J}')} c_i\Big]\Big] \\
&= \mathbb{E}\big[\max\big\{0,\, z_1, (z_2-c), (z_3-c), \cdots, (z_{N-1}-c), z_N, \\
&\qquad (z_1+z_2), (z_1+z_3-c), \cdots, (z_{N-1}+z_N-c), \cdots \\
&\qquad \cdots, (z_1+z_2+\cdots+z_N+c)\big\}\big] \quad \text{where } z_j = d_j - c.
\end{aligned} \tag{9}$$

where $P_{1NF}(\bar{J}')$ represents the subset of suppliers for demands in $\bar{J}'$ after a node failure. Due to the loss of supply node S_1, all the shortfalls from the cuts including D_1 have increased by c. Compared to the single link failure case, the number of cuts joining the 0^{th} order set $\mathbb{Z}^{0}_{1NF}(N)$ increases as more terms are affected by the loss of a node. We also note that the last term has a mean of c due to the loss of a supply node. By combining the terms with means less than 0 into the set $\mathbb{Z}^{-0}_{1NF}(N)$, we similarly rewrite the expected shortfall due to a single node failure as follows:

$$\begin{aligned}
\mathbb{E}\big[SF_{1NF}(N)\big] = \mathbb{E}\big[\max\big\{\, & 0, \\
& z_1, z_N, \\
& (z_1+z_2),\ (z_1+z_N),\ (z_{N-1}+z_N), \\
& (z_1+z_2+z_3), (z_1+z_2+z_N), (z_1+z_{N-1}+z_N), (z_{N-2}+z_{N-1}+z_N), \\
& \cdots\cdots, \\
& (z_1+z_2+\cdots+z_N+c),\ \mathbb{Z}^{-0}_{1NF}(N)\big\}\big].
\end{aligned} \tag{10}$$

We propose a similar, but slightly refined, approximation scheme for the node failure case. Note that in $SF_{1NF}(N)$ as expressed in (10), the cut corresponding to the second to last term, $(z_1 + \cdots + z_N + c)$, has the highest mean among all the cuts. This term would typically dominate all other terms for a balanced system. However, this term is not likely to be the dominant term as excess capacity and the size of network N increases. In such cases, one of the early terms in (10), in fact, 0, will most likely achieve the maximum since the random variable z_j has a negative mean. To capture both of these possibilities in one approximation, we include the first term, 0, along with $(z_1 + \cdots + z_N + c)$. The shortfall for the single node failure from (10) is approximated as

$$\mathbb{E}\big[\widetilde{SF}_{1NF}(N)\big] = \mathbb{E}\big[\max\big\{0,\, (z_1+\cdots+z_N+c)\big\}\big]. \tag{11}$$

Again, by using the random walk, $\{R_n, n \geq 0\}$ where $R_0 = 0$ and $R_n = \sum_{i=0}^{n} z_i$, from (6) and (11), we finally obtain the approximate expected fragility of a single node failure in a network of size N as $\mathbb{E}\big[\widetilde{\mathcal{F}}_{\mathit{1NF}}(N)\big] = \mathbb{E}\big[\widetilde{SF}_{\mathit{1NF}}(N)\big] - \mathbb{E}\big[\widetilde{SF}(N)\big] = \mathbb{E}\big[\max\big\{0, R_N + c\big\}\big] - \mathbb{E}\big[\max\big\{0, R_N\big\}\big]$.

B.2. The Performance of the Random Walk Approximation

Table 4: Exact and approximate fragility with different levels of capacity

	Link Failure								Node Failure							
	BS		10% EC		20% EC		30% EC		BS		10% EC		20% EC		30% EC	
N	$\mathbb{E}[\mathcal{F}]$	$\mathbb{E}[\widetilde{\mathcal{F}}]$	$\mathbb{E}[\mathcal{F}]$	$\mathbb{E}[\widetilde{\mathcal{F}}]$	$\mathbb{E}[\mathcal{F}]$	$\mathbb{E}[\widetilde{\mathcal{F}}]$	$\mathbb{E}[\mathcal{F}]$	$\mathbb{E}[\widetilde{\mathcal{F}}]$	$\mathbb{E}[\mathcal{F}]$	$\mathbb{E}[\widetilde{\mathcal{F}}]$	$\mathbb{E}[\mathcal{F}]$	$\mathbb{E}[\widetilde{\mathcal{F}}]$	$\mathbb{E}[\mathcal{F}]$	$\mathbb{E}[\widetilde{\mathcal{F}}]$	$\mathbb{E}[\mathcal{F}]$	$\mathbb{E}[\widetilde{\mathcal{F}}]$
2	2.70	2.40	1.73	1.76	0.77	0.82	0.20	0.19	89.85	89.92	86.90	86.98	78.48	79.49	69.96	69.97
4	5.87	5.87	4.03	3.98	1.33	1.34	0.24	0.22	85.43	85.85	68.08	68.02	42.24	42.12	19.85	19.68
6	8.83	8.61	5.29	5.34	1.46	1.47	0.22	0.22	82.59	82.75	51.34	51.09	18.11	17.30	3.01	2.62
8	11.70	10.95	6.12	6.00	1.51	1.45	0.22	0.22	80.62	80.33	38.89	37.18	7.76	5.95	0.67	0.22
10	14.20	13.40	6.66	6.56	1.51	1.48	0.22	0.22	79.05	78.54	30.22	26.96	4.27	1.88	0.55	0.05
12	16.36	15.42	7.07	6.88	1.50	1.47	0.23	0.21	77.97	77.01	24.71	19.47	3.39	0.53	0.50	0.02
14	18.21	17.13	7.25	7.16	1.50	1.49	0.23	0.22	77.62	75.72	21.02	14.24	3.16	0.29	0.48	0.01
16	19.71	18.94	7.44	7.18	1.52	1.45	0.23	0.21	77.63	74.56	18.89	10.37	3.05	0.15	0.47	0.00
18	21.11	20.68	7.57	7.33	1.55	1.46	0.23	0.21	77.61	73.65	17.45	7.78	3.02	0.08	0.46	0.00
20	22.32	22.19	7.62	7.45	1.52	1.48	0.23	0.21	77.59	72.84	16.66	5.99	3.01	0.03	0.46	0.00
22	23.44	23.96	7.66	7.51	1.53	1.46	0.23	0.21	77.63	72.41	16.25	4.70	3.00	0.01	0.46	0.00
24	24.49	25.01	7.69	7.51	1.53	1.46	0.22	0.21	77.65	71.76	15.95	3.89	3.00	0.00	0.46	0.00
26	25.50	26.20	7.68	7.42	1.57	1.45	0.22	0.21	77.76	71.49	15.72	3.30	3.00	0.00	0.46	0.00
28	26.33	27.74	7.68	7.53	1.60	1.48	0.22	0.21	77.85	71.22	15.60	3.01	3.00	0.00	0.46	0.00
30	27.01	28.65	7.67	7.57	1.67	1.48	0.22	0.21	78.01	70.81	15.49	2.84	3.00	0.00	0.46	0.00

C. Regression Results for Generalized Network

We have 6 basic configurations, $G(2|12)$, $G(2,2,2,3,3)$, $G(3|12)$, $G(4|12)$, $G(6|12)$, and $G(12)$. Then we superpose $G(2|12)$ on $G(4|12)$, $G(2|12)$ on $G(6|12)$, $G(2|12)$ on $G(12)$, $G(2,2,2,3,3)$ on $G(6|12)$, $G(2,2,2,3,3)$ on $G(12)$, $G(3|12)$ on $G(6|12)$, $G(3|12)$ on $G(12)$, $G(4|12)$ on $G(12)$, and $G(6|12)$ on $G(12)$ to generate 9 new configurations. Then another 9 configurations that are variants of the 6 basic configurations (by adding bridge links that join two separate groups) are generated by adding 3, or 4, or 5 links to $G(2|12)$, adding 3, or 4 links to $G(2,2,2,3,3)$, adding 2, or 3 links to $G(3|12)$, adding 2 links to $G(4|12)$, and adding 1 link to $G(6|12)$.

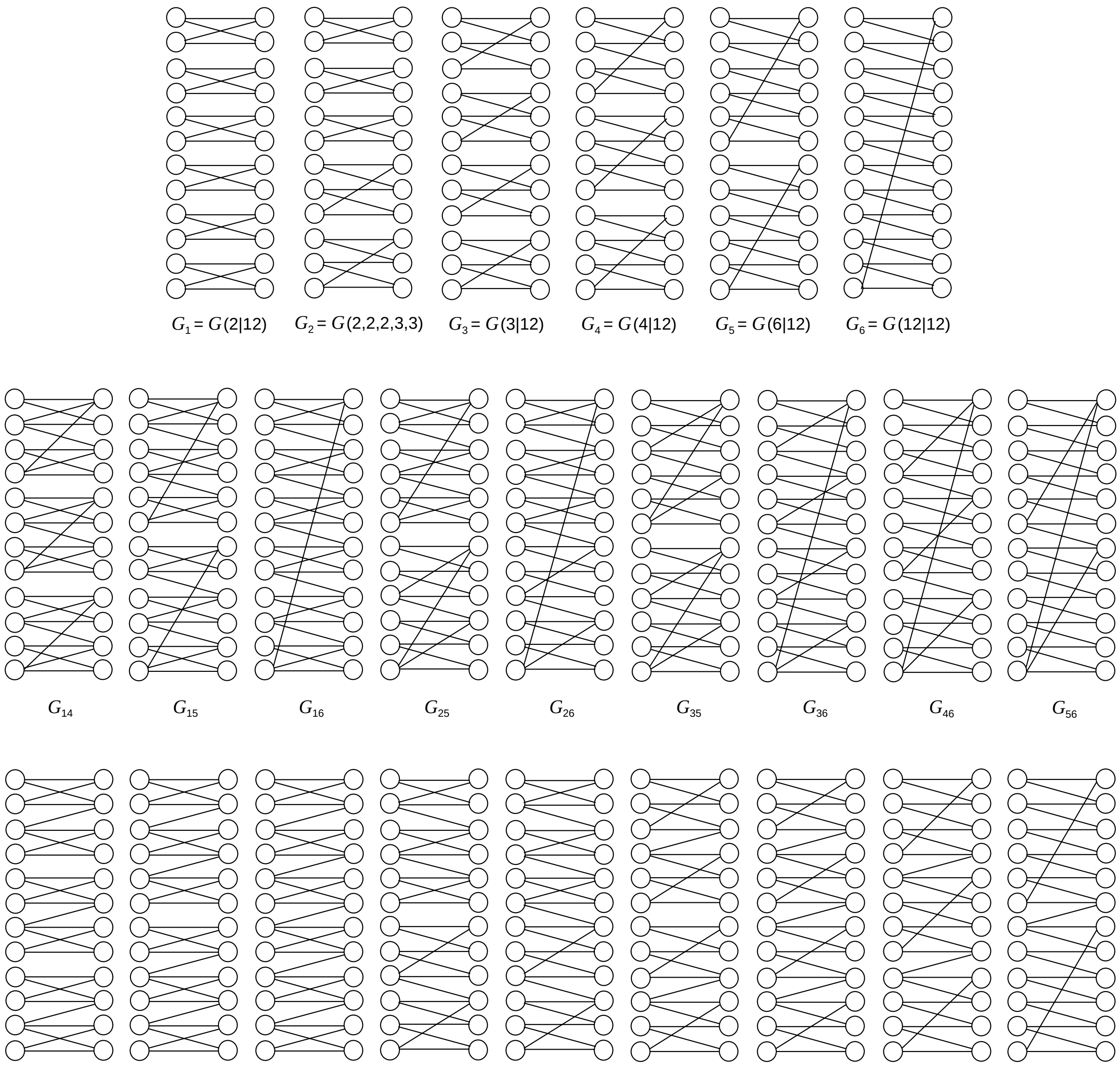

Figure 12: Configurations tested for expected fragility nnnn regression with $N = 12$

Table 5: Effect of single link and node failure on a multiple unidentical chain network

	Basic chains						Superposed chains					
Config.	G_1	G_2	G_3	G_4	G_5	G_6	G_{14}	G_{15}	G_{16}	G_{25}	G_{26}	G_{35}
GS	2	2.5	3	4	6	12	4	6	12	6	12	6
MinCS	2	2	3	4	6	12	2	2	2	2	2	3
MaxCS	2	3	3	4	6	12	4	6	12	6	12	6
$\mathbb{E}[\mathcal{F}_{1LF}(G)]$	2.08	2.99	4.08	5.67	8.71	15.17	2.49	3.69	6.29	5.01	6.92	4.47
$\mathbb{E}[\mathcal{F}_{1NF}(G)]$	89.79	88.48	87.11	85.33	82.73	78.29	85.32	82.74	77.99	80.55	77.98	82.74

	Superposed chains (cont.)			Chains with joint links								
Config.	G_{36}	G_{46}	G_{56}	$G_{1(3)}$	$G_{1(4)}$	$G_{1(5)}$	$G_{2(3)}$	$G_{2(4)}$	$G_{3(2)}$	$G_{3(3)}$	$G_{4(2)}$	$G_{5(1)}$
GS	12	12	12	4	6	12	6	12	6	12	12	12
MinCS	3	4	6	2	2	2	2	2	3	3	4	6
MaxCS	12	12	12	2	2	2	3	3	3	3	4	6
$\mathbb{E}[\mathcal{F}_{1LF}(G)]$	7.01	7.83	9.78	2.48	2.92	4.31	3.58	4.71	4.42	5.63	7.09	9.43
$\mathbb{E}[\mathcal{F}_{1NF}(G)]$	77.84	77.75	77.89	87.96	86.35	82.50	85.89	82.87	85.39	81.72	81.15	79.95