Group: ______________ ______________ ______________ Name: _______________

Math 231E. Worksheet 2A. September 4, 2018
Calculating the uncalculatable! (using Taylor series)

It is common in science and engineering that we have a mathematical model of an object or process of interest. It is typical that this model does not explicitly give us the solution we are looking for, but gives the solution only implicitly. And, in the end, it is sometimes impossible to solve our model exactly. However, Taylor series (and other kinds of approximation techniques) allow us to get **close** to a solution without having to know it exactly!

Problem 1. *For an object falling through the air in the absence of air friction the velocity $v(t)$ satisfies*

$$m\frac{dv}{dt} = -mg \tag{1}$$

1. *Write down all solutions to this equation in the form $v(t) = \dots$. Notice that the solution to this problem involves an arbitrary constant. What is the physical interpretation of this constant?*

2. *The constant g is approximately $g = 9.8m/s^2$ near the Earth's surface. Assume that the initial velocity of the object is $0m/s$ and compute the velocity at $t = 10s, 50s$. What changes if the initial velocity of the object is $-1m/s$? Again compute the velocity at $t = 10s, 50s$.*

Problem 2. *Now consider the case where there is friction due to air resistance.* If the velocity of the object is negative*, then the relevant equation is*

$$m\frac{dv}{dt} = -mg + \frac{1}{2}\rho A C_d v^2. \tag{2}$$

where A is the cross-sectional area, ρ is the density of air, and C_d is a dimensionless constant called the coefficient of drag.

1. *In the case where there is air friction the velocity no longer grows without bound, but tends to a fixed velocity called "terminal velocity", which we write as v_∞. Find the terminal velocity v_∞ in terms of m, g, A, ρ, C_d. (***Hint:*** *If the object is moving at velocity v_∞, then its velocity will no longer change.)*

2. *Show that equation (2) can be written in the form*

$$\frac{dv}{dt} = -g\left(1 - \frac{v^2}{v_\infty^2}\right). \tag{3}$$

3. *Now make the guess that the formula for $v(t)$ is of the form*

$$v(t) = v_0 + v_1 t + v_2 t^2 + O(t^3). \tag{4}$$

Obtain a formula for dv/dt by differentiating Equation (4). Be careful: what is the order of the correction? Compute v^2 by multiplying series together. Plug all of these into Equation (3). What are the units of v_1, v_2?

4. *How does the sign of the acceleration v_1 depend on v_0, v_∞? Does this make sense physically?*

5. *The density of air (at sea level and at 25 °C) is about $\rho = 1.225\,\mathrm{kg/m^3}$. The typical area of a parachute is about $A = 40\,\mathrm{m}^2$ and the coefficient of drag is about $C_d = 1.5$. Assume that a parachutist with equipment masses about $m = 100\,\mathrm{kg}$. What is her terminal velocity? If the parachutist is travelling at $30\,\mathrm{m/s}$ when her chute opens what is her acceleration (expressed in multiples of g)? (Notice that for small t, the formula (4) is very close to $v(t) = v_0 + v_1 t$.)*