University of Illinois Integration Bee Qualifying Exam

5th April 2019

This is the qualifying test for the 2019 Integration Bee. Finalists will be notified. You have 30 minutes to solve as many of the given 15 integrals as you can. The first ten integrals are worth 2 points, the next four are worth 5 points and the final problem is worth 10 points. In order to receive full credit, you must express your answer in terms of x for indefinite integrals or simplified expressions in terms of constants for definite integrals, and you must record your answers on the answer sheet. There is no partial credit. The "log" symbol denotes the natural logarithm. 'i' is the imaginary unit. In your answers, it is not necessary to include the arbitrary constant C nor the absolute value sign around the argument of a logarithm. There is a final bonus problem, which if solved, ensures direct qualification. However, you must show full work for this problem.

1.

$$\int_{\pi^e}^{e^\pi} 1^x dx$$

2. If

$$\int_{\phi e}^{10\pi} \frac{\sin(x)}{x} dx = A, \frac{dA}{dx} = ?$$

3.

$$\int \frac{61}{60\sin(x) + 11\cos(x)} dx$$

4.

$$\int \frac{1}{x^{1729} + x} dx$$

5.

$$\int \sin(\sin(\sin(...(x))))dx$$

6.

$$\int \frac{2000x^{2018} - 18}{x^{2019} + x} dx$$

7.

$$\int \frac{x^{\frac{-7}{12}}}{\sqrt[3]{x} + \sqrt[4]{x}} dx$$

8.

$$\int \frac{\sec(x - A)}{\sqrt{1 - sin^2(x - B)}} dx$$

9.

$$\int cosec^3(x)\sec(x)dx$$

10.

$$\int \frac{x \log x sech^2(x) - \tanh(x)}{x[\log x]^2} dx$$

11.Prove

$$\int e^{nix}dx = \frac{1}{n}[\sin(nx) - i\cos(nx)],\ \forall\, n \in N.$$

12. Given

$$\sum_{j=1}^{n} x^{j-1} = f(x);$$

$$f(x) = \frac{1-x^n}{1-x};$$

$$\sum_{k=1}^{n}[kx^{k-2} - \frac{h'(x)}{(k+1)x^2}] = g(x);$$

$$h(x) = x^{k+1}$$

Evaluate

$$1 + \int [g(x)dx].$$

13.

$$\int_3^7 \frac{\log(x+2)}{\log(24+10x-x^2)}dx$$

14.

$$\int_0^1 \frac{(x-x^2)^4}{1+x^2}dx$$

15. Show that $\int_{-\infty}^{\infty} e^{b+at-t^2}dt$ is always equal to some real, positive multiple of $\sqrt{\pi}$. ($a, b \in \mathbf{R}$) [Given erf(∞) = 1, where 'erf' denotes the Gauss error function]

BONUS

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)^2}dx$$