RESEARCH STATEMENT

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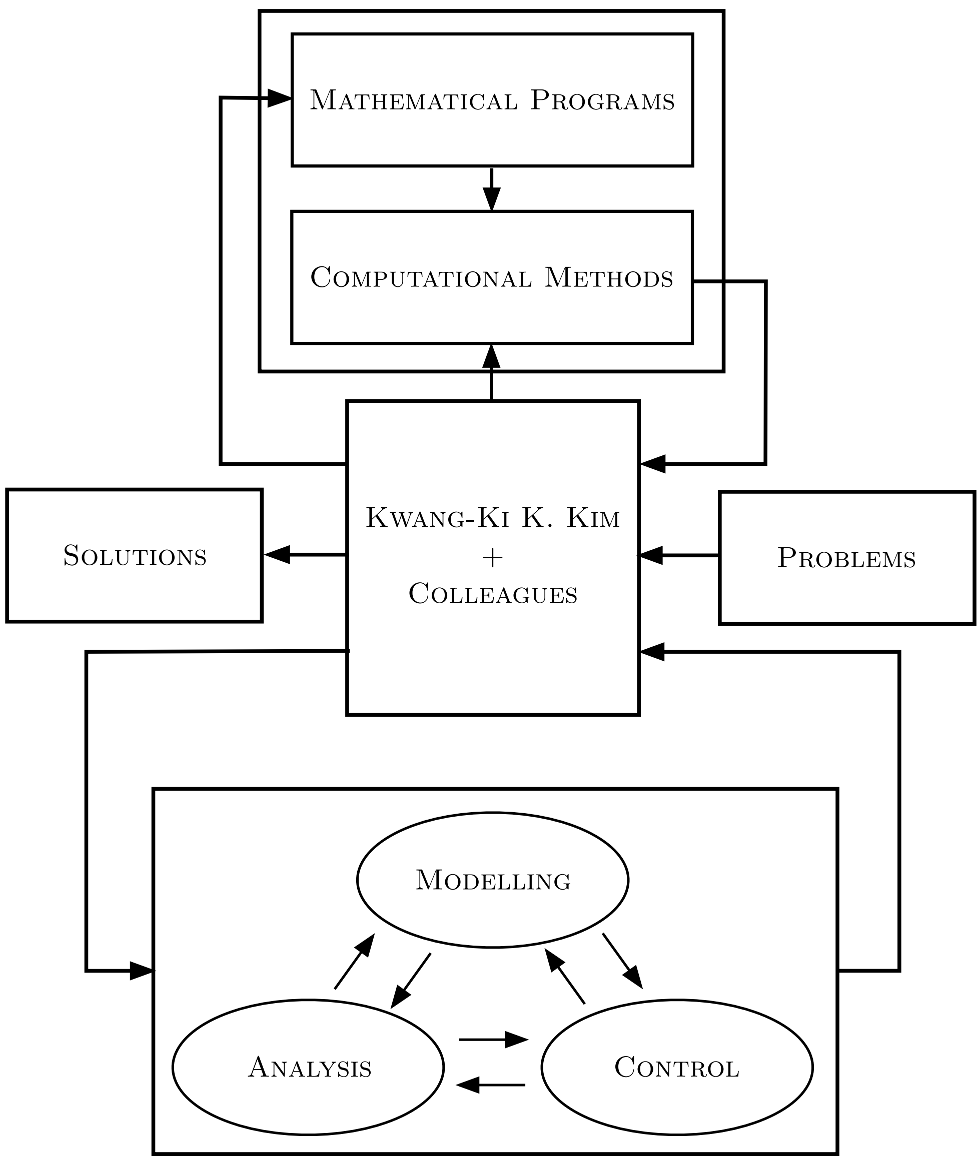

Research Diagram of Kwang-Ki K. Kim

Abstract

My research interests span the areas of systems and control theory, theoretical and computational optimization, and theoretical and analytical physics. A common thread in my research is in understanding physical and cyber-physical processes in system levels and using systems and control theoretic interpretations and mathematical tools to tackle synthetic design problems for which mathematical models of real processes are used to predict process behaviors. To understand systems of physics and cyber-physics, we resort to system models that can be obtained from the first principle or system identification with model validation/invalidation. Such system models inevitably contains parametric and non-parametric uncertainties for which one enables to build set-valued system descriptions. In model-based analysis and control, a main challenge of research is how to deal with uncertainties and the research philosophy is to fully exploit system model structure and accessible knowledge on the elements of structured and parameterized models. To improve and facilitate the decision-making process, properties and characteristics of a system under study need to be understood and updated whenever new information is accessible. Most of approaches for analysis and control of uncertain systems can be categorized by (a) *Robust* or *Adaptive Control* and (b) *Deterministic* or *Stochastic Control*, which have been on stream of my core research.

1 Completed and Ongoing Work

Uncertainties are ubiquitous in mathematical models of complex systems and decision-making under such uncertain circumstances requires a guide in which the followings need to be brought to the forefront: (i) how to quantify and estimate uncertainty propagation through mathematical models, (ii) how to adjust your decision accordingly by considering uncertainties in prediction, and (iii) how to optimize the expected cost or reward in the presence of uncertainties and unknowns. Answers to those questions can be solutions to mathematical programs for which systems of functions and functionals are used to mathematically represent uncertainty propagation, cost-to-go in prediction, the effects of manipulated variables.

Mathematical Programs for Robust and Adaptive Control For control problems in the presence of uncertainty, a common approach is to consider (a) *Robust Control*, (b) *Stochastic Control*, (c) *Stochastic Adaptive Control*, and their variations. A goal of my research is to develop systematic processes of decision-making for those control problems for which mathematical models of system are used and contain uncertainties and unknowns. The aforementioned control problems can be formulated as mathematical programs and in particular, I have been focusing mostly on finite-dimensional mathematical programs. For deterministic robust control, consider *Robust Programming* which has a standard form

$$\begin{aligned} &\min_{x\in\mathcal{X}} \max_{\delta\in\Delta} \ f(x,\delta) \\ &\qquad \text{s.t. } g_i(x,\delta) \geq 0,\ i=1,\ldots,m,\ \forall\delta\in\Delta, \end{aligned} \tag{1}$$

where the space of decision variables $\mathcal{X}$ and the space of uncertain parameters Δ are assumed to be compact, and the functions $f : \mathcal{X}\times\Delta \to \mathbb{R}$ and $g_i : \mathcal{X}\times\Delta\to\mathbb{R}$ are well-defined. For stochastic robust control, consider *Stochastic Programming* which has a standard form

$$\begin{aligned} &\min_{x\in\mathcal{X}} \ f(x) \\ &\text{s.t. } \Pr[g_i(x,\delta)\geq 0] \geq \beta_i,\ i=1,\ldots,m,\ \delta\sim p_\delta, \end{aligned} \tag{2}$$

where the space of decision variables $\mathcal{X}$ is assumed to be compact and p_δ refers to known probability distribution of a random vector δ whose support is not necessarily compact. The objective function can have a probability representation such as $\Pr[f(x,\delta) \geq \gamma]$ for a constant $\gamma \in \mathbb{C}$. For stochastic adaptive control, consider *Stochastic Adaptive Programming* which has a standard form

$$\begin{aligned} &\min_{x \in \mathcal{X}} f(x) \\ &\text{s.t. } \Pr[g_i(x,\delta) \geq 0] \geq \beta_i,\ i = 1, \ldots, m,\ \delta \sim \hat{p}_\delta, \end{aligned} \tag{3}$$

where the space of decision variables $\mathcal{X}$ is assumed to be compact and $\hat{p}_\delta$ refers to a Bayesian estimate of the true probability distribution p_δ that is unknown. Note that $\hat{p}_\delta$ in general is a function of the observables which are dependent of the decision variables $x \in \mathcal{X}$. Similar to stochastic programming, the support Δ might not be compact and the objective function can have a probability representation such as $\Pr[f(x,\delta) \geq \gamma]$ for a constant $\gamma \in \mathbb{C}$.

1.1 Robust Control and Absolute Stability with Applications

Related Publications: [4–7, 9, 10, 12, 15, 17, 20, 28]

Absolute Stability and Nonlinear Internal Model Control Some of nonlinear system identifications have a certain type of Luré systems and some of existing robust and absolute stability conditions can be applied to and further developed for those system models. In [28], a nonlinear internal model control (IMC) procedure was presented for stable Wiener systems that ensures robustness of closed-loop stability and performance to uncertainties in the inversion of the static nonlinearity, while having polynomial-time computational cost. Several more general observations can be made based on a pH control case study. Assuming perfect nonlinearity inversion when controlling pH processes led to overly optimistic predictions on the achievable closed-loop performance, which indicates that the commonly made assumption of perfect nonlinearity inversion can produce poor results in practical applications. A comparison of pH controllers designed to satisfy robust stability or disturbance suppression constraints showed that the closed-loop response speed could significantly change depending on the design criteria. A comparison of the sufficient robust stability condition with a necessary condition showed that the sufficient robust stability condition was nonconservative for this particular application. The nonlinear IMC procedure is applicable to stable Wiener systems with unstable zero dynamics, unmeasured states, disturbances, and measurement noise. This is in contrast to many nonlinear control methods that require stable zero dynamics and/or ignore disturbances and measurement noise. The generalization of the approach to Hammerstein and Sandwich models is straightforward, and can be used to explicitly incorporate actuator constraints into the nonlinear controller design, by combining these static nonlinearities with any other static nonlinearity associated with the input to the process. The approach can also be combined with directionality compensation, which can improve the closed-loop dynamics for ill-conditioned processes.

Absolute Stability and Neural Network Models In [7], a comprehensive overview of theory on the use of dynamic artificial neural network models for identification and control problems is provided. The three main classes of dynamic artificial neural network models for identification of nonlinear dynamical systems are reviewed: (a) neural state-space models, (b) global input-output models, (c) dynamic recurrent neural network models. The presentation of their mathematical models and architectures are followed by their representations in terms of consistent block diagrams convenient for stability and performance analyses and argued to have potential benefits for system identification and control. The classes of nonlinear dynamical systems that can be universally approximated by such models are characterized, with rigorous upper bounds on the approximation errors. While many of the results are available in the literature, it was the first to fully develop and clearly explain these models and their interrelationships to provide a broader perspective, and presents some new results to fill in the gaps in the literature. In [6], new computationally efficient stability conditions for parameterized nonlinear system models based on artificial neural networks are presented. We present a unified framework and convex conditions for stability analysis of these three classes of dynamic artificial neural network models. Each parameterized nonlinear model is transformed into a standard nonlinear operator form for which stability criteria based on linear matrix inequalities (LMIs) are derived. This approach is computationally efficient and reliable, equipped with well-developed computational algorithms for semidefinite programming (SDP).

Robust Absolute Stability and Control: SDP Approaches In [4], two LMI-based procedures are proposed for the design of observer-based output feedback controllers for a Luré-type system with conic-sector-bounded slope-restricted nonlinearities. Observer design methods are proposed for two different strategies: (a) based on an observer-controller separation and (b) based on simultaneous design derived from the variable reduction lemma (aka Finsler's lemma). Both sets of LMIs are easily solved using existing interior-point methods. Their robustness against model uncertainty and insensitivity to output disturbance were also investigated. In [5], design methods of static and fixed-order dynamic output feedback controllers are proposed for discrete-time parametric uncertain Luré systems with sector-bounded nonlinearities in which parametric uncertainties are described by polytopes. Controller design equations are derived for systems with multiple states, outputs, and nonlinearities in terms of iterative LMIs, which come up with solving semidefinite programs (SDPs). The proposed design methods are developed from stability conditions using parameter-dependent Lyapunov functions (PDLFs) and existing iterative numerical methods such as cone complementary or alternating projection algorithm are adopted to solve certain classes of nonconvex optimization problems.

Generalized D-stability and its Applications In [9], generalized notions of D-stability are studied in terms of their relations, implications, and applications. Necessary and sufficient conditions for continuous- and discrete-time D-stability are proposed in terms of structured singular values of related matrices. It is shown that D-stability is not preserved under bilinear transformation between continuous- and discrete-time autonomous LTI systems, whereas diagonal stability is preserved. In particular, continuous-time D-stability implies discrete-time D-stability under bilinear transformation, but the converse is not generally true. It is shown that, for a certain class of interconnected systems, diagonal stability and D-stability are equivalent and the optimization of diagonal scaling gives a necessary and sufficient condition for stability of those systems. In addition to the aforementioned results on diagonal stability and D-stability, diagonal stability, multiplicative D-stability, and additive D-stability are discussed as well as applications to stability and stabilization of a distributed power control algorithm and the study of reaction-diffusion models. The approaches are based on structured singular value theory incorporated with loop and matrix transformations. It is shown that some of existing results on robust power distribution control and additive D-stability are special cases of the results presented in [9]. Furthermore, robust optimal control design for power distribution control are formulated as linear programs (LPs) and SDPs for which there are efficient (polynomial-time) numerical algorithms such as primal-dual interior point methods.

Stability Analysis and Control of Large-scale Interconnected Systems In [10,12], stability of large-scale interconnected or networked systems is investigated. Because of substantial demands on computation and lack of exact solutions (see [15] for an overview of computational methods and their complexity of robust analysis and control problems), developing tractable and scalable sufficient conditions for stability of such systems is a crucial problem in mathematical systems theory. The conference proceedings [10, 12] present verifiable convex sufficient stability conditions for certain classes of interconnection structure and subsystem dynamics. The proposed conditions are obtained by applying the concept of topological graph separation and require for certain related transfer functions to be positive real. The resultant positive realness conditions can be transformed into linear matrix inequality conditions in terms of a state-space realization of subsystem dynamics, a Lyapunov matrix, and multipliers such that the proposed stability tests are convex problems and computationally tractable. It is also shown that those conditions can be extended to design of decentralized controllers that optimize local objectives written in terms of the LQR sense and stabilize the overall system. The latter results do not explicitly consider uncertainties in the subsystem (agent) dynamics, but use the spectrum or the numerical range of the interconnection matrix to derive stability criteria in terms of robust topological graph separation. While such robust topological graph separation approaches to stability analyses guarantee certain levels of robustness against uncertainties in subsystem dynamics, time delay in communication, and interconnection structure, an objective is to develop scalable and explicit conditions for robust stability for large-scale interconnected/networked systems. These results are used to investigate a communication link control problem, to maximize the robustness while retaining a certain level of convergence rate of consensus or stability.

1.2 Model-based Spectral Methods for Analysis and Control of Stochastic Uncertain Systems: Polynomial Chaos

Related Publications: [11, 14]

Stochastic Uncertainty Quantification for Control Problems Uncertainties are ubiquitous in mathematical models of complex systems and a method of spectral approximation called generalized polynomial chaos (gPC) expansions can be used for uncertainty propagation and quantification that are significantly important in robust analysis and control. In [14], gPC expansions are shown to be more computationally efficient than Monte Carlo simulations for quantifying the influence of stochastic parametric uncertainties on the system behaviors and properties. Approximate surrogate models based on generalized polynomial chaos expansions are applied to design optimal controllers by solving stochastic optimizations in which the control laws are suitably parameterized, and the cost functions and probabilistic (chance) constraints are approximated by spectral representations. The approximation error is shown to converge to zero as the number of terms in the generalized polynomial chaos expansions increases. The analysis of uncertainty propagation and quantification in models has several applications in systems engineering. The validation/invalidation of models against experiment data must take into account the effects of model uncertainties and measurement noise. In the presence of stochastic uncertainties, it is important to determine the probabilities of the system properties exceeding specified critical values or operation limits, and such evaluation can be used to conduct reliability and risk analyses. When the system parameters are assumed to be random variables and the exogenous disturbance are random processes, the solution and output trajectories are stochastic and controller design reduces to solving stochastic optimization problems with probabilistic constraints, also known as chance constraints. PC expansion approaches can be used to approximate chance constraints and the convergence of the approximation is guaranteed. In a gPC approach for addressing these problems, the system model is replaced by a surrogate model whose solutions are represented by a gPC expansion and the surrogate model analyzed or simulated to quantify the propagation of uncertainty through the system.

Approximate Stochastic Receding Horizon Control In [11], model predictive control problems are considered with dynamical systems subject to stochastic parametric uncertainty due to plant/model mismatches and exogenous disturbance that corresponds to uncertain circumstance in operation of the system. Parametric uncertainty propagation and quantification are approximated using a spectral method using gPC expansions and exogenous disturbance is assumed to be an additive Gaussian random process. With Gaussian approximation of resulting solution trajectory of a stochastic differential equation using a gPC expansion, one needs to solve convex finite-horizon model predictive control problems that are amenable to online computation of a stochastically robust control policy over the time-horizon. The proposed approach to chance-constrained model predictive control provides an explicit way to handle a stochastic system model in the presence of both model uncertainty and exogenous disturbances. Probabilistic constraints are replaced by convex deterministic constraints that approximate the probabilistic violations with a user-defined confidence level.

1.3 Optimal Experiment/Input Design for System Identification and Control

Related Publications: [1, 2, 13, 16, 21]

Convex Relaxation of Sequential Input Design Mathematical modeling of dynamical systems is ubiquitous in systems and control theory, and indispensable for prediction of system behaviors and controller design. The identification of mathematical models, also known as system identification, has been extensively in systems and control science literature and many different approaches to input (or experimental) design for system identification have been pursued and are resurging due to advances in optimization theory and algorithm, especially in convex optimization, and in model-based control (e.g., model predictive control (MPC), internal model control (IMC), robust and adaptive control, etc.) that is also closely related to identification for control (also known as dual control).

In most of the current literature of optimal input design for system identification, the input design is performed as a batch process. In other words, the problems ignore the availability of extra information such as online intermittent measurements and assume the nominal values of parameters are close to the true parameter values, which is somewhat paradoxical. In [21], the parameter values used to evaluate the Fisher information matrix is updated whenever we can access new information or measurements. Widely used optimization problems for input/experimental design for system identification are known to be nonconvex and NP-hard, and the problem data such as the Fisher information matrix depend on the true system parameters that are unknown a priori. To overcome such difficulties, this paper uses convex relaxations to compute suboptimal solutions for the original nonconvex problems and parameter adaptation methods to update Fisher information matrix with the estimated parameter that is time-varying and obtained from the available observables. We establish a strategy of iterative input design that is to compute an optimal input sequence within a finite horizon of prediction and use a receding horizon scheme for computing suboptimal input sequences. This design procedure can be considered as an iterative approximate dynamic programming for maximizing the performance of parameter estimation.

In addition, it is shown that similar design procedures using convex relaxation and receding horizon methods can be extended to frequency response estimation for which mixed time- and frequency-domain constraints over various types of estimation qualities are explicitly considered. To our best knowledge, this is the first work on unified optimal input design methods with which one can manage both time- and frequency-domain quality measures for system identification, solve multi-objective optimality criteria, and incorporate various types of input and output constraints for both time- and frequency-domain into the resultant convex optimization to ensure operation safety and consider physical limitation on capability of actuators. The degree of sub-optimality can be quantified for some cases. For the case of the input power constrained optimization, the relaxation is shown to be lossless, i.e., there is no (dual) gap. It is expected that the optimum/sup-optimum (dual) input design methods presented in [21] make strong contributions to system identification and model-based controller design, especially to real-time model-based optimal/sub-optimal control such as MPC and approximate dynamic programming.

Convex Relaxation of Sequential Experiment Design for Process Gain Estimation In [16], optimal input design for a class of structured large-scale systems in which the input-output directionality is independent of frequency is studied. For maximizing the information contained in experimental data collected from applying inputs to the process, manipulated variables are computed from solving constrained optimizations for which the payoff function is related to the covariance of estimation or a user-specified quality measure of estimation and the constraints correspond to requirements for operational safety and actuator limitations. The methods are applied to process gain estimation for a simulated blown film process to illustrate the input design procedure and compare the results for two different types of constraint sets. In addition, closed-form solutions are computed for the optimal input design associated with signal-to-noise ratio (SNR), D-optimality, and A-optimality measures in the presence of an input sum-of-squares constraint. A new measure for sensitivity of optimality criteria to the change in input direction is introduced and computed for the three aforementioned optimality criteria.

1.4 Model-based Fault Tolerant and Reliable Control

Related Publications: [20, 22, 27]

Robust Reliable Control An inevitable consequence of industrial practice is that actuators and sensors can become faulty or fail, which motivates the development of methods to evaluate the reliability of the closed-loop system to such imperfect operations. A feedback-controlled system is said to be *reliable* if it is guaranteed to retain desired closed-loop system properties while tolerating faults or failures of actuators and/or sensors. Maximizing the reliability of a system concerns minimizing its potential performance degradation while retaining closed-loop stability when a fault or failure occurs in a control/measurement channel. In addition to the possibility of actuator/sensor faults or failures, plant-model mismatches are also inevitable, which motivates their incorporation into reliability and integrity analysis. This paper is motivated by the need for nonconservative testing conditions to ensure closed-loop stability and to retain a satisfactory closed-loop performance in the presence of both plant-model mismatches and actuator/sensor faults or failures.

In [20], decentralized controlled systems and their *robust reliable stability and performance* are studied in the presence of possible actuator/sensor faults or failures with consideration of the overall plant-model mismatches that are described in terms of bounded set-valued linear operators. The main contribution of [20] is to derive necessary and sufficient conditions for various types of *robust reliable stability and performance* of a set-valued plant model that is described by a linear fractional transformation (LFT) with structured uncertainties. It is assumed that any failure of a local controller is detected and the controller is taken out of service whenever a failure occurs, so that any undesirable propagation of local failures to other parts of the system can be avoided. Even though it is mostly concentrated on the analysis of decentralized control systems, the proposed approach does not depend on the structure of the selected control schemes and can be applied to any type of linear controller and actuator-sensor selection. The approaches proposed in [20] are based on the structured singular value (μ) and a standard representation of uncertain systems known as the LFT. Robust reliable control problems for large-scale systems with decentralized control are reformulated in terms of robustness analysis based on μ to model the effects of faults. Structures of interconnected sensors and actuators as well as structures of uncertainties can be fully exploited to perform nonconservative or less conservative analysis. An efficient framework for the analysis and synthesis of robust reliability is presented. Faults and failures in process components are treated as parametric uncertainties that are compatible with μ. Although the main focus of this paper is on decentralized control problems, the methodology is not restricted to decentralized control and the results can be extended to general control structures in a straightforward manner.

Active FDD and Controller Reconfiguration In [22], statistical approaches to stochastic robust fault tolerant model predictive control are studied. Bayesian inference is a natural approach to fault detection and diagnosis (FDD) that is compatible with stochastic model predictive control (MPC), since the predicted controlled trajectories in the stochastic model predictive control framework is intrinsically random processes whose probability spaces are associated with stochastic uncertainties, disturbances, and noises. We consider two different measures of statistical distance between two probability distributions to quantify detectability of a fault. A set of models that correspond to multi scenarios of faults is used for Bayesian hypothesis testing for which each model is considered as a competing model and the FDD problem is to find an optimal model of a hypothesized fault explaining the observable data at each instance of fault monitoring. A novel *active* fault detection and isolation method is proposed as an optimization for which an optimal input distinguishing fault scenarios consistent with the current observables is computed to actively search the true fault [22, 27]. The presented statistical inference method for multiple hypothesized models of fault scenarios is essentially compatible with stochastic MPC design problems and can be incorporated with statistical state estimation such as Bayesian state estimation including Kalman filter and its variations, and moving-horizon estimation (MHE). The results in [22, 27] should be considered as a promising start to build an integrated suboptimal control design algorithm for *Stochastic Robust Fault Tolerant Model Predictive Control*.

1.5 Robust Statistical Inference for Identification of Reaction Networks

Related Publications: [18]

Analytical Solutions to CMEs and Sparse Network Identification Determining the interaction networks of bio-chemical reactions and genetic regulation has significantly important applications, especially in a large-scale interconnected system. Most of existing approaches to bio-chemical interaction networks are based on the continuum model of dynamical systems in which the state variables are concentrations of each species. In [18], our methods of reaction network identification use the chemical master equations (CMEs) in which the discrete Markov processes characterize the transition probabilities between populations of species and the transition probabilities are functions of the reaction rates and the interconnection networks. To incorporate observation mechanism into the associated CMEs, we use hidden Markov models (HMMs) with which Bayesian inference problems are formulated for determination of the parameters of the HMMs and result in the maximum likelihood estimate (MLE). Due to the presence of measurement noise and sensor errors, the regulation procedure is indispensable and can be performed by introducing a penalty term into the MLE:

$$\begin{aligned} \max\ & L\left(\{y_i\}_{i=t-t_{\rm w}}^{t} | A\right) - \gamma\phi(A) \\ \text{s.t.}\ & A \in \mathcal{S} \end{aligned} \tag{4}$$

where $L(\cdot|A) : \mathcal{Y}_{t_{\rm w}} \to [0,1]$ refers to the likelihood function, $\mathcal{S}$ denotes the structure constraint on the network that is associated with priori knowledge, $t_{\rm w}+1$ is the length of observation window corresponding to the measurement sequence $\{y_i\}_{i=t-t_{\rm w}}^{t}$ that could depend on the current time t, $\phi : \mathcal{S} \to \mathbb{R}_+$ refers to the penalty term that encourages sparsity of the network interconnection, and the constant $\gamma \geq 0$ refers to the user-defined weight imposing sparsity on the interconnection, which characterizes the trade-off between model complexity and prediction error associated with the likelihood function. Extensions of this work include sequential identification incorporated with uncertainty quantification and optimal experiment design, and the problems of model invalidation and discrimination for which experiments must be designed to discriminate between multiple hypothesized models.

1.6 Controller Synthesis Problems for $\mathcal{L}_1$ Adaptive Control

Related Publications: [3, 24–26]

Multi-objective Filter Design for $\mathcal{L}_1$ Adaptive Control In [24–26], the similarities and connections between disturbance observer and $\mathcal{L}_1$ adaptive control are investigated. $\mathcal{L}_1$ adaptive output–feedback control framework is studied and it is shown that the $\mathcal{L}_1$ reference controller is equivalent to a certain type of internal model control called the disturbance observer. Using this fact, several properties of the disturbance observer architecture are investigated, leading to various filter design methods towards verification of the stability conditions for the $\mathcal{L}_1$ adaptive output-feedback controller.

Analysis of the Architecture of $\mathcal{L}_1$ Adaptive Control In [3], the limiting behavior of $\mathcal{L}_1$ adaptive controllers in the presence of fast adaptation is analyzed. For linear systems it is shown that the $\mathcal{L}_1$ adaptive controllers approximate an *implementable non-adaptive* linear controller, when the adaptation rate approaches infinity. This implies that both the $\mathcal{L}_1$ adaptive and the limiting non-adaptive control systems achieve the same control objective. The essential difference between the two systems is in the synthesis of the feedback signal. In the limiting non-adaptive system the feedback signal makes explicit use of the system inverse, while the fast estimation loop of the $\mathcal{L}_1$ adaptive controller approximates this inverse map, avoiding the explicit system inversion. This property of the adaptive architecture is essential for its extension to nonlinear systems and also for accommodating various hardware constraints.

1.7 Cone-invariant Systems and Partial Order Preserving Operators

Related Publications: [19, 23]

Linear Lyapunov Functional for Cone-invariant LTI Systems Mathematical models of dynamical systems in which the state trajectories remain in a cone are prevalent in many systems and control problems and their application to switched positive systems has become a popular research topic. For a continuous-time LTI system $\dot{x} = Ax$ with an initial condition $x(0) = x_0$, it is well-known that an equilibrium state is globally asymptotically stable (g.a.s) if and only if there exists a positive definite solution $P \succ 0$ of the Lyapunov inequality $PA + A'P \prec 0$. This stability condition can be generalized for a cone invariant LTI system, namely, the state transition map corresponding to the system $\dot{x} = Ax$ ensures that $x(0) \in \mathcal{C}$ implies $x(t) \in \mathcal{C}$ for all $t \geq 0$, where $\mathcal{C} \in \mathbb{R}^n$ is a cone. It is straightforward to show that the aforementioned cone invariant LTI system is g.a.s. if and only if there exists $P \succ_{\mathcal{C}} 0$ such that $PA + A'P \prec_{\mathcal{C}} 0$, where $P \succ_{\mathcal{C}} 0$ is equivalent to $x'Px > 0$ for all $x \in \mathcal{C} \setminus \{0\}$ and $P \prec_{\mathcal{C}} 0$ is equivalent to $-P \succ_{\mathcal{C}} 0$. This stability condition for the $\mathcal{C}$-invariant LTI system is called the *copositive Lyapunov inequality*. For the copositive Lyapunov inequality and function of a polyhedral cone invariant LTI system, an open question was raised:

Problem 1. Consider a matrix $A \in \mathbb{R}^{n\times n}$ and the cone $\mathcal{C} \triangleq \{x \in \mathbb{R}^n : Cx \geq 0\}$. Determine necessary and sufficient conditions for the existence of a symmetric matrix $P \succ_{\mathcal{C}} 0$ such that $PA + A'P \prec_{\mathcal{C}} 0$.

In [19], conic Lyapunov stability conditions of cone invariant linear time-invariant (LTI) systems that leave a proper cone positively invariant. Geometric algebraic conditions for the stability of an equilibrium state are established from using the concepts of dual and polar cones. Namely, the existence of a dual variable in the interior of the dual cone such that the adjoint operator maps the dual variable into the interior of the polar cone is a necessary and sufficient condition for the stability of the cone invariant LTI system, where the cone can be an arbitrary proper cone in the state space. Spectral properties of the corresponding transfer function are investigated, to which the spectral radius of the system matrix is explicitly related. In addition, robustness analysis with respect to certain types of parametric uncertainties and disturbances is presented. A main contribution of the paper [19] is to provide convex conditions for the stability of general cone invariant LTI systems and present a fundamental answer to the stability of general cone invariant LTI systems, which generalizes the answer to Problem 1.

Linear Storage Functions for Analysis and Control of Cone-invariant Uncertain Systems Methods for computing quadratic Lyapunov solutions for the copositive Lyapunov inequality are under development [23] that exploit the topological properties of proper cones with the associated dual and polar cones. Furthermore, a linear Lyapunov functional can be used as a storage function that defines dissipation inequalities with respect to linear supply rates over the cone. This implies that input-output characteristics of a cone-invariant system can be analyzed in terms of linear conic programs that are computationally attractive.

2 New Research Agenda

2.1 Model-based Robust Approximate Dynamic Programming

Related Publications: [8]

Modified Sequential Rollout Algorithm Since the optimal cost-to-go (value) function is not generally available, one natural approach to solving the optimal control problem is to generate an approximation within a parameterized class of functions such as statistical regression or machine learning. For such a functional approximation, one need to determine a parameterized class of functions $\hat{V} : \mathbb{X} \times \mathbb{R}^s \to \mathbb{R}$ and then compute an optimal parameter vector $r \in \mathbb{R}^s$ to fit the optimal cost-to-go function. For an effective approximation, there are two main steps of formalism: (a) selection of features associated with an approximation architecture and (b) optimal fitting of appropriate parameter values. Several different forms of regression models can be considered:

$$
\begin{aligned}
&\text{(Linear Regression) } \hat{V}(x,r) = \sum_{i=1}^{s} r_i \psi_i(x), \\
&\text{(Kernel Regression) } \hat{V}(x,r) = \sum_{i=1}^{s} r_i K(x,x_i), \\
&\text{(Piecewise Quadratic) } \hat{V}(x, r=\{Q,q,q_0\}) = \min_i \{\psi_i(x) = x'Q_i x + q'x + q_0\}, \\
&\text{(Piecewise Linear) } \hat{V}(x, r=\{q,q_0\}) = \max_i \{\psi_i(x) = q'x + q_0\}.
\end{aligned} \tag{5}
$$

Note that this is, of course, impossible without knowing the optimal policy $\mu^\star$. Without knowing the exact optimal policy, a typical approach for ADP is to iterate value function approximations and policy improvements. To denote the policy-dependence of approximate value function, use $V^\mu(x,r)$. A value function approximation is followed by policy evaluation and improvement:

$$
\mu^{(j+1)}(x) := \arg \min_{u \in \mathcal{U}(x) \cap \mathcal{M}(x)} \left\{ \mathbf{E}\left[\hat{V}^{\mu^{(j)}}(Ax + Bu + Ew, r^{(j)}) \right] + \gamma \ell(x,u) \right\} \tag{6}
$$

where $r^{(j)} \in \mathcal{R}_j$[1] is an optimal fitting parameter vector of $\hat{V}^{\mu^{(j)}}$ that is an approximation to the true value function $V^{\mu^{(j)}}$ corresponding to the policy $\mu^{(j)}$, i.e,

$$
r^{(j)} := \arg \min_{r \in \mathcal{R}_j} \| V^{\mu^{(j)}} - \hat{V}^{\mu^{(j)}}(\cdot, r) \| \tag{7}
$$

where $\|\cdot\|$ is a properly chosen functional norm. In particular, we might consider $\|V\| = \int_{x \in \mathbb{X}} V(x) d\nu(x)$ where $\nu : \mathbb{X} \to \mathbb{R}$ defines a measure space. The multi-dimensional integral can be approximated by an importance sampling method, i.e., $\|V\| \approx \sum_{i=1}^{N_s} \nu_i V(x_i)$ where N_s is the number of generated samples and ν_i denote the importance weights corresponding to the state x_i.

To tackle state constrained dynamic programming problems, we consider the modified rollout algorithms [8] below. The idea is to solve one-stage optimization for which the cost-to-go function is replaced by the value function associated with a predetermined base policy and a penalty term corresponding to the constraint violations of the controlled trajectory generated by a base policy is introduced.

Modified rollout algorithm: In the presence of state/output constraints having forms of chance constraint, it might be hard to find a feasible solution trajectory (denoted by the tube $\mathcal{F}$) that satisfies such constraints for all future times in determining a predictive optimal or suboptimal control policy. For a start, a base policy is used as a pseudo-active control policy and evaluate its probabilistic feasibility under stochastic circumstances. Current control policy can be obtained from solving one-stage stochastic programming in which the optimal cost-to-go function is replaced by the value function corresponding to a base policy and a penalty term associated with the probabilistic state constraint violations computed from a base policy is introduced with a (predetermined) weighting factor.

1. For a given base policy μ^b, simulate the model and compute the corresponding average cost $V^{\mu^b}(x)$ for all $x \in \mathcal{X}_s \subset \mathbb{X}$ and $|\mathcal{X}_s| < \infty$.
2. Approximate $V^{\mu^b}(x)$ by $\hat{V}^{\mu^b}(x;r)$.

[1] The set of candidate parameters at each cost approximation step can be different. In particular, one can use nested sets $\mathcal{R}_j \subseteq \mathcal{R}_{j+1}$ for all $j = 0, \ldots$.

3. Evaluate the expected number of constraint violations

$$N_v^{\mu^b}(x) := \mathbf{E}\left[\sum_{t=1}^{\infty} I_{\{x_t \notin \mathcal{F}\}} | x_0 = x\right] \tag{8}$$

and compute a function $P^{\mu^b}(x)$ that is monotonically increasing in $N_v^{\mu^b}(x)$.

4. Solve

$$\begin{aligned} \min_{u_t \in \mathcal{U}_t(x_t)} \ & \mathbf{E}\left[\ell(x_t, u_t) + \hat{V}^{\mu^b}(x_{t+1}; r) + \lambda P^{\mu^b}(x_{t+1})\right] \\ \text{s.t.} \ & x_{t+1} = A_t x_t + B_t u_t + E_t w_t, \\ & x_{t+1} \in \mathcal{F}, \\ & w_t \sim p_w,\ x_t \sim p_{x_t}. \end{aligned} \tag{9}$$

2.2 Distributed Analysis and Control of Complex Uncertain Systems

In the course of my research, I have noticed that the conservatism inherent in deterministic worst-case analysis and control might be unrealistic in the sense that it might result in small performance guarantees or even no feasible solutions for robust control. In stochastic analysis and control of uncertain systems, representations of system model uncertainty can be more informative in the sense that probabilistic uncertainty quantification and propagation enable one to exploit both subjective and objective information about uncertain models that are used for prediction of the true system properties and behaviors. I envisage the field of robust approximate dynamic programming created from the ground up, building upon the foundations of a number of different research fields including statistical and machine learning, convex relaxation, and stochastic programming. In the near future, I am interested in the principles involved in distributed and decentralized robust approximate dynamic programming. They include model-based predictive suboptimal control for large-scale interconnected or networked uncertain systems in which subsystems share information with their neighbors and update the concurrently associated optimizations to achieve cooperative or non-cooperative control objectives. Decentralized or distributed optimal control problems are formulated as mathematical programs that can be approximately solved by using regression models of value functions, Lagrange relaxation, generalized S-procedure, etc. Simultaneously, I intend to understand how subsystems of large-scale interconnected or networked systems can cooperate to achieve a common control objective. My research will focus on developing integration protocols for distributed and decentralized suboptimal control that are based on stochastic robust predictive suboptimal control for which the interconnection or networking structures are exploited to derive decomposed design criteria with which online computational demands can be reduced. In particular, examples of applications that I have a keen interest in are power distribution control of smart grid in the presence of communication delays and uncertain demands for electricity.

2.3 Decision Making under Stochastic Uncertainty

Decision making under uncertain circumstances has been one of the most important problems in many research fields including control, optimization, operation research, etc. In most of research problems, one is required to make a decision based on prediction and forecasting before uncertainties are realized. There can be two different approaches to deal with such stochastic uncertainties–(a) stochastic robust analysis and control and (b) stochastic adaptive analysis and control. Both approaches have their own pros and cons and our goal is not to compare them but to determine which approach is preferably applicable to a given (specific) problem and develop generalizable problem-dependent solutions. Optimality criteria can be the minimum response time for uncertain demands, maximizing the probability of stable resource allocations, etc. In addition to centralized decision making, team-decision making under uncertainty has been a significantly active research field for several decades. Information exchange between team members undergoes certain types of limitations such as delay in communication and capacity limitation, as well as stochastic uncertainties in system models.

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