Multi-area Nonlinear State Estimation using Distributed Semidefinite Programming

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Outline

- Motivation and context
- Centralized state estimation (SE) using SDP
- Distributed SDP-SE formulation
- Iterative ADMM solver
- Numerical tests
- Concluding remarks
Context

- State estimation (SE) for interconnected systems [Gomez-Exposito et al’11]
  - Central processing vulnerable to unreliable telemetry
  - High computational costs related to the full system
  - Data privacy concerns of regional operators

- Distributed (D-) SE evolves with parallel computation techniques
  - Sequential/hierarchical structures [Schweppe et al’70], [Zhao et al’05], [Korres’11]
  - Decentralized implementations [Falcao et al’95], [Conjeno et al’07]
  - Fully D-SE using distributed optimization tools [Xie et al’11], [Kekatos et al’12]

- **Goal:** develop efficient/fully D-SE for nonlinear measurements
  - Existing approaches limited to (approximate) linear SE models
  - Convex relaxation approach to solving nonlinear SE [Zhu-Giannakis’11]
State Estimation

- Transmission network $\mathcal{N} := \{1, \ldots, N\}$ with given admittance matrix $Y$
- $K$ interconnected areas as $\mathcal{N} = \bigcup_{k=1}^{K} \mathcal{N}_k$
  Each $\mathcal{N}_k$ has its own control center
- SE: obtain bus voltage $\mathbf{v} := [V_1, \ldots, V_N]^T \in \mathbb{C}^N$
- $\mathcal{N}_k$ collects $M_k$ measurements in $\mathbf{z}_k := [z^1_k, \ldots, z^{M_k}_k]^T \in \mathbb{R}^{M_k}$
  - $P_n (Q_n)$: real (reactive) power injection at bus $n$;
  - $P_{mn}(Q_{mn})$: real (reactive) power flow from bus $m$ to bus $n$;
  - $|V_n|$: voltage magnitude at bus $n$.

$$P_n = \sum_{m=1}^{N} P_{mn} = \sum_{m=1}^{N} |V_n||V_m| (G_{nm} \cos \theta_{nm} + B_{nm} \sin \theta_{nm})$$

$$Q_n = \sum_{m=1}^{N} Q_{mn} = \sum_{m=1}^{N} |V_n||V_m| (G_{nm} \sin \theta_{nm} - B_{nm} \cos \theta_{nm})$$
Centralized SE

- **Measurement model** for the \(l\)-th meter at the \(k\)-th area

  \[ z_k^l = h_k^l(v) + \epsilon_k^l, \quad \forall k, \ell \]

  where \( \epsilon_k^l \sim \mathcal{N}(0, \sigma^2) \) accounts for the measurement noise

- **Centralized SE**: least-squares (LS) estimator

  \[
  \hat{v} := \arg \min_v \sum_{k=1}^{K} \sum_{\ell=1}^{M_k} [z_k^l - h_k^l(v)]^2 \tag{C-SE}
  \]

- **Nonlinear** LS optimization problem is nonconvex
  - Gauss-Newton via iterative linear approximations [Abur-Exposito ‘04]
  - Sensitive to initial guesses: local optimum; convergence?
Convexifying C-SE

- Formulate **(C-SE)** to a semidefinite program (SDP)
  - Quadratic measurement \( z_k^\ell \) linear in the outer-product \( V := vv^H \)

\[
z_k^\ell = h_k^\ell(v) + \epsilon_k^\ell = \text{Tr}(H_k^\ell V) + \epsilon_k^\ell
\]

\[
\hat{V} := \arg\min_V \sum_{k=1}^K \sum_{\ell=1}^{M_k} \left[ z_k^\ell - \text{Tr}(H_k^\ell V) \right]^2
\]

s.to \( V \succeq 0 \), and \( \text{rank}(V) = 1 \) (C-SDP)

- Convexify SE by dropping the rank constraint [Zhu-Giannakis’11]
  - Performance guarantees for several nonconvex problems [Luo et al’10]
  - Polynomial complexity and global optimality regardless of initial guesses

**Goal**: develop **distributed** SDP that scales with the size of each control area, while minimizing communication costs and preserving privacy
Cost Decomposition

- Augment $\mathcal{N}_{(k)}$ to capture tie-line buses
  \[ z_k^\ell = \text{Tr}(H_{(k)}^\ell V_{(k)}) + \epsilon_k^\ell \]
- Define the LS cost per area $k$
  \[ f_k(V_{(k)}) := \sum_{\ell=1}^{M_k} \left[ z_k^\ell - \text{Tr}(H_{(k)}^\ell V_{(k)}) \right]^2 \]

\[
\hat{V} := \arg\min_V \sum_{k=1}^{K} \sum_{\ell=1}^{M_k} \left[ z_k^\ell - \text{Tr}(H_{(k)}^\ell V) \right]^2 \\
\text{s.to} \quad V \succeq 0
\]

\[
\hat{V} := \arg\min_V \sum_{k=1}^{K} f_k(V_{(k)}) \\
\text{s.to} \quad V \succeq 0
\]

**Challenge**: as the augmented sets $\{\mathcal{N}_{(k)}\}$ partially overlap, the PSD constraint couples local matrices $\{V_{(k)}\}$
Constraint Decoupling

Q: Can the overall $\mathbf{V} \succeq \mathbf{0}$ decouple to $\mathbf{V}_{(k)} \succeq \mathbf{0}$ per area $k$?

A: Yes! “Chordal” graph property for completing PSD matrix [Grone et al’84]

- **(as1)** The communication graph among all control areas form a tree.
- **(as2)** Each control area has at least one bus non-overlapping with other areas.

**Proposition:** Under (as1)-(as2), the edges corresponding to entries in $\{\mathbf{V}_{(k)}\}$ form a chordal graph, and thus (C-SDP) and (D-SDP) are equivalent.
Distributed SDP-SE

- Each control area $k$ only solves for its local matrix $\mathbf{V}^{(k)}$

$$
\hat{\mathbf{V}} := \arg\min_{\mathbf{V}} \sum_{k=1}^{K} f_k(\mathbf{V}^{(k)})
\text{s.t.} \quad \mathbf{V}^{(k)} \succeq 0, \quad \forall k
$$

$$
\{\hat{\mathbf{V}}^{(k)}\} := \arg\min_{\{\mathbf{V}^{(k)}\}} \sum_{k=1}^{K} f_k(\mathbf{V}^{(k)})
\text{s.t.} \quad \mathbf{V}^{(k)} \succeq 0, \quad \mathbf{V}^{(i)} = \mathbf{V}^{(j)}
$$

- Alternating Direction Method-of-Multiplier (ADMM) [Bertsekas et al.'97]
  - Introduce Lagrangian multiplier variable $\Lambda_{k,j}$ per equality constraint
  - Iterative updates between local matrix variables and multiplier variables

- Asymptotic convergence $\mathbf{V}^{(k)(i)} \rightarrow \hat{\mathbf{V}}^{(k)}$ as iteration number $i \rightarrow \infty$
  - Resilient to noisy and asynchronous communications [Zhu et al.'09]
ADMM Convergence

- IEEE 14-bus system with 4 areas
- 5 power flow meters on the tie-lines

- Matrix errors $\|V_{(k)}(i) - \hat{V}_{(k)}\|$ asymptotically vanishes, convergent!
Estimation Error

- Local estimation error $\|v^{(k)}(i) - \hat{v}^{(k)}\|$ of more interest

- Local error achieves the estimation accuracy within 40 iterations!
Concluding summary

- **D-SE for interconnected power system regional operators**
  - Minimal computational and communication costs
  - Local areas keep data private from the external system

- **Reformulating to a distributed optimization problem**
  - Convex SDP relaxation to tackle the measurement nonlinearity
  - Exploit chordal graph property for overall matrix decomposition
  - Iterative ADMM updates that scale with the size of local areas

- **Future research directions**
  - Efficient SDP implementations for larger power systems
  - Generalize the distributed SDP framework for other power system optimization problems