Cash-settled options for wholesale electricity markets

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Premise of this work

- Energy is typically procured in advance to meet the demand requirements
- Renewable resources are uncertain, intermittent, and non-dispatchable
- With renewable resources, prediction errors can increase from 1-3% to 12% [Bird et al., 2013]
- Deepening penetration of variable renewable supply results in high volatility of payments to market participants
- How to mitigate increased financial risks of market participants?

Our answer: *Centralized cash-settled options market*

A benchmark forward market design

Dispatch model + **Payment scheme**

Certainty equivalent dispatch

Locational marginal pricing

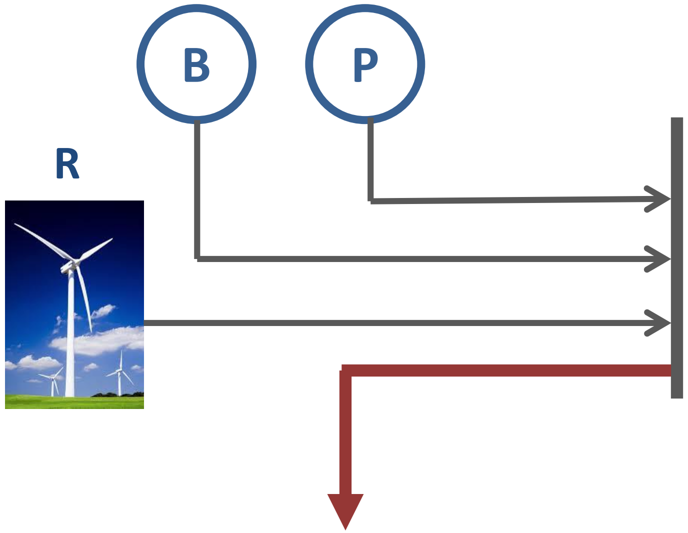
- Forecast uncertain variables
- Dispatch forward against that forecast
- Re-dispatch to balance attending deviation in real-time

Outline

- Illustrate financial risks with the benchmark example
- Show how *bilateral* call options can help
- Propose a *centralized* option market mechanism
- Generalizations, challenges, … etc
- Model risk-aversion of market participants

Benchmark example

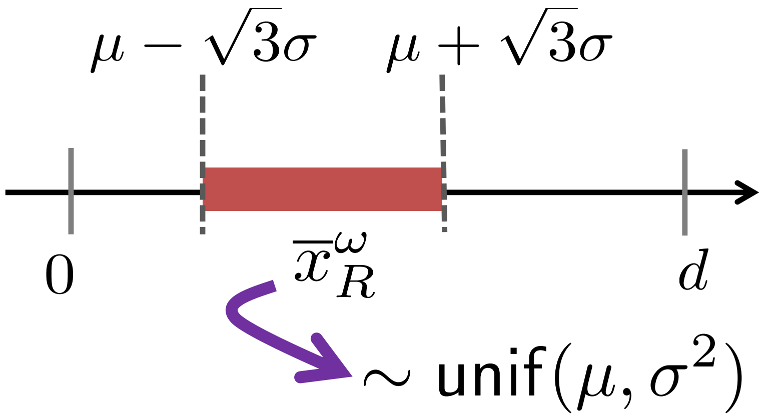
$\Omega =$ Scenarios of available renewable supply



$d > 0$

Uncontrollable but predictable demand

Renewable power producer

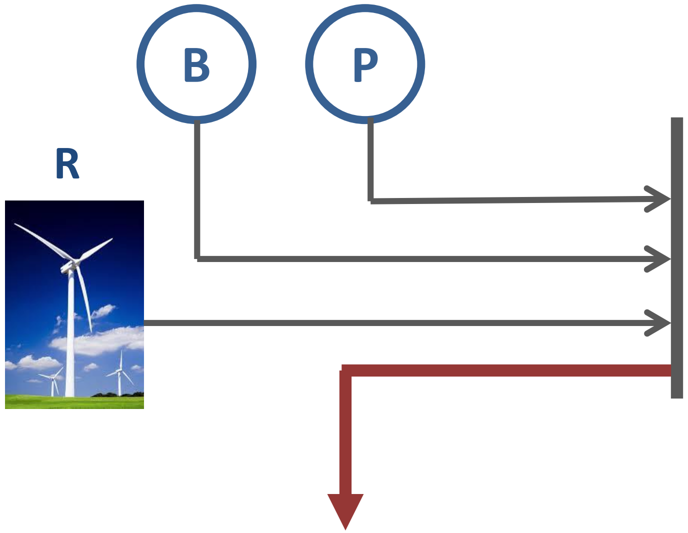


$$0 \leq x_R^\omega \leq \overline{x}_R^\omega$$

cost $= 0.$

Benchmark example

$\Omega =$ Scenarios of available renewable supply



$d > 0$

Uncontrollable but predictable demand

Baseload generator

$$0 \leq x_B^\omega \leq \infty$$

Forward set point

$$|x_B^\omega - X_B| = 0$$

cost $= x_B^\omega$

$\rho > 1$

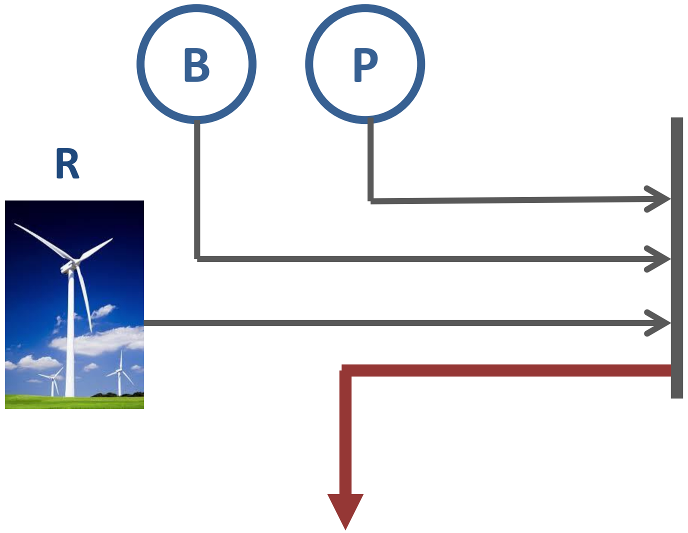
Peaker power plant

$$0 \leq x_P^\omega \leq \infty$$

$$|x_P^\omega - X_P| \leq \infty$$

cost $= \rho x_P^\omega$

Benchmark example



$$\underbrace{d > 0}$$

Uncontrollable but predictable demand

A certainty-equivalent based forward dispatch

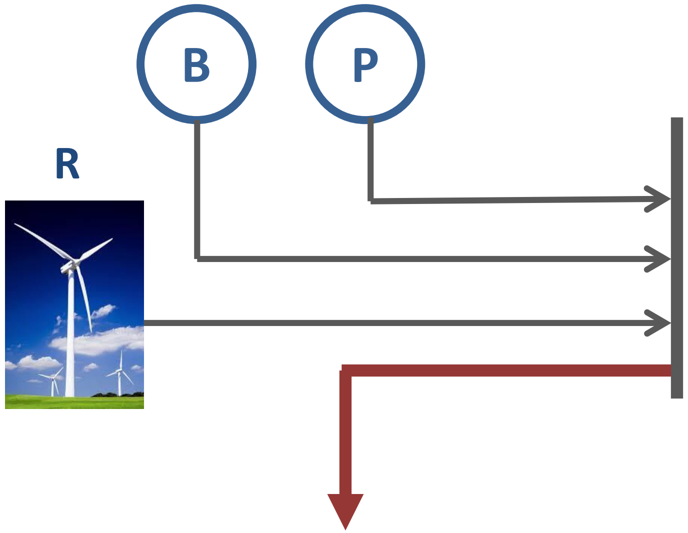
$$\begin{aligned}
\underset{X_B, X_P, X_R}{\text{minimize}} \quad & 1 \cdot X_B + \rho \cdot X_P + 0 \cdot X_R, \\
\text{subject to} \quad & X_B + X_P + X_R = d, \\
& X_B \geq 0, X_P \geq 0, \\
& 0 \leq X_R \leq \mu.
\end{aligned}$$

Forward price = optimal Lagrange multiplier P^*

Summary of forward market clearing:

$$\begin{aligned}
X_B^* &= d - \mu, \\
X_P^* &= 0, \\
X_R^* &= \mu, \\
P^* &= 1.
\end{aligned}$$

Benchmark example



$$\underbrace{d > 0}_{\text{Uncontrollable but predictable demand}}$$

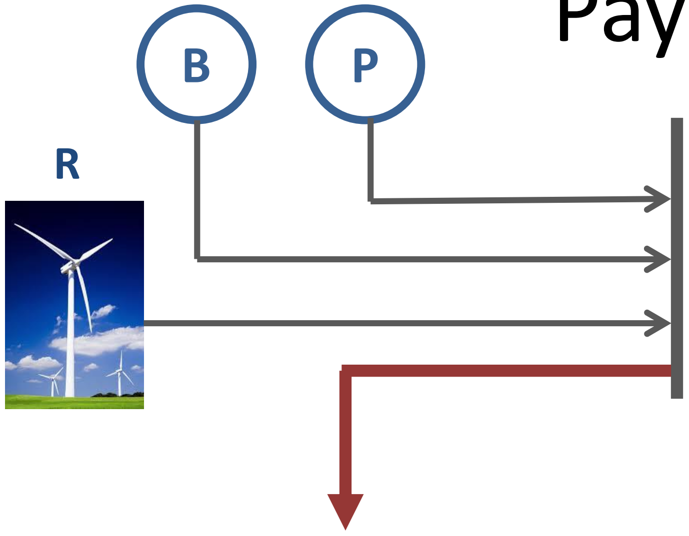
Real-time balancing,
given forward dispatch

$$
\begin{array}{ll}
\underset{x_B^\omega, x_P^\omega, x_R^\omega}{\text{minimize}} & 1 \cdot x_B^\omega + \rho \cdot x_P^\omega + 0 \cdot x_R^\omega, \\
\text{subject to} & x_B^\omega + x_P^\omega + x_R^\omega = d, \\
 & |x_B^\omega - X_B| = 0, \\
 & x_P^\omega \geq 0, \\
 & 0 \leq x_R^\omega \leq \overline{x}_R^\omega.
\end{array}
$$

Summary of real-time market clearing:

$$
\begin{aligned}
x_B^{\star\omega} &= d - \theta, \\
x_P^{\star\omega} &= (\theta - \omega)^+, \\
x_R^{\star\omega} &= \min\{\omega, \theta\}, \\
p^{\star\omega} &= \rho \cdot \mathbb{1}\{\omega < \theta\}
\end{aligned}
$$

Payments can be highly volatile!



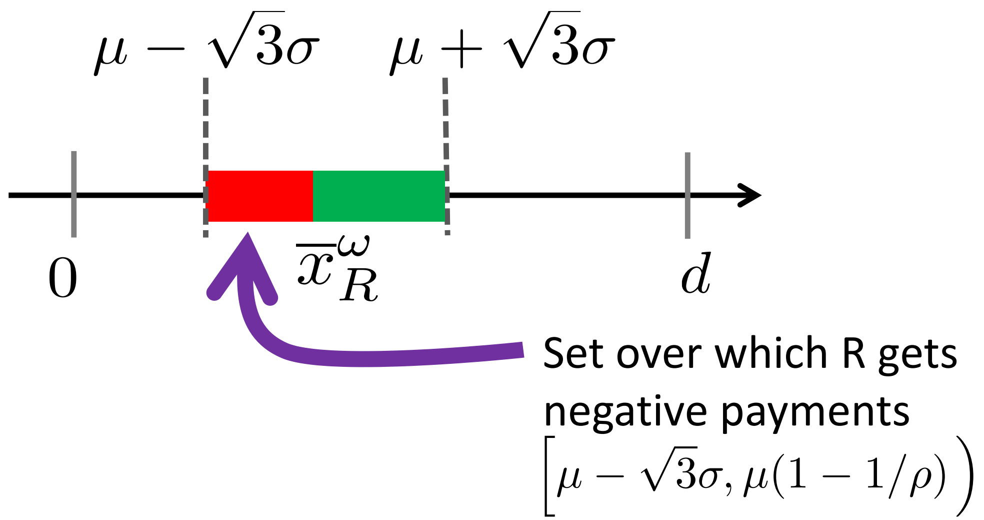
$d > 0$

Payments to market participants:

$$\pi_B^\omega = d - \mu,$$
$$\pi_P^\omega = \rho(\mu - \omega)^+,$$
$$\pi_R^\omega = \mu - \rho(\mu - \omega)^+.$$

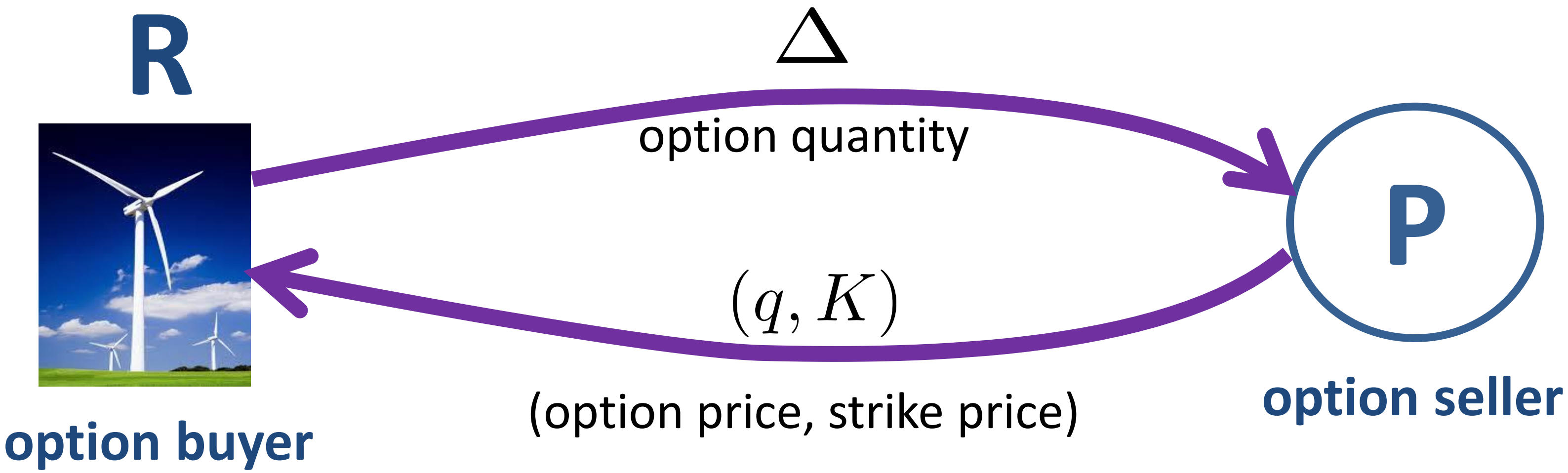
$\text{var}[\pi^\omega]$: a measure of volatility

Q1. Can we reduce this volatility?



Q2. Can we shrink this set?

A remedy: *bilateral* cash-settled call option

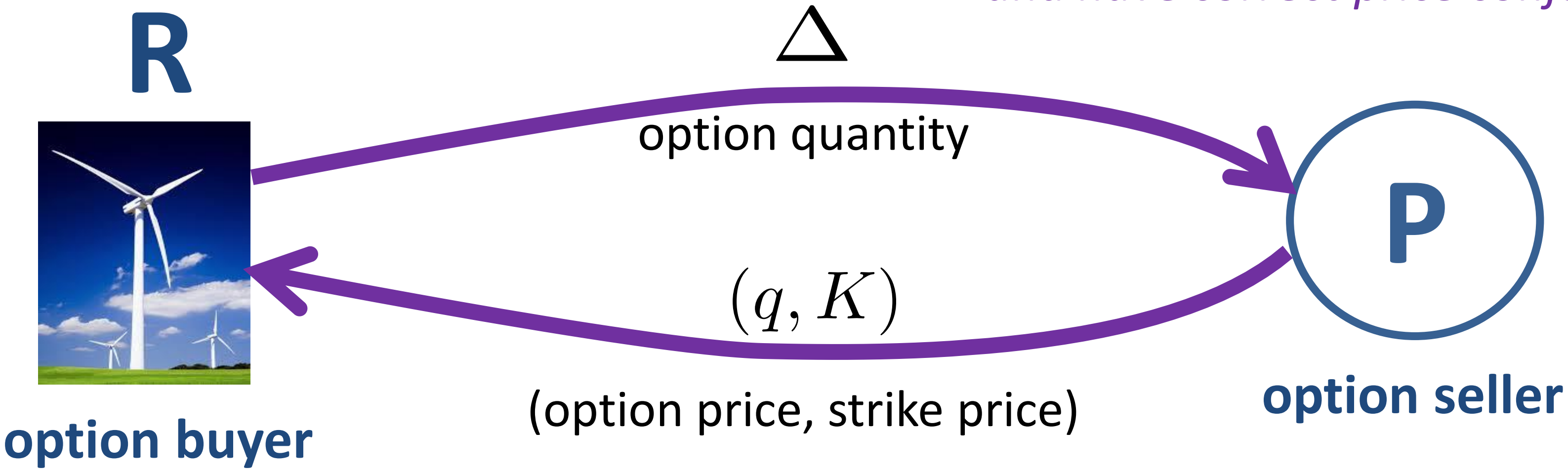


What is a cash-settled call option?

A derivative that allows the holder the right to claim a monetary reward equal to the positive difference between the real-time price and the strike price for an upfront fee.

A remedy: *bilateral* cash-settled call option

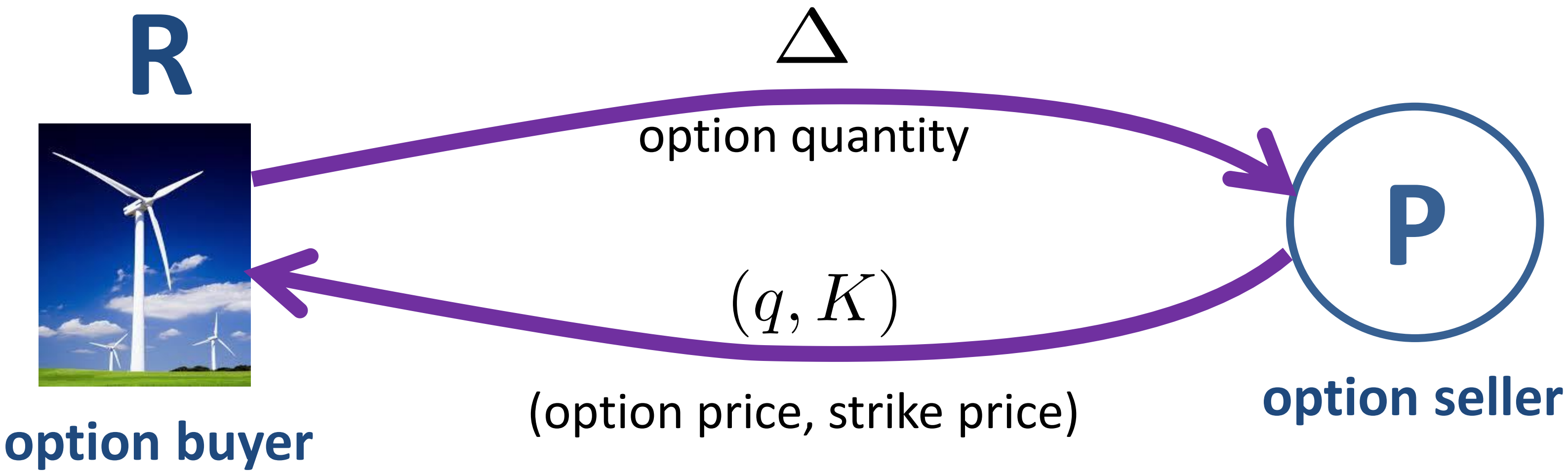
Assume that players are risk-neutral, and have correct price conjectures.



$$\Pi_R^\omega(q, K, \Delta) := \pi_R^\omega - q\Delta + (p^{\omega,*} - K)^+ \Delta,$$

$$\Pi_P^\omega(q, K, \Delta) := \underbrace{\pi_P^\omega}_{\text{Payment from electricity market}} + \underbrace{q\Delta - (p^{\omega,*} - K)^+ \Delta}_{\text{Payment from option trade}}.$$

Robust Stackelberg game



A Stackelberg equilibrium $(q^*, K^*, \Delta^*(q^*, K^*))$ satisfies

$$\mathbb{E}\left[\Pi_P^{\omega}(q^*, K^*, \Delta^*(q^*, K^*))\right] \geq \mathbb{E}\left[\Pi_P^{\omega}(q, K, \Delta^*(q, K))\right],$$

$\Delta^* : \mathbb{R}_+^2 \to [0, \sqrt{3}\sigma]$ is the best response of R, which satisfies

$$\mathbb{E}\left[\Pi_R^{\omega}(q, K, \Delta^*(q, K))\right] \geq \mathbb{E}\left[\Pi_R^{\omega}(q, K, \Delta(q, K))\right],$$

for a given (q, K).

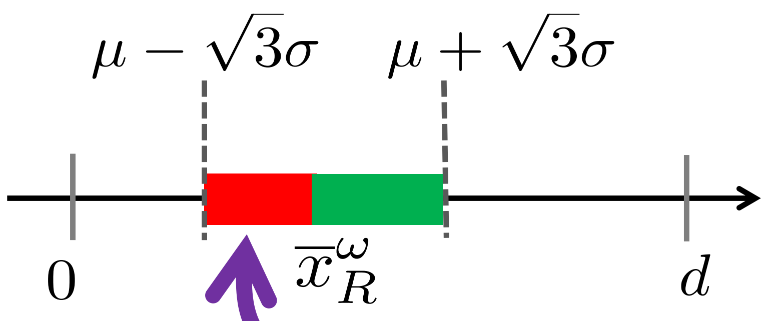
Proposition. The nontrivial Stackelberg equilibria are given by

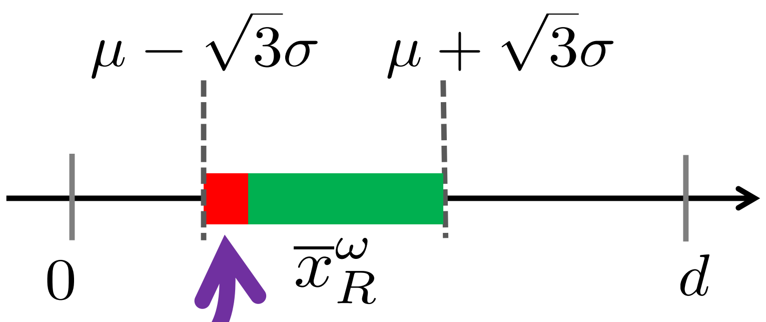
$$\mathcal{N} = \left\{ (q, K, \Delta) \mid (q, K) \in \mathbb{R}_+^2,\ \Delta : \mathbb{R}_+^2 \to [0, \sqrt{3}\sigma],\ 2q + K = \rho \quad \right\}.$$

For $i \in \{R, P\}$ and any $(q^*, K^*, \Delta^*(q^*, K^*) = \sqrt{3}\sigma) \in \mathcal{N}$, we have

$$\text{var}\left[\Pi_i^\omega(q^*, K^*, \Delta^*(q^*, K^*))\right] - \text{var}\left[\pi_i^\omega\right] = -3K^*\sigma^2/2 < 0.$$

measure of "uncertainty"





Set over which R gets negative payments

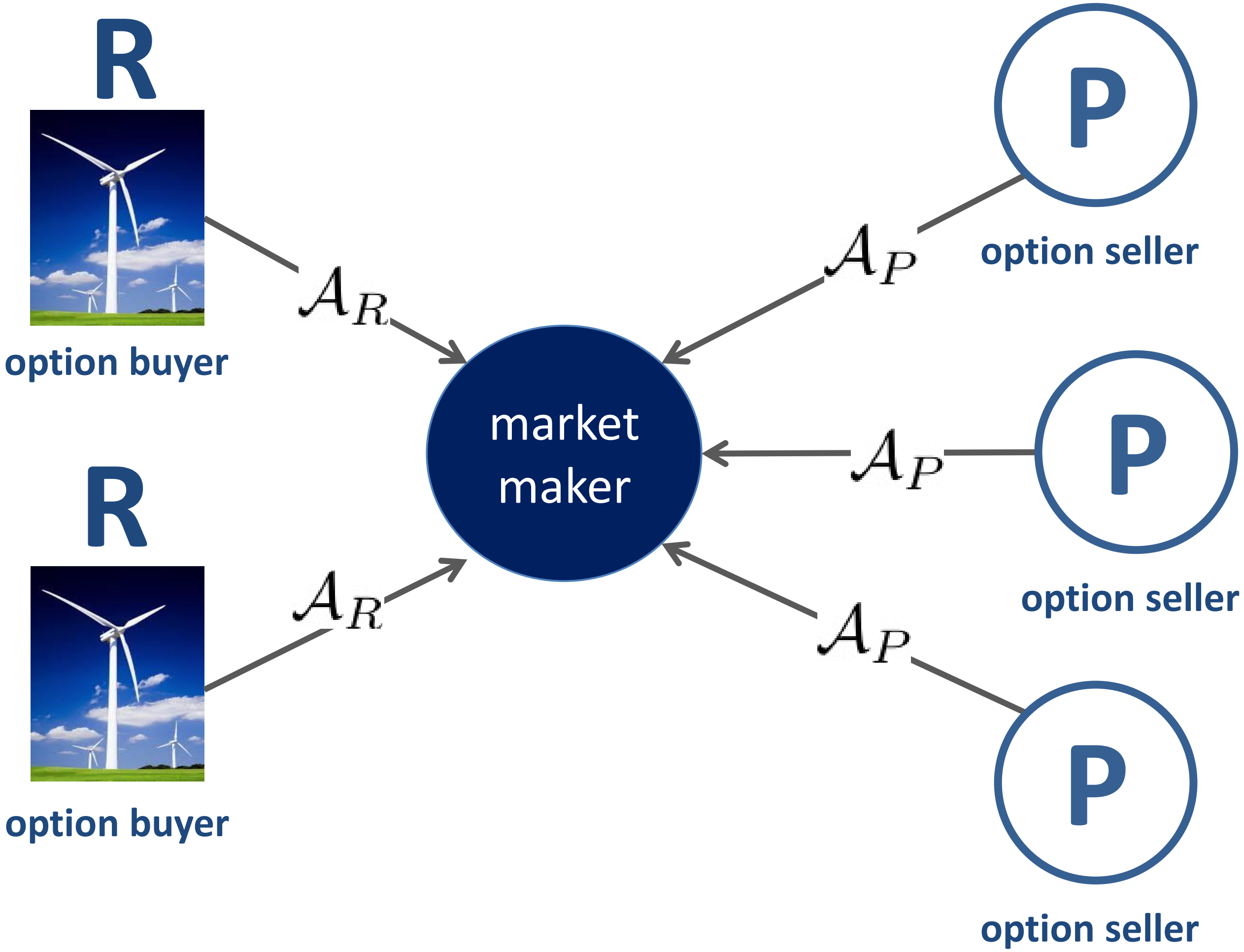
$\left[\mu - \sqrt{3}\sigma, \mu(1 - 1/\rho)\right)$ shrinks to $\left[\mu - \sqrt{3}\sigma, \mu(1 - 1/\rho) - \sqrt{3}q^*\sigma/\rho\right)$

A partial remedy: *bilateral* cash-settled call option

Needs multiple such trades with a collection of dispatchable and renewable power producers

Key idea: introduce an intermediary, whom we call the market maker

Market maker buys from option sellers and sells them to option buyers



$\mathcal{A}$ defines the set of acceptable trades (q, K, Δ) for each participant.

Market maker buys from option sellers and sells them to option buyers

…a centralized option trading mechanism

…generalizes the bilateral trade case

Option market clearing via stochastic optimization:

$$
\begin{aligned}
\text{maximize} \quad & \mathbb{E}[\mathsf{MS}^{\omega}], \\
\text{subject to} \quad & \sum_{P\in\mathfrak{P}} \Delta_P = \sum_{R\in\mathfrak{R}} \Delta_R, \\
& (q_P, K_P, \Delta_P) \in \mathcal{A}_P, \ (q_R, K_R, \Delta_R) \in \mathcal{A}_R, \\
& \delta_P^{\omega} \in [0, \Delta_P], \\
& \sum_{P\in\mathfrak{P}} \delta_P^{\omega} = \sum_{R\in\mathfrak{R}} \Delta_R \mathbb{1}\{p^{\omega,*} \geq K_R\} \\
& \text{for each} \ \ P \in \mathfrak{P}, \ R \in \mathfrak{R}, \ \omega \in \Omega
\end{aligned}
$$

Market maker may have a different objective.

Can encode risk aversion in the set of acceptable trades.

$$
\mathsf{MS}^{\omega} := \sum_{R\in\mathfrak{R}} \left[q_R \Delta_R - (p^{\omega,*} - K_R)^{+} \Delta_R \right] - \sum_{P\in\mathfrak{P}} \left[q_P \Delta_P - (p^{\omega,*} - K_P)^{+} \delta_P^{\omega} \right].
$$

Bilateral trade in benchmark example = special case of the centralized mechanism

Proposition. The triple $(q_i^*, K_i^*, \Delta_i^*) \in \mathbb{R}_+^3$ given by

$$2q_i^* + K_i^* = rho \quad \text{and} \quad \Delta_i^* \in (0, \sqrt{3}\sigma]$$

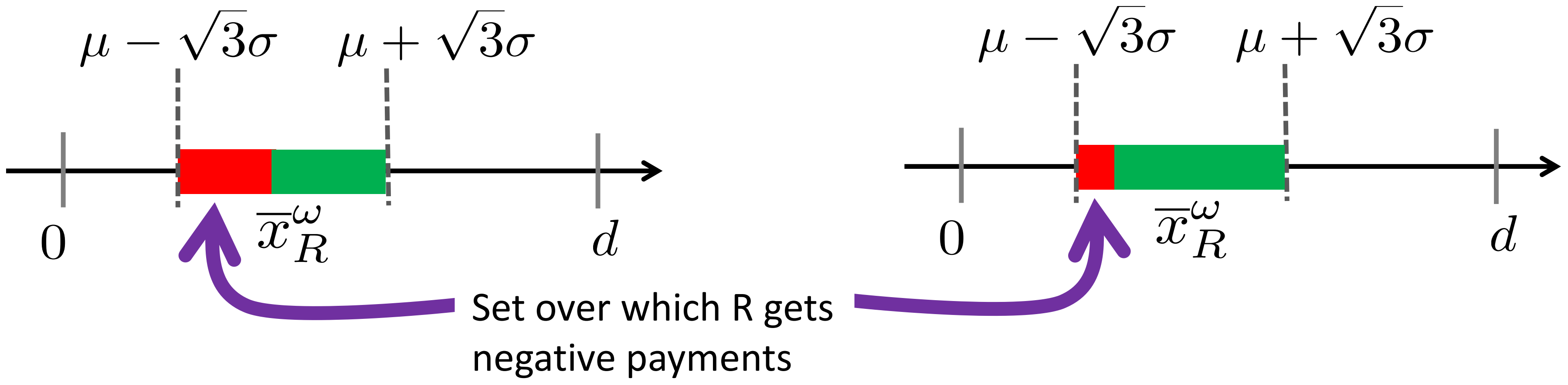
for each $i \in \{R, P\}$, constitutes the optimal solution of the centralized market. Moreover, at $\Delta_i^* = \sqrt{3}\sigma$, the variances of the payments satisfy

$$\mathsf{var}\left[\Pi_i^\omega(q^*, K^*, \Delta^*(q^*, K^*))\right] - \mathsf{var}\left[\pi_i^\omega\right] = -3K_i^*\sigma^2/2 < 0$$

The merchandising surplus of M is given by

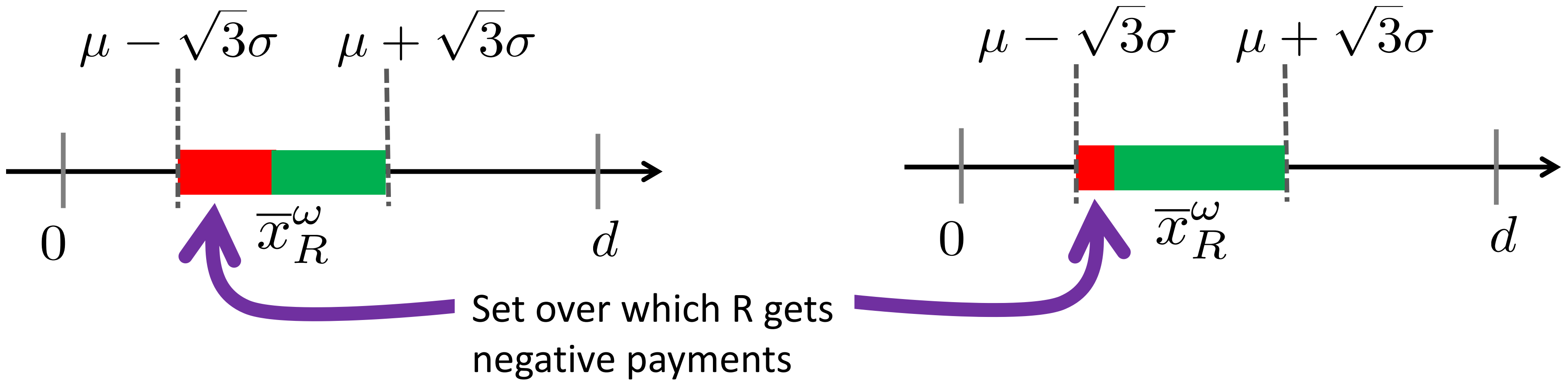
$$\mathsf{MS}^{\omega,*} = \begin{cases} (q_P^* - q_R^*)\sqrt{3}\sigma, & \text{if } \ \omega \leq \mu, \\ (q_R^* - q_P^*)\sqrt{3}\sigma, & \text{otherwise.} \end{cases}$$

Q. What changes if the market maker is a social benefactor?



With a social market maker…

- market clearing problem = system of nonlinear equations
- lends itself to solution via Newton-Raphson's method.



With a profit maximizer market maker…

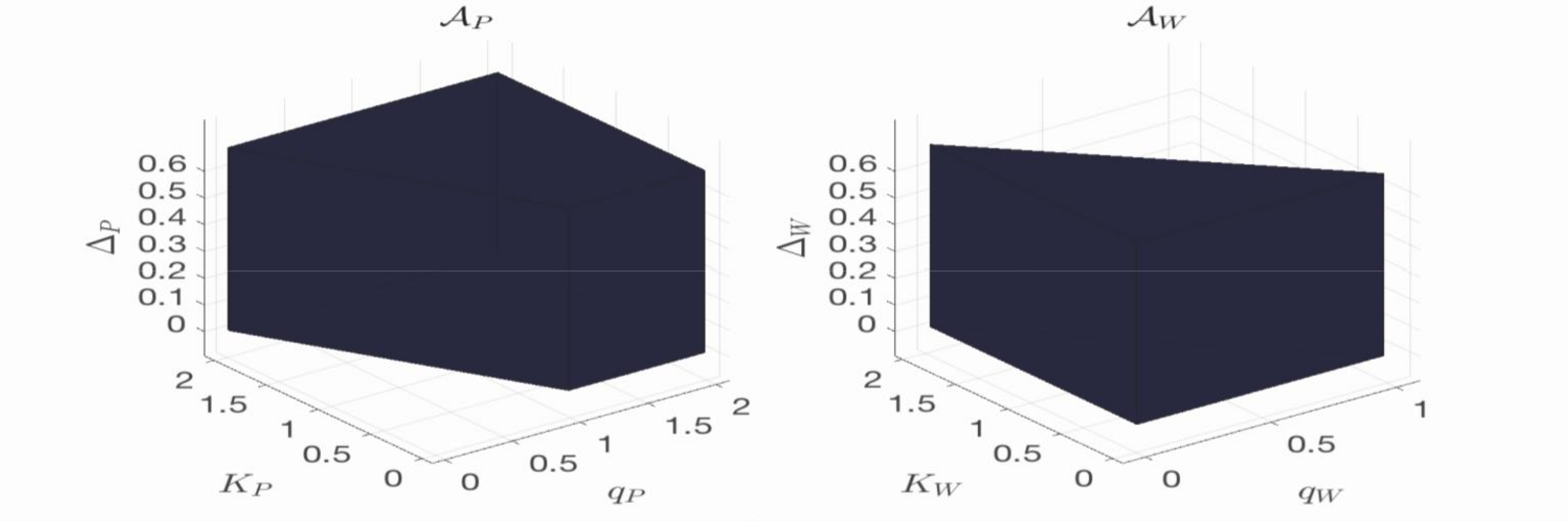
- market clearing problem is non-convex and the objective is non-differentiable

Acceptable trades for risk-neutral players

A risk-neutral seller P will accept the trade defined by (q_P, K_P, Δ_P), if

$$\mathbb{E}[\Pi_P^\omega(q_P, K_P, \Delta_P)] \geq \mathbb{E}[\pi_P^\omega],$$

Note: $\Pi_P^\omega := \pi_P^\omega + q_P\Delta_P - (p^{\omega,*} - K_P)^+\Delta_P$



Encoding risk-aversion in acceptable trades

A risk-averse market participant $i \in \mathfrak{P} \cup \mathfrak{R}$ finds a trade triple (q_i, K_i, Δ_i) acceptable, if

$$\mathsf{CVaR}_{\alpha_i}[-\Pi_i^{\omega}(q_i, K_i, \Delta_i)] \leq \mathsf{CVaR}_{\alpha_i}[-\pi_i^{\omega}]$$

where

$$\mathsf{CVaR}_{\alpha}[z^{\omega}] := \min_{t \in \mathbb{R}} \{t + \frac{1}{1-\alpha}\mathbb{E}[(z^{\omega} - t)^{+}]\} \quad \text{and} \quad \alpha \in [0,1]$$