Appendix D

Lists of Common Distributions

In this appendix, we provide a short list of common distributions. For each distribution, we note the expression where the pmf or pdf is defined in the text, the formula for the pmf or pdf, its mean and variance, and its mgf. The first list contains common discrete distributions, and the second list contains common continuous distributions.

List of Common Discrete Distributions

Bernouli $0 < p < 1$	(3.1.1) $p(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$ $\mu = p, \quad \sigma^2 = p(1-p)$ $m(t) = [(1-p) + pe^t], \quad -\infty < t < \infty$
Binomial $0 < p < 1$ $n = 1, 2, \ldots$	(3.1.2) $p(x) = \binom{n}{x} p^x(1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$ $\mu = np, \quad \sigma^2 = np(1-p)$ $m(t) = [(1-p) + pe^t]^n, \quad -\infty < t < \infty$
Geometric $0 < p < 1$	(3.1.5) $p(x) = p(1-p)^x, \quad x = 0, 1, 2, \ldots$ $\mu = \frac{p}{q}, \quad \sigma^2 = \frac{1-p}{p^2}$ $m(t) = p[1-(1-p)e^t]^{-1}, \quad t < -\log(1-p)$
Hypergeometric (N, D, n) $n = 1, 2, \ldots, \min\{N, D\}$	(3.1.7) $p(x) = \frac{\binom{N-D}{n-x}\binom{D}{x}}{\binom{N}{n}}, \quad x = 0, 1, 2, \ldots, n$ $\mu = n\frac{D}{N}, \quad \sigma^2 = n\frac{D}{N}\frac{N-D}{N}\frac{N-n}{N-1}$ The above pmf is the probability of obtaining x Ds in a sample of size n, without replacement.
Negative Binomial $0 < p < 1$ $r = 1, 2, \ldots$	(3.1.4) $p(x) = \binom{x+r-1}{r-1} p^r(1-p)^x, \quad x = 0, 1, 2, \ldots$ $\mu = \frac{rp}{q}, \quad \sigma^2 = \frac{r(1-p)}{p^2}$ $m(t) = p^r[1-(1-p)e^t]^{-r}, \quad t < -\log(1-p)$
Poisson $m > 0$	(3.2.1) $p(x) = e^{-m}\frac{m^x}{x!}, \quad x = 0, 1, 2, \ldots$ $\mu = m, \quad \sigma^2 = m$ $m(t) = \exp\{m(e^t - 1)\}, \quad -\infty < t < \infty$

List of Common Continuous Distributions

beta $\alpha > 0$ $\beta > 0$	(3.3.5) $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1$ $\mu = \frac{\alpha}{\alpha+\beta}, \quad \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$ $m(t) = 1 + \sum_{i=1}^{\infty}\left(\prod_{j=0}^{k-1}\frac{\alpha+j}{\alpha+\beta+j}\right)\frac{t^i}{i!}, \quad -\infty < t < \infty$
Cauchy	(1.9.1) $f(x) = \frac{1}{\pi}\frac{1}{x^2+1}, \quad -\infty < x < \infty$ Neither the mean nor the variance exists. The mgf does not exist.
Chi-squared, $\chi^2(r)$ $r > 0$	(3.3.3) $f(x) = \frac{1}{\Gamma(r/2)2^{r/2}}x^{(r/2)-1}e^{-x/2}, \quad x > 0$ $\mu = r, \quad \sigma^2 = 2r$ $m(t) = (1-2t)^{-r/2}, \quad t < \frac{1}{2}$ $\chi^2(r) \Leftrightarrow \Gamma(r/2, 2)$ r is called the degrees of freedom.
Exponential $\lambda > 0$	(3.3.2) $f(x) = \lambda e^{-\lambda x}, \quad x > 0$ $\mu = \frac{1}{\lambda}, \quad \sigma^2 = \frac{1}{\lambda^2}$ $m(t) = [1 - (t/\lambda)]^{-1}, \quad t < \lambda$ Exponential(λ) $\Leftrightarrow \Gamma(1, 1/\lambda)$
F, $F(r_1, r_2)$ $r_1 > 0$ $r_2 > 0 > 0$	(3.6.6) $f(x) = \frac{\Gamma[(r_1+r_2)/2](r_1/r_2)^{r_1/2}}{\Gamma(r_1/2)\Gamma(r_2/2)}\frac{(x)^{r_1/2-1}}{(1+r_1x/r_2)^{(r_1+r_2)/2}}, \quad x > 0$ If $r_2 > 2$, $\mu = \frac{r_2}{r_2-2}$. If $r > 4$, $\sigma^2 = 2\left(\frac{r_2}{r_2-2}\right)^2\frac{r_1+r_2-2}{r_1(r_2-4)}$. The mgf does not exist. r_1 is called the numerator degrees of freedom. r_2 is called the denominator degrees of freedom.
Gamma, $\Gamma(\alpha, \beta)$ $\alpha > 0$ $\beta > 0$	(3.3.1) $f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}, \quad x > 0$ $\mu = \alpha\beta, \quad \sigma^2 = \alpha\beta^2$ $m(t) = (1-\beta t)^{-\alpha}, \quad t < \frac{1}{\beta}$

Continuous Distributions, Continued	
Laplace $-\infty < \theta < \infty$	(2.2.1) $f(x) = \frac{1}{2}\, e^{-\lvert x-\theta\rvert}, \quad -\infty < x < \infty$ $\mu = \theta, \quad \sigma^2 = 2$ $m(t) = e^{t\theta}\frac{1}{1-t^2}, \quad -1 < t < 1$
Logistic $-\infty < \theta < \infty$	(6.1.8) $f(x) = \frac{\exp\{-(x-\theta)\}}{(1+\exp\{-(x-\theta)\})^2}, \quad -\infty < x < \infty$ $\mu = \theta, \quad \sigma^2 = \frac{\pi^2}{3}$ $m(t) = e^{t\theta}\Gamma(1-t)\Gamma(1+t), \quad -1 < t < 1$
Normal, $N(\mu, \sigma^2)$ $-\infty < \mu < \infty$ $\sigma > 0$	(3.4.6) $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, \quad -\infty < x < \infty$ $\mu = \mu, \quad \sigma^2 = \sigma^2$ $m(t) = \exp\{\mu t + (1/2)\sigma^2 t^2\}, \quad -\infty < t < \infty$
t, $t(r)$ $r > 0$	(3.6.1) $f(x) = \frac{\Gamma[(r+1)/2]}{\sqrt{\pi r}\Gamma(r/2)} \frac{1}{(1+x^2/r)^{(r+1)/2}}, \quad -\infty < x < \infty$ If $r > 1$, $\mu = 0$. If $r > 2$, $\sigma^2 = \frac{r}{r-2}$. The mgf does not exist. The parameter r is called the degrees of freedom.
Uniform $-\infty < a < b < \infty$	(1.7.4) $f(x) = \frac{1}{b-a}, \quad a < x < b$ $\mu = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}$ $m(t) = \frac{e^{bt}-e^{at}}{(b-a)t}, \quad -\infty < t < \infty$