Group: _____ _____ _____ Name: _____

Math 231E Fall 2013. Worksheet 1A. August 29, 2017

Constructive and Destructive Interference

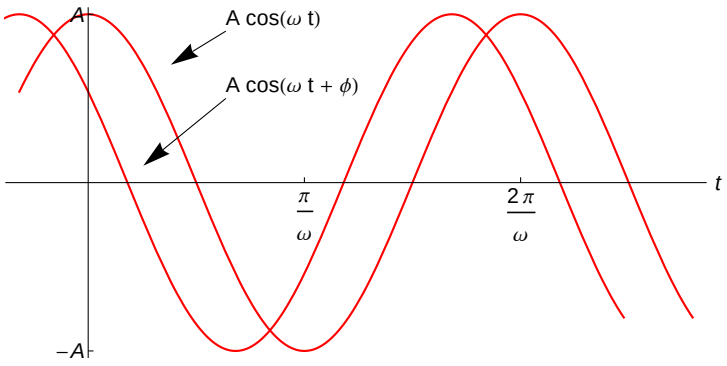
Have you ever wondered how "noise-canceling" headphones, or other types of "active cancellation" work? In principle, the electronics determine the signal received by the user, and then add another signal to cancel it. The engineering using this principle makes your flights quieter and your cell phone conversations clearer. Today we examine the mathematics behind the canceling of signals.

1. Recall the fundamental formula $e^{i\theta} = \cos\theta + i\sin\theta$.

(a) $\mathrm{Re}[e^{i\theta}] =$ _____? $\mathrm{Im}[e^{i\theta}] =$ _____? (b) Show that $e^{-i\theta} = \cos\theta - i\sin\theta$.

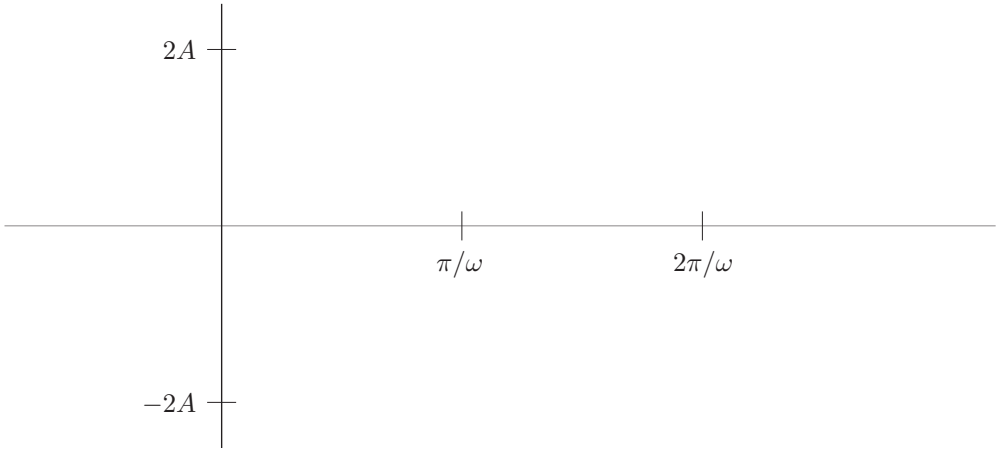
c) $e^{-i\theta} + e^{i\theta} =$ _____?

2. Consider the wave defined by $A\cos(\omega t)$ whose graph is shown in the figure below. The amplitude is A and the frequency is ω. Also shown is $A\cos(\omega t + \phi)$, which has the same amplitude and frequency, but phase shift ϕ.



The sum (or superposition) of these two waves is $A\cos(\omega t) + A\cos(\omega t + \phi)$.

a) On the axes below, sketch this sum when $\phi = 0$ (constructive interference), and also when $\phi = \pi$ (destructive interference).



b) Show that $\text{Re}\left[Ae^{i\omega t} + Ae^{i(\omega t+\phi)}\right] = A\cos(\omega t) + A\cos(\omega t + \phi)$.

c) Show that $Ae^{i(\omega t+\phi/2)}(e^{-i\phi/2} + e^{i\phi/2}) = Ae^{i\omega t} + Ae^{i(\omega t+\phi)}$.

d) Use part (c) to show that $Ae^{i\omega t} + Ae^{i(\omega t+\phi)} = 2A\cos(\phi/2)e^{i(\omega t+\phi/2)}$.

e) Use parts (b) and (d) to find a formula for the sum $A\cos(\omega t) + A\cos(\omega t + \phi)$ that is written in the form of a wave with an amplitude and a phase shift.

f) What is the amplitude of your answer in (e)? (Recall that amplitude is, by definition, positive!!)

g) Find the largest value achieved by the amplitude in (e), as the phase shift ϕ varies between 0 and π. State the value of ϕ at which this largest amplitude occurs. Then find the smallest amplitude, and state its corresponding value of ϕ.

Does your answer make sense in terms of the graphs you drew in part (a)? Explain why or why not.