Group: ______________ ______________ ______________ Name: _______________

Math 231E. Worksheet 1B. 8/30/18
Taylor series and approximations

Taylor series, and the truncated Taylor polynomials, approximate a general function with a polynomial. In many cases, polynomials are easier to work with, so this simplification helps. Of course, it comes at a cost: when we approximate we introduce some error. However, these approximations are frequently good enough to be useful when numerically computing the values of a function. In this worksheet, we examine the error introduced by Taylor series approximations.

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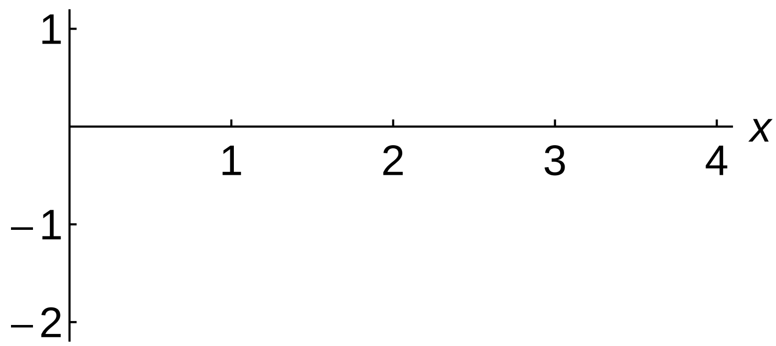
1. Use the series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + O(x^6)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + O(x^8)$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + O(x^9)$$

to verify the formula $e^{i\theta} = \cos\theta + i\sin\theta$.

2. Give the first four nonzero terms in the Taylor series for $f(x) = \ln x$ at the point $a = 1$. **Also** compute the order of the neglected terms in the expansion.

3. Write out $T_0(x), T_1(x), T_2(x)$. Sketch $\ln(x)$ and all three polynomials on the graph below.



4. a) Use the formula from Problem 2 to approximate $\ln(1.1)$. Using that $\ln(1.1) \approx 0.0953102$, compute the **relative** error in this approximation.

b) Use the formula from Problem 2 to approximate $\ln(1.9)$. Using that $\ln(1.9) \approx 0.641854$, compute the **relative** error in this approximation.

c) Which of the approximations in parts (a) and (b) is more accurate? Why do you think this happens?