Distributed Frequency Control Over Time-Varying Communication Networks

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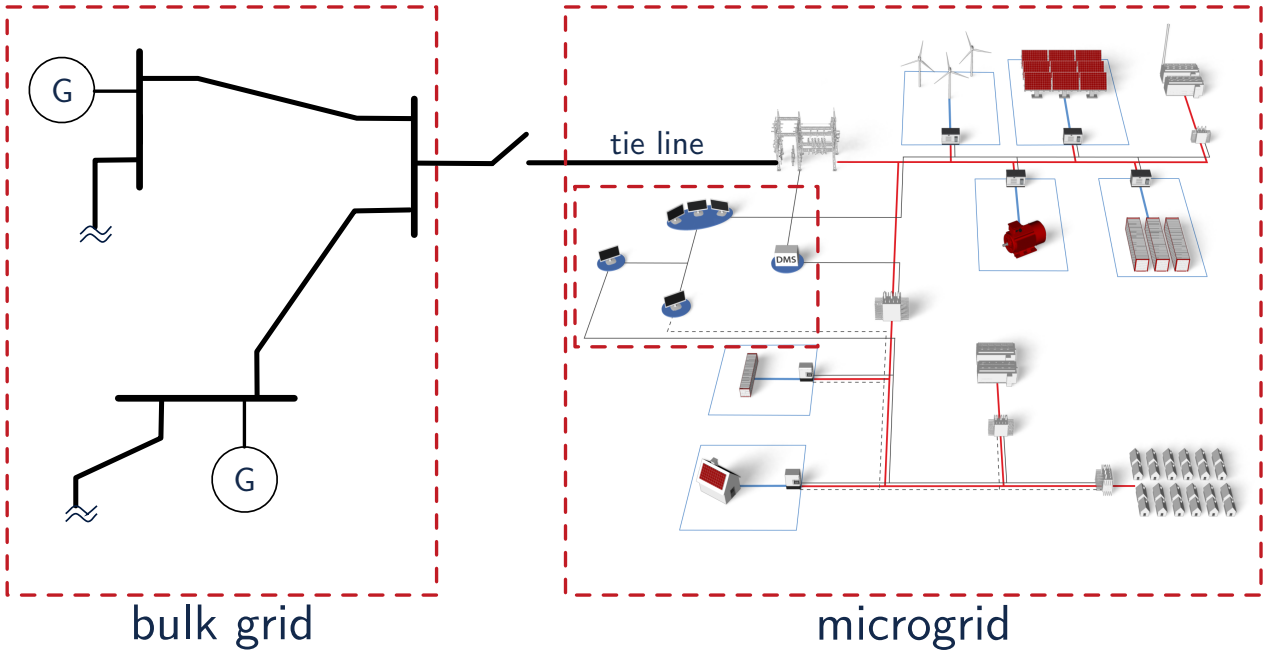
March 12, 2018

Outline

1. Introduction
2. Preliminaries
3. Frequency Controller
4. Simulation Results
5. Concluding Remarks



- Islanded mode: voltages and frequency of the microgrid are no longer supported by the bulk grid
- Inverter-interfaced distributed energy resources (DERs): low rotational inertia

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Microgrid Model

$$\text{ODE model: } \begin{cases} \dot{\theta}^{(I)}(t) = u(t) \\ \dot{P}^{(I)}(t) = S(\theta^*)u(t) \end{cases}$$

- $|\theta_i^* - \theta_j^*| < \tan^{-1}(|\frac{B_{ij}}{G_{ij}}|) \rightarrow S(\theta^*)$ is row diagonally dominant

$\theta^{(I)}(t)$	vector of phase angles at inverter buses
$P^{(I)}(t)$	net active power injections at the inverter buses

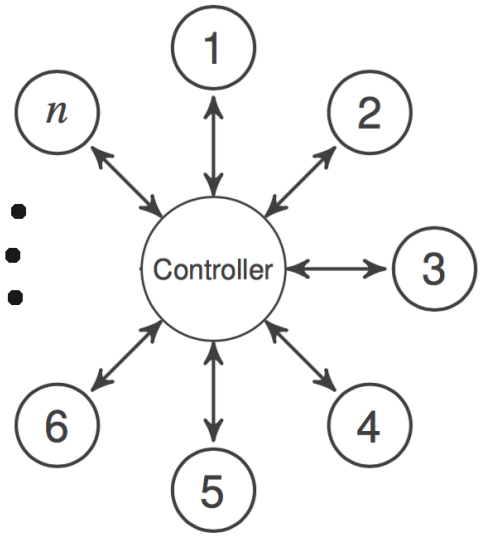
Communications Network

Centralized:

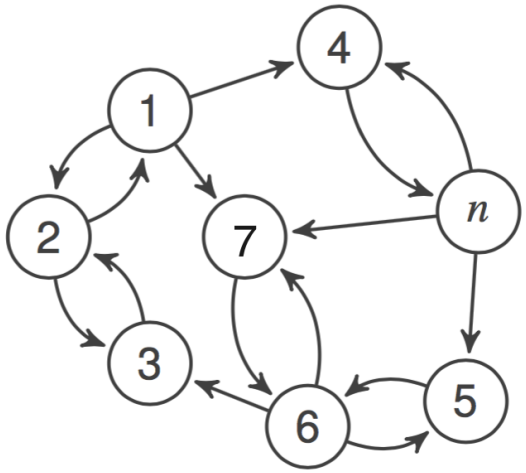
- Failure of a central processor might cause the breakdown of the whole microgrid (single point of failure)
- May require communication over prohibitively large distances

Distributed:

- Resilient to a single point of failure
- Neighbor-to-neighbor communication



Centralized



Distributed

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Multiagent Optimization

$$\mathbf{P}_0 : \min_x \; f(x) := \sum_{i=1}^{n} f_i(x),$$
$$\text{subject to } x \in \mathbb{R}^n,$$

Agent-related constraints:

- $f_i(x)$ is a local cost of agent i unknown to other agents
- two agents can communicate if they are physically close to each other

Common approach:

- Let each agent estimate x^*, solution of $\mathbf{P}_0$, using gradient descent
- Neighbors average their estimates via communications

Includes two steps:

- Averaging step ensures that estimates converge to a common point (consensus)
- Gradient descent step steers the consensus point towards the optimal set

Multiagent Optimization Techniques

- Undirected communication graph:
  - *Distributed subgradient method*:
    Agent i estimate of x^*

$$x^{(i)}[k+1] = \underbrace{\sum_{j=1}^{n} a_j^{(i)}[k] x^{(j)}[k]}_{\text{distributed averaging}} - \underbrace{\alpha[k] g_i[k]}_{\text{gradient descent}}$$

- Directed communication graph:
  - *Subgradient-push method*:

$$w^i[k+1] = \sum_{j \in \mathcal{N}_i^-[k]} \frac{z^{(j)}[k]}{d_j[k]+1}$$

$$y^i[k+1] = \sum_{j \in \mathcal{N}_i^-[k]} \frac{y^j[k]}{d_j[k]+1}$$

$$x^i[k+1] = \frac{w^i[k+1]}{y^i[k+1]}$$

$$z^{(i)}[k+1] = w^i[k+1] - \alpha[k+1] g_i[k+1]$$

Control Objectives as Multiagent Optimization Problem

O1. achieve desired active power load sharing among generators

- $P_i^{(I)} \to \frac{D_i}{\sum_j D_j} \sum_j P_j^{(I)}$
  D_i's are computed by the tertiary control layer

O2. maintain frequency at each bus at the nominal frequency

Closed-loop microgrid dynamics need to solve

$$\mathbf{P}: \min_{P^{(I)}} \sum_{i=1}^{m} \frac{\beta}{2D_i} \left(P_i^{(I)} - \chi_i \sum_{j=1}^{m} P_j^{(I)}[0] \right)^2$$

$$\text{subject to } P^{(I)} \in \mathbb{R}^m$$

where $\chi_i := \frac{D_i}{\sum_{j=1}^{m} D_j}$.

Distributed-Averaging-PI (DAPI) Controller

$$\dot{\theta}^{(I)} = -W(P^{(I)} - P^r)$$

$$W\dot{P}^r = -\underbrace{\beta(P^r - P^{(I)})}_{\text{frequency error}} - \underbrace{L^{(I)}WP^r}_{\text{load sharing}}$$

Closed-loop microgrid dynamics ($\xi[k] := WP^r[k]$):

$$P^{(I)}[k+1] = P^{(I)}[k] - \underbrace{\alpha[k]S(WP^{(I)}[k] - \xi[k])}_{\text{gradient descent}}$$

$$\xi[k+1] = \underbrace{(I - \alpha[k]L^{(I)})\xi[k]}_{\text{distributed averaging}} - \underbrace{\alpha[k]\beta(W^{-1}\xi[k] - P^{(I)}[k])}_{\text{gradient descent}}$$

Communication Network:

- Bidirectional communication links
- Simultaneous information exchange

Practical communication network is time-varying:

- Packet drops
- Communication delays

Frequency Control over Undirected Communication Graphs

Way to resolve communication delays:

- neighbors can average their estimates whenever they exchange information
  - $L_{ij}^{(I)}[k] = L_{ji}^{(I)}[k] > 0$ if generators i and j simultaneously exchange information during time interval (t_k, t_{k+1})
  - $L_{ii}^{(I)}[k] = -\sum_{j \neq i} L_{ij}^{(I)}[k]$ for ensuring $(I - \alpha[k]L^{(I)}[k])$ is doubly stochastic.

$$P^{(I)}[k+1] = P^{(I)}[k] - \alpha[k]S(WP^{(I)}[k] - \xi[k])$$
$$\xi[k+1] = (I - \alpha[k]L^{(I)}[k])\xi[k] - \alpha[k]\beta(W^{-1}\xi[k] - P^{(I)}[k])$$

Limitation:

- simultaneous information exchange

Frequency Control over Directed Communication Graphs

Apply subgradient-push method to design frequency controller:

$$w_i[k+1] = \sum_{j \in \mathcal{N}_i^-[k]} \frac{x_j[k]}{d_j[k]+1}$$

$$y_i[k+1] = \sum_{j \in \mathcal{N}_i^-[k]} \frac{y_j[k]}{d_j[k]+1}$$

$$\xi_i[k] = \frac{w_i[k+1]}{y_i[k+1]}$$

$$x_i[k+1] = w_i[k+1] - \alpha[k]\beta(D_i\xi_i[k] - P_i^{(I)}[k]),\ i \in \mathcal{V}^{(I)}$$

$$\omega[k] = -W(P^{(I)}[k] - D\xi[k])$$

Assumption:

- Each node j knows its out-degree $d_j[k]$ (number of agents which received information from node j at time k)

Multiagent Optimization Interpretations For Frequency Control

- $\tilde{p}$ and ζ are the scaled versions of $P^{(I)}$ and $\frac{\sum_{j=1}^{m} P_j^{(I)}[0]}{\sum_{j=1}^{m} D_j}$

$$\mathbf{P}_1 : \min_{\tilde{p},\zeta} \sum_{i=1}^{m} \frac{\beta}{2D_i} \left(P_i^{(I)}[0] + c(S^{\frac{1}{2}})_i \tilde{p} - D_i \zeta \right)^2$$

$$\text{subject to } \tilde{p} \in \mathbb{R}^m,\ \zeta \in \mathbb{R}$$

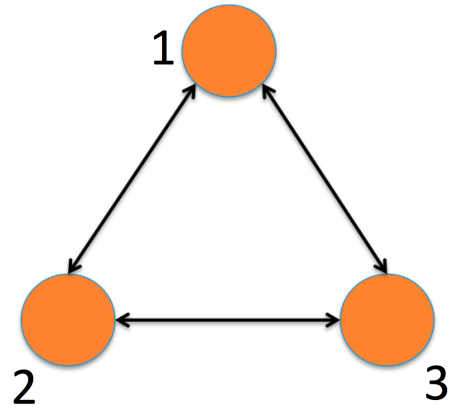
$(\tilde{p}^*, \zeta^*)$ is a solution of $\mathbf{P}_1$

Observations:

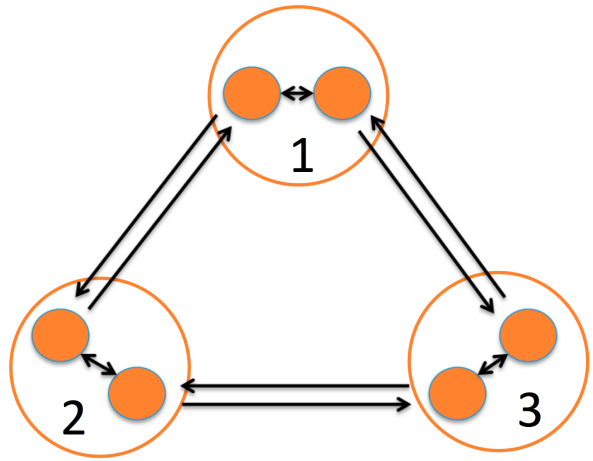
- $\xi_i[k]$ is agent i estimate of ζ^*
- $\tilde{p}$ - average process induced in microgrid (average estimate of $\tilde{p}^*$)

Frequency Control over Undirected Communication Graphs

- Distributed Subgradient Method with Simultaneous Averaging



- Distributed Scaled Subgradient Method with Alternating Averaging
  - allows agents to exchange information in asynchronous fashion



  - Node i maintains $|\mathcal{N}_i|$ estimates of ζ^*, each estimate is attributed to a communication link
  - Additional gradient terms are included into gradient descent step to drive all local estimates of node i to a common point

Frequency Control over Undirected Communication Graphs

$$\mathbf{P}_2 : \min_{\tilde{p},z} \; f(\tilde{p},z) := \sum_{i=1}^{m} f_i(\tilde{p},z),$$

$$\text{subject to } \tilde{p} \in \mathbb{R}^m, \; z \in \mathbb{R}^{|\mathcal{E}_\infty^{(I)}|},$$

$z := [\{z_{ij}\}_{\{i,j\}\in\mathcal{E}_\infty^{(I)}}]^\top$, z_{ij} - analog of ζ corresponding to link $\{i,j\}$

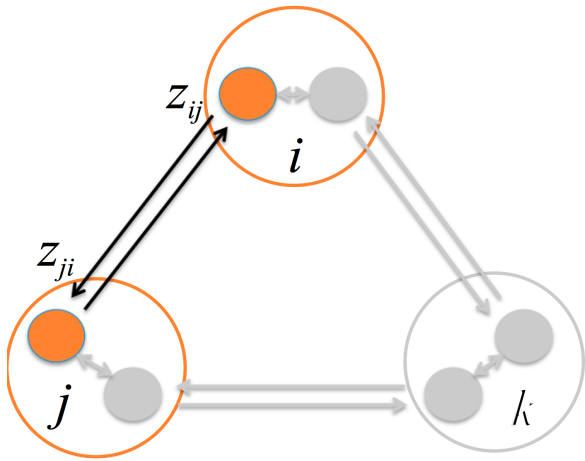
$$f_i(\tilde{p},z) := \underbrace{\frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i}\frac{\beta}{2D_i}\left(P_i^{(I)}[0] + c(S^{\frac{1}{2}})_i\tilde{p} - D_i z_{ij}\right)^2}_{\text{frequency regulation objectives}} \leftarrow \text{what we had before}$$

$$+ \underbrace{\frac{1}{2}\sum_{j\in\mathcal{N}_i}\sum_{l\in\mathcal{N}_i} b_{jl}(z_{ij}-z_{il})^2}_{\text{local averaging objective}} \leftarrow \text{what we added}$$

Frequency Control over Undirected Communication Graphs

$$g_{ij}^{(i)}[k] := \frac{\beta}{|\mathcal{N}_i|}(D_i z_{ij}[k] - P_i^{(I)}[k]) + \sum_{l\in\mathcal{N}_i} b_{jl}[k](z_{ij}[k] - z_{il}[k])$$

$$\begin{aligned}
z_{ji}[k_{2t}+1] &= \big((1-a_{ji})z_{ji}[k_{2t}] + a_{ji}z_{ij}[k_{2t}]\big) \\
&\quad - \alpha[k]K_{22}g_{ji}^{(j)}[k] \\
z_{ji}[k+1] &= z_{ji}[k] - \alpha[k]K_{22}g_{ji}^{(j)}[k] \\
z_{ij}[k_{2t+1}] &= \Big[(1-a_{ij})z_{ij}[k_{2t}] \\
&\quad + a_{ij}\big((1-a_{ji})z_{ji}[k_{2t}] + a_{ji}z_{ij}[k_{2t}]\big)\Big] \\
&\quad - \alpha[k]K_{11}\sum_{s=0}^{\Delta k_{2t}-1} g_{ij}^{(i)}[k_{2t}+s] \\
z_{ij}[k+1] &= z_{ij}[k] - \alpha[k]K_{11}g_{ij}^{(i)}[k]
\end{aligned}$$



The closed-loop system: $P^{(I)}[k+1] = P^{(I)}[k] - \alpha[k]S(WP^{(I)}[k] - \xi[k])$

$$\xi_i[k] = \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i} z_{ij}[k],\ i \in \mathcal{V}^{(I)}$$

Message Passing Protocol for Alternating Averaging

Consider a link (i, j)

- Node i maintains acknowledgement variable ν_{ij}
- When node i receives a new and different value of ν_{ji} from j (this means j has performed averaging):

  1. node i performs averaging
  2. ν_{ij} is set to a different value (let node j know that node i has performed averaging)
  3. node i sends z_{ij} and ν_{ij} to node j

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- frequency controllers for undirected and directed time-varying communication graphs

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- frequency controllers for undirected and directed time-varying communication graphs
- for undirected graphs, distributed scaled subgradient method with alternating averaging ensures convergence when:
  - information exchange between neighbor agents is not necessarily simultaneous
  - there are packet drops

Concluding Remarks

Questions?