Traditional Design of Cage Rotor Induction Motors

Ronald G. Harley and Yao Duan

Georgia Institute of Technology

November, 2009

Rating considerations

Dimensions of a machine depend on

- Torque at a specific speed
- How intensively the magnetic circuit is used.
- How intensively the electric circuit is used
- The type of enclosure
- Type of cooling
- The duty cycle of the load
- The frequency of starting and stopping

$S = 3(4.44 K_w f T_{ph} I_{ph} \Phi_m)$ volt amperes

$B_g = 2p\, \Phi_m / (\pi D L)$ Tesla (average magnetic flux density over air-gap surface)

$ac = 3(2 T_{ph} I_{ph})/(\pi D)$ *amp. cond. per m air-gap circumference*

$f = pn,$ where p = pole pairs, and n = speed in revs per second

Hence

$$S = 11 k_w * B_g * ac * D^2 * L * n$$

Rating and dimensions

1. So D^2Ln = volume x speed = $S/(11K_w B_g ac)$

Get S from shaft output power (hp or kW), efficiency and power factor.

B_g = specific magnetic loading

ac = specific electric loading

- Select $\boldsymbol{B_g}$ from **experience** (limited by losses in the teeth and magnetizing current). Determines how heavily the magnetic core material is utilized. High $\boldsymbol{B_g}$ means less magnetic material but higher magnetic losses. Select magnetic material also based on frequency. Cooling.

- Select specific electric loading ***ac*** (ampere conductors per meter of air gap circumference) from **traditional Tables**. Determines how heavily the electric material is utilized. High ***ac*** means less electric material but higher electric losses. Cooling.

Rating and dimensions (continued)

- Trade offs depend on **objectives** – low volume and weight, high losses and low efficiency, versus high volume and weight, low losses and high efficiency.
- ***B*** and ***ac*** values also depend on duty cycle, ambient temp.

D m.	$\frac{L}{D}$ max.	Slip-ring			Cage		
		$\overline{B}$ Wb./m.^{2}	ac amp.-cond./m.	δ A./mm.^{2}	$\overline{B}$ Wb./m.^{2}	ac amp.-cond./m.	δ A./mm.^{2}
0·1	0·8	0·3	6 000	3·8	0·3	11 000	4·0
0·15	0·75	0·35	10 000	3·6	0·35	15 000	3·8
0·2	0·7	0·4	13 000	3·4	0·4	18 000	3·6
0·3	0·65	0·43	17 500	3·3	0·43	22 500	3·5
0·4	0·62	0·45	21 500	3·2	0·45	26 000	3·5
0·5	0·6	0·46	25 000	3·2	0·46	29 000	3·5
0·75	0·5	0·47	30 000	3·2	0·47	33 000	3·5
1·0	0·42	0·48	32 500	3·2	0·48	35 000	3·5
1·5	0·33	0·5	34 000	3·2			
2·0	0·3	0·51	35 000	3·2			
3·0	0·3	0·53	37 000	3·2			

Ref. [3]

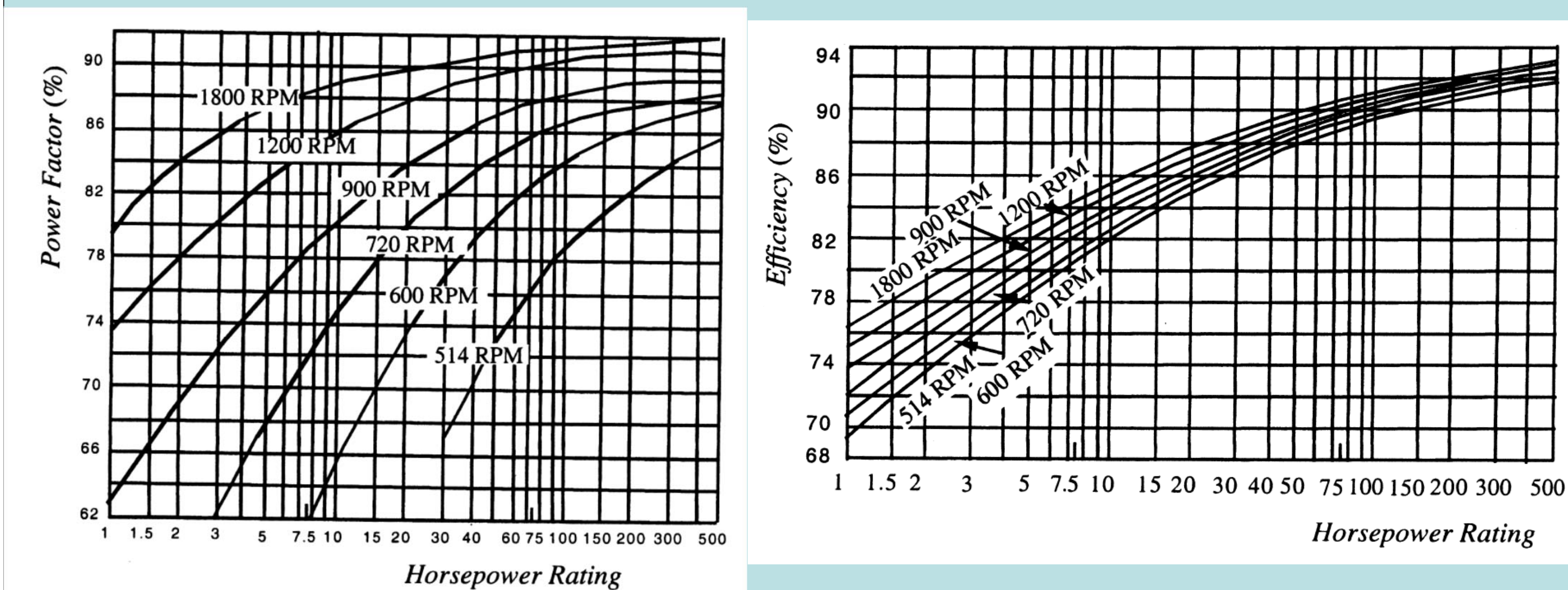
Say

[3] M. G. Say, *Performance and design of AC machines*: Pitman, London, 1970.

Efficiency and power factor

2. **Assume efficiency and power factor** (from experience) to convert shaft power to input power, then compute rotor volume that is (rotor diameter D)^2 (rotor length L).

Typical power factor and efficiency of three phase 60 Hz NEMA B induction machines Ref. [2] Lipo

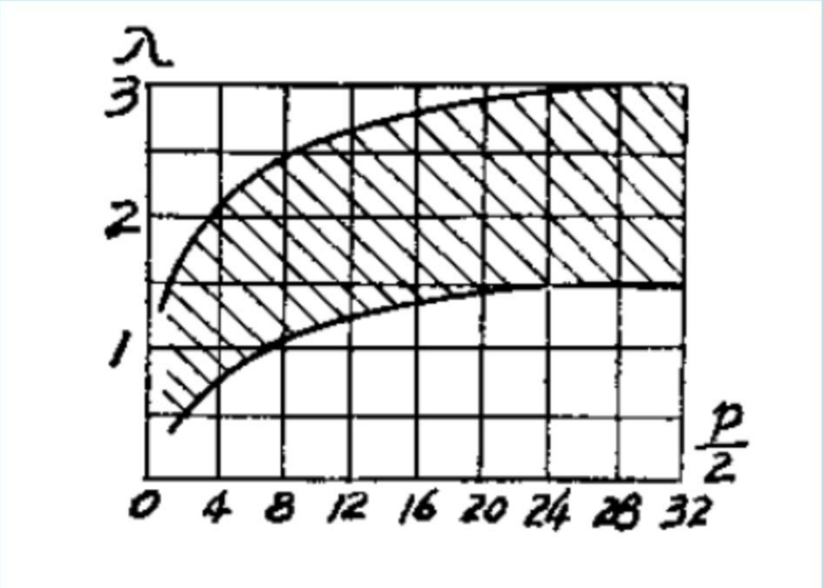


[2] T. A. Lipo, *Introduction to AC machine design*, 2 ed.: University of Wisconsin-Madison, 2004.

Aspect ratio

- Ratio of D/L determines the shape of a pole, square or rectangular. Select shape from Tables (experience) and **calculate D and L.**

$$\lambda = \frac{L}{\pi D / p} = \frac{L}{Y}$$



Ref. [4] Fu

p

[4] F. Fu and X. Tang, *Induction machine design handbook*: China Machine Press, 2002.

Air gap length

3. **Air gap length** from empirical formula. Depends on several factors.

Electromagnetic factors: magnetizing current, pulsation losses
Mechanical factors: mechanical tolerances, bearing, shaft deflection, unbalanced magnetic pull

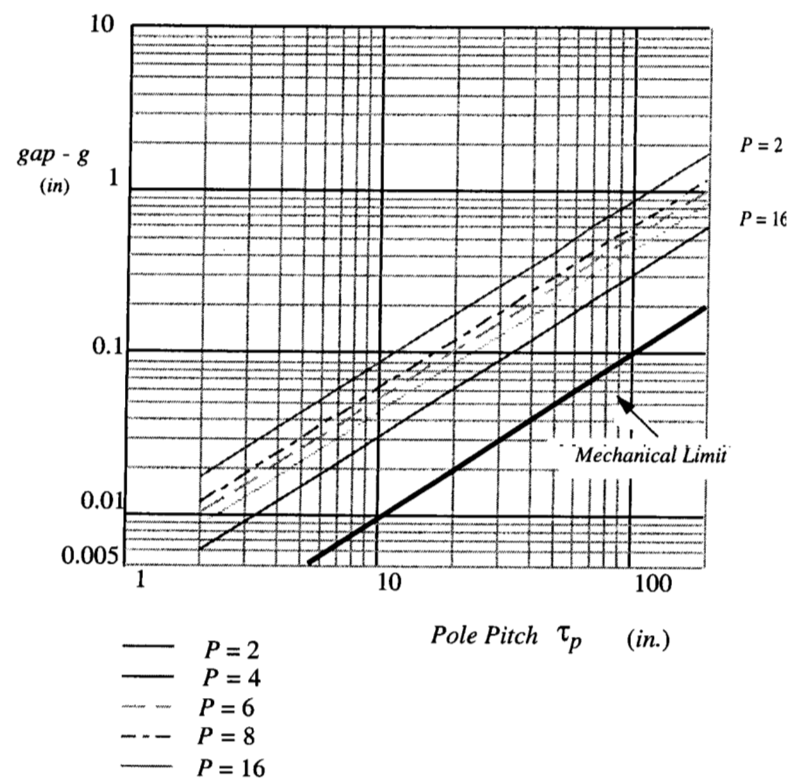
Different versions of empirical formulas:

$$g = 5*10^{-3}(\frac{D}{2}\tau_p)^{1/2}$$ Ref. [2] Lipo

$$g = 9*10^{-3} r(p)^{-1/2}$$ Ref. [2] Lipo

$$g = 3*10^{-3}\tau_p(p)^{1/2}$$ Ref. [2] Lipo

$$g = 0.2 + 2\sqrt{DL}$$ Ref. [3] Say



$$\tau_p = \frac{\pi D}{p}$$ pole pitch p: pole number

Calculate number of turns

4. Calculate number of stator turns per phase depending on previous *B, D, L,* supply voltage (math) and assumed flux density shape factor α_i .

Flux per pole $B_g = 2p\,\Phi_m/(\pi DL)$ to find Φ_m

Back EMF factor

$$K_E = \frac{E_1}{V_{1ph}}$$

K_E: typically 0.85-0.95, higher for large power rating or small pole number [4] Fu.

Turns per phase

$$T_{ph} = \frac{K_E V_{1ph}}{4 K_f K_{w1} f \phi}$$

K_f: form factor, typically assumed = 1
K_{w1} : winding factor for fundamental = typically 0.955
f: fundamental frequency

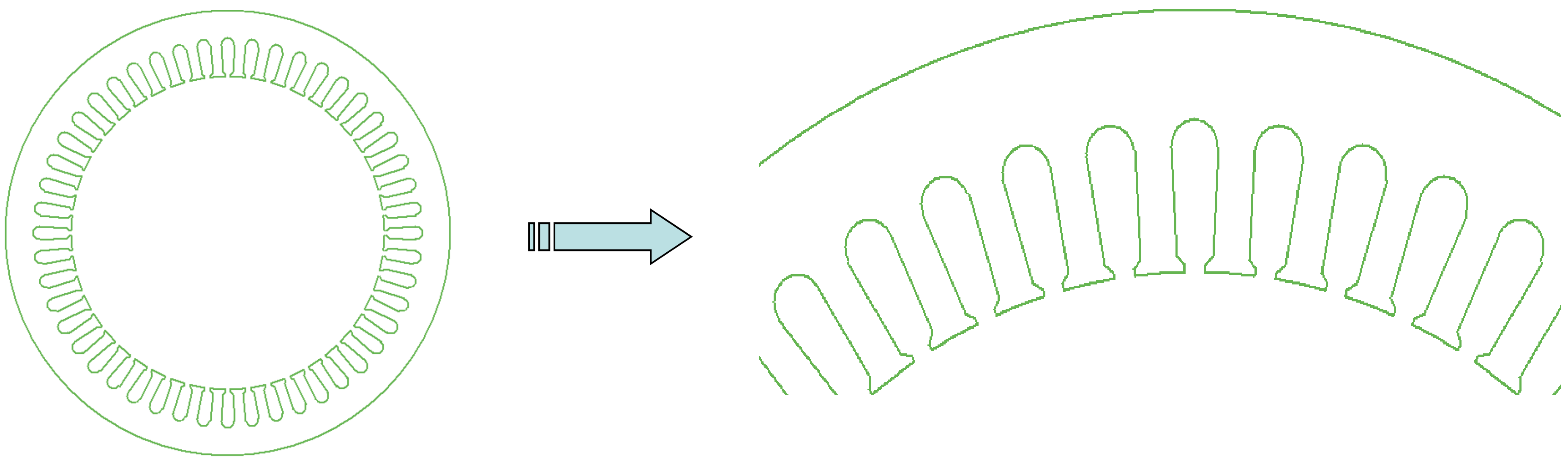
Select number of stator slots

5. Select **number of stator slots** and suitable three phase winding layout (experience).

   Less slots: 1)less cost **2)** less space lost due to insulation and slot opening;

   More slots: 1) smaller leakage inductance and larger breakdown torque **2)** small MMF harmonics **3)** better cooling

Typically, stator teeth width between ¼" and 1", ratio of slot width to slot pitch between 0.4 and 0.6 (Ref [2] Lipo)



Stator slot geometry

- In small motors with small diameters the taper on the tooth or slot is significant and tapered slots (parallel sided teeth) are used. This gives maximum area of slot for given tooth flux density. Round wires of small gauge are used since they are easy to wind and do not mind the taper of the slot.

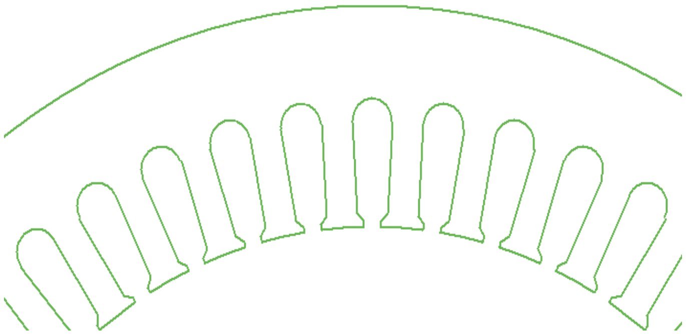
- In larger machines with larger diameters, the tooth taper is much less and often strip conductors are used which need parallel sided slots, thus tapered teeth.

Stator slot sizing

6. Select **stator current density (**experience but this value depends on ambient temp, cooling conditions, and duty cycle), and find stator **conductor size.**

> **Enclosed fan-cooled: 5 to 6.5 A/mm^2, larger for 20kW below**
> **Closed frame, no fan: 10-15% lower (Ref [4] Fu)**

7. Then check that initial value chosen for ***ac*** is approximately correct. If not, **return to step (1),** select a different value for ***ac*** and repeat steps (2) to (5).
8. Select **stator tooth width** depending on mechanical strength without teeth flux density being too high.
9. Assume a **fill factor (experience)** for stator slots, pack in conductors, and find outer diameter of slots.



Select flux density

10. Select suitable values of **flux density** in stator back iron and compute stator outer diameter. (for 60 Hz, ordinary electric steel, lower for higher frequencies)

Position	Typical flux density range (Ref. [3] Say)	Maximum flux density (Ref. [2] Lipo)
Airgap Bg	0.65 – 0.82 T (ave.)	
Stator yoke	1.1 – 1.45 T (peak)	1.7 T
Stator teeth	1.4 – 1.7 T	2.1 T
Rotor yoke	1.2 T	1.7 T
Rotor teeth	1.5 – 1.8 T	2.2 T

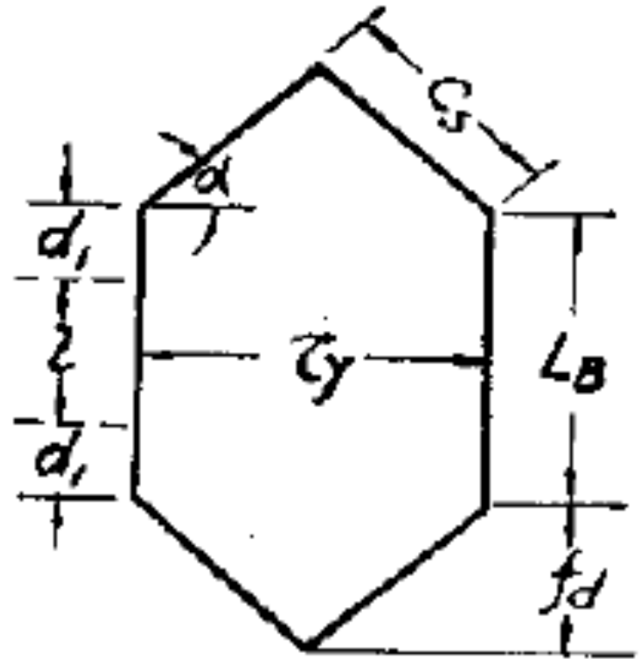
Calculate stator winding resistance

11. Calculate **stator winding resistance (approx. math – end turns)**

Resistively of conductors ρ_{C0}

Estimate end length l_{end}

Conductor cross sectional area (standard wire gauge) $A_{c0} = \frac{I_{phase}}{J}$



Stator resistance $R_s = \rho_{c0} \frac{2(L + l_{end})T_{ph}}{A_{c0}}$

Select number of rotor slots

12. Select **number of rotor slots**. Ratio to stator slot number is important to avoid cogging torque (experience but based on space harmonics).
13. Decides on **rotor skew**

Combinations To avoid (P=pole number) (Ref. [2] Lipo)

Noisy or vibrations

$$S_1 - S_2 = \pm 2$$
$$= \pm(P \pm 1), \text{ and}$$
$$= \pm(P \pm 2).$$

Cusps in torque speed curve (due to MMF harmonics)

$$S_1 - S_2 = \pm P$$
$$= -2P, \text{ or}$$
$$= -5P$$

Cogging problem

$$S_1 - S_2 = 0 \text{ or } = \pm mP$$

Recommended combination (Ref. [2])

Preferred combinations in smaller sizes have S1-S2 = + or - 2P with 1 rotor slot skew to reduce cusps and cogging

Pole Number	Stator/Rotor Slot Number				
2	36/28	48/38	54/46	60/52	
4	48/40	48/56	60/44	60/76	72/58
6	54/42	54/66	72/88	72/54	72/84
8	54/70	72/58	72/88		
10	72/88	72/92			
12	72/92				

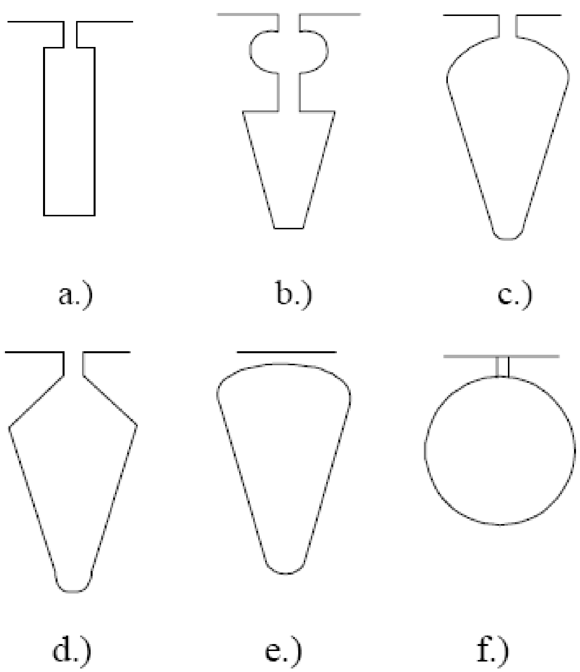
Rotor bar

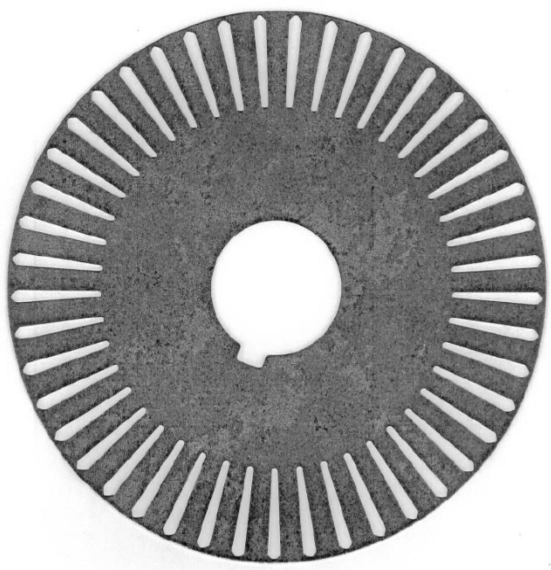
14. Select **current density** in rotor bars and end rings (depends on ambient temp, cooling conditions, and duty cycle), and from rotor bar and end ring currents get their cross sectional areas.

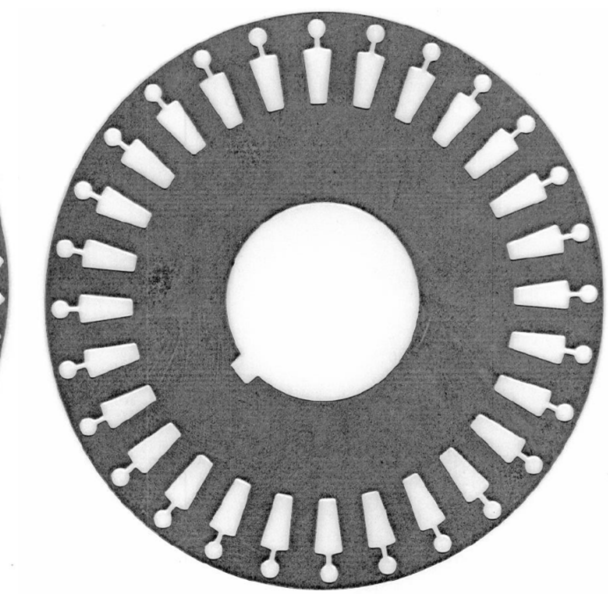
> For aluminum bar, 2.2 to 4.5 A/mm^2, lower value for small motors
> For deep bar rotor, 5.5 to 7.5 A/mm^2
> For load with large inertia and high rated speed, not exceed 6.5 to 7 A/mm^2

Ref. [4] Fu

15. Rotor bar (width to depth) geometry now depends on what torque-speed characteristic and starting torque is needed. **Trial and error and experience**.







Skin effect

16. Calculate **rotor** bar and end ring **resistances** and hence the conductor losses (math and approximations, skin effect coefficients).

Skin effect causes non-uniform distribution of current in the conductor

Current density in the rotor bar is higher closer to air-gap.

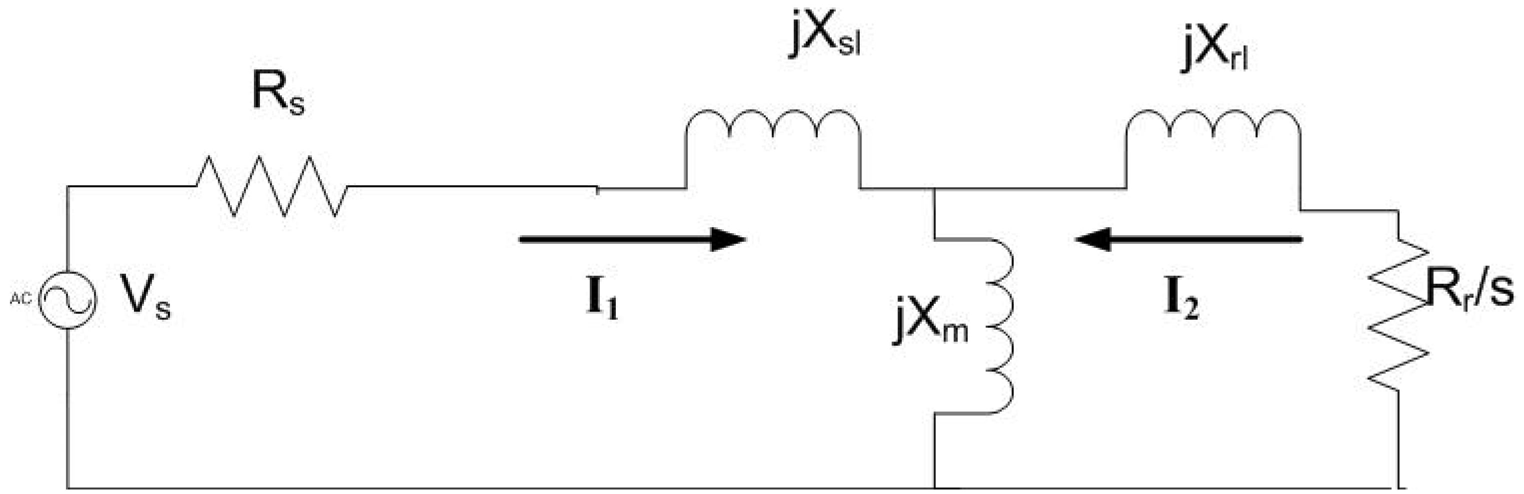
In traditional designs of 60 Hz line-fed induction machines, skin effect is represented by **correction coefficients K_R** and K_X for bar resistance and slot leakage inductance. (Ref. [1] Boldea)

K_R and K_X depend on the shape and size of the rotor slot, the conductor material and the rotor current frequency. Typically K_R is in the range of 1 to 5, and K_X is in the range of 0.2 to 1. (Ref. [1] Boldea)

$$K_R = \frac{\text{rotor ac resistance}}{\text{rotor dc resistance}} \qquad K_X = \frac{\text{rotor ac slot leakage reactance}}{\text{rotor dc slot leakage reactance}}$$

Skin effect may not be neglected in line-start motors when assessing the starting, or breakdown torque. The larger the motor power, the more severe this phenomenon. (Ref. [1] Boldea)

Equivalent circuit calculation



Calculate magnetizing current

17. Calculate magnetizing inductance

Magnetizing MMF $$F_{1m} = 2(K_c g \frac{B_g}{\mu_0} + F_{mts} + F_{mtr} + F_{mcs} + F_{mcr})$$

K_c Carter coefficient to account for the effective airgap length increase due to slot opening. Usually in the range of 1-1.5 (Ref [1-4])

$F_{mts}, F_{mtr}, F_{mcs}, F_{mcr}$ MMF drop along stator teeth, rotor teeth, stator core and rotor core, estimated from assigned flux density and B-H curve

$$1 + K_{sd} = 1 + \frac{F_{mts} + F_{mtr}}{F_{mg}} = 1 + \frac{F_{mts} + F_{mtr}}{K_c g \frac{B_g}{\mu_0}}$$

Teeth saturation coefficients, need to agree with the value selected in step 1

Magnetizing current

$$I_{mag} = \frac{\pi p F_{1m}}{3\sqrt{2} T_{ph} K_{w1}}$$

Calculate stator leakage inductance

18. Calculate the **leakage reactance** consisting of several components by using some equations and some empirical formulas (very approximate).

$$X_{sl} = 2\pi\mu_0 f_1 L \frac{T_{ph1}^2}{pq}(\lambda_{sls} + \lambda_{ds} + \lambda_{ecs})$$

q: Stator slots/pole/phase

λ_{sls} Stator slot leakage coefficients

λ_{ds} Stator differential leakage coefficients

λ_{ecs} Stator end leakage coefficients

$$X_{sl} = 2\pi\mu_0 f_1 L \frac{T_{ph1}^2}{pq}(\lambda_{sls} + \lambda_{ds} + \lambda_{ecs}) = C_s(\lambda_{sls} + \lambda_{ds} + \lambda_{ecs})$$

$$= X_{sls} + X_{ds} + X_{ecs}$$

X_{sls} Stator slot leakage reactance

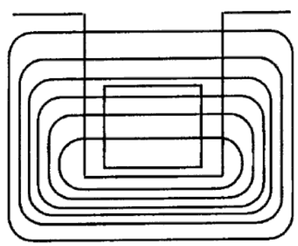
X_{ds} Stator differential leakage reactance

X_{ecs} Stator end leakage reactance

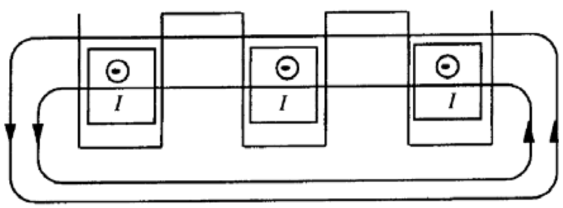
Slot leakage coefficients

$$X_{sl} = X_{sls} + X_{ds} + X_{ecs} \qquad X_{sls} = C_s \lambda_{sls}$$

Slot leakage flux in a single slot



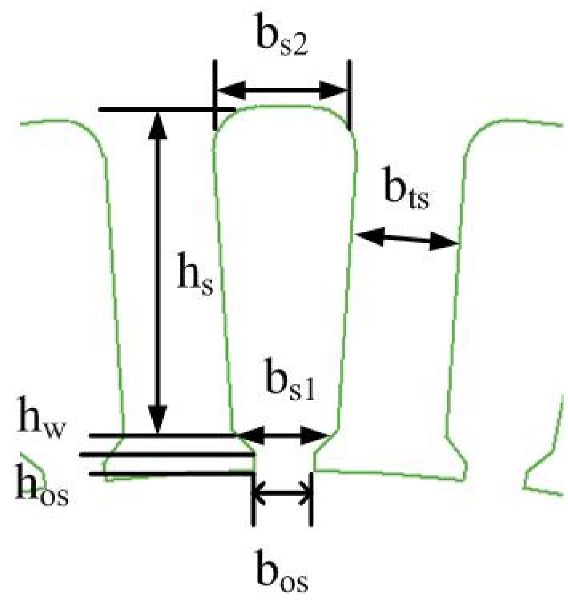
Slot leakage flux in a phase belt



$$\lambda_{sls} = \left[\frac{2}{3}\frac{h_s}{(b_{s1}+b_{s2})} + \frac{2h_w}{(b_{os}+b_{s1})} + \frac{h_{os}}{b_{os}}\right]\left(\frac{1+3\beta}{4}\right)$$

β : (coil pitch) / (pole pitch)

Ref. [1] Boldea



Deeper slot, larger slot leakage reactance

Wider slot, larger slot opening, smaller leakage reactance

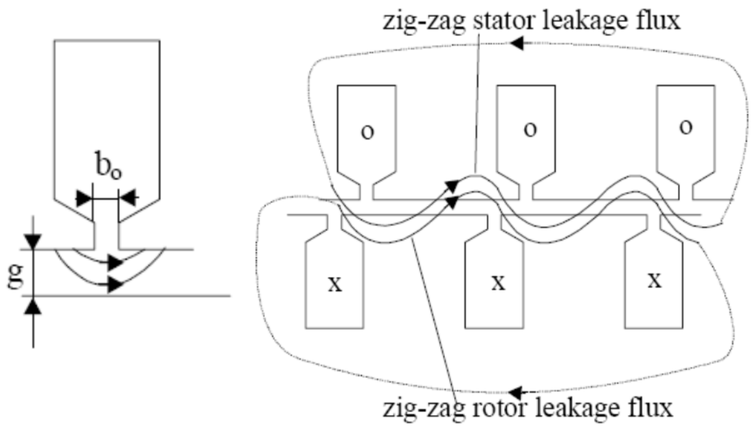
Differential leakage coefficients

$$X_{sl} = X_{sls} + X_{ds} + X_{ecs} \qquad X_{ds} = C_L \lambda_{ds} = C_L(\lambda_{zgs} + \lambda_{bts}) = X_{zgs} + X_{bts}$$

The total reactance due to all harmonic fields of both stator and rotor is called *differential reactance*.

Differential reactance has two components: **zigzag**(X_{zgs}) and **belt** (X_{bts})

zigzag



$$\lambda_{zgs} = \frac{5gK_c / b_{os}}{5 + 4gK_c / b_{os}} \frac{3\beta + 1}{4}$$ Ref. [1]

β : (coil pitch) / (pole pitch)

K_c: Carter coefficients

belt Ref. [1] Boldea

$$\sigma_{bts} = \frac{X_{bts}}{X_m} = \sum_{\nu=1} \left(\frac{K_{dp\nu}^{\;2}}{\nu^2 K_{dp1}^{\;2}}\right) \frac{K_s}{K_{s\nu}}$$

X_{bts}: belt leakage reactance

X_m: magnetizing reactance

$K_{dp\nu}$: winding factor for ν^{th} harmonic

$K_{s\nu}$: saturation factor for ν^{th} harmonic,can be approximated by K_{sd} in step 17

End leakage coefficients

$$X_{sl} = X_{sls} + X_{ds} + X_{ecs}$$

$$X_{ecs} = C_s \lambda_{ecs}$$

An approximate expression

$$\lambda_{ecs} = 0.34 \frac{q}{L} (l_{end} - 0.64 \beta \tau_p)$$

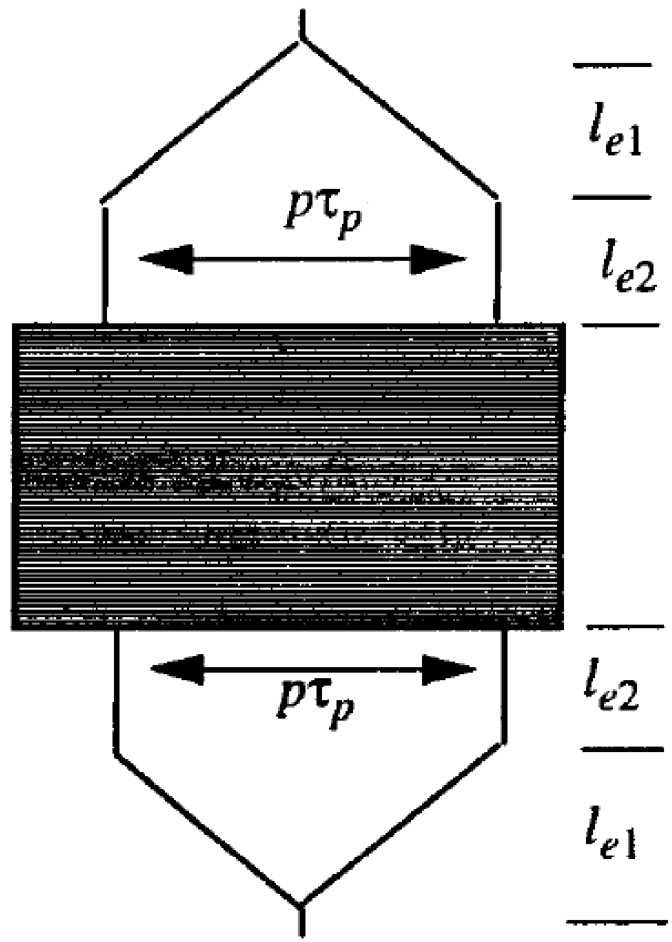
Ref. [1] Boldea

q: Stator slots/pole/phase

β: (coil pitch) / (pole pitch)

l_{end}: End connection length of a coil

L: Machine axial length



Calculate rotor leakage inductance

19. Calculate the **leakage reactance** consisting of several components by using some equations and some empirical formulas (very approximate).

$$X_{rl} = 2\pi\mu_0 f_1 L(\lambda_{slr} K_X + \lambda_{dr} + \lambda_{er}) = C_r(\lambda_{slr} K_X + \lambda_{dr} + \lambda_{er})$$

$$X_{rl} = X_{slr} + X_{dr} + X_{er}$$

λ_r Rotor slot leakage coefficients, similar to stator slot leakage

λ_{dr} Rotor differential leakage coefficients

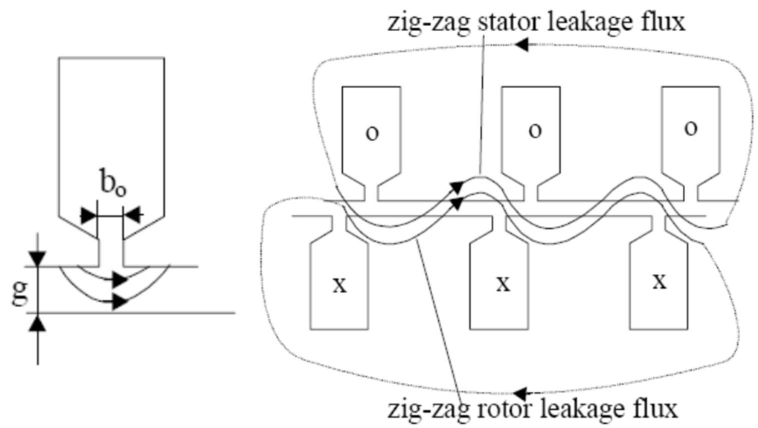
λ_{er} Rotor end leakage coefficients

K_X Skin effect coefficients, described in step 16

Rotor differential inductance

$$X_{rl} = X_{slr} + X_{dr} + X_{er} \qquad X_{dr} = C_r \lambda_{dr} = C_r(\lambda_{zgr} + \lambda_{btr}) = X_{zgr} + X_{btr}$$

Zigzag X_{zgr}



$$\lambda_{zgr} = \frac{5gK_c / b_{or}}{5 + 4gK_c / b_{or}} \frac{3\beta_y + 1}{4}$$

$\beta_y = 1$ for cage rotors

Ref. [1] Boldea

g: Airgap length

K_c: Carter's coefficients

b_{or}: Rotor slot opening

belt X_{btr}

$$\lambda_{btr} = \frac{0.9\tau_r \gamma_{btr}}{K_c g}\left(\frac{N_r}{12p}\right)$$

$$\gamma_{btr} = 9\left(\frac{12p}{N_r}\right)10^{-2}$$

Ref. [1] Boldea

p: Pole number

N_r: Number of rotor slots

τ_r: Rotor slot pitch

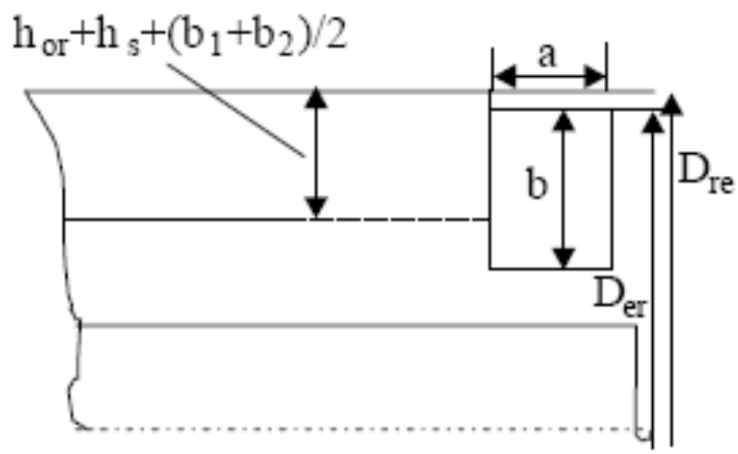
Rotor end leakage inductance

$$X_{rl} = X_{slr} + X_{dr} + X_{er} \qquad X_{er} = C_r \lambda_{er}$$

$$\lambda_{er} = \frac{2.3(D_{er} - b)}{N_r L * 4\sin^2(\frac{2\pi p}{N_r})} \log \frac{4.7(D_{er} - b)}{b + 2a}$$

Ref. [1] Boldea

Rotor end-ring cross section



p: Pole number

N_r: Number of rotor slots

L: Machine axial length

a, b: Endring ring width and height

D_{re}: Rotor outer diameter

D_{er}: End-ring outer diameter

Finite Element Analysis (FEA) calculation

- FEA is based on numerical solution of the magnetic field. The FEA calculation is not based on analytical theories, such as the classical equivalent circuit shown before.
- Designer's **input to FEA** is the physical geometry of the machine, material properties, the excitation applied to the winding (current source or voltage source), and the load of the machine.
- **Output of FEA** is the overall performance of machine, such as winding current (if voltage source applied), shaft torque, rotor speed at a certain mechanical load.
- Copper loss is calculated **off-line** from the FEA solution of current and the calculated resistance by the designer.
- Core loss is mostly approximated from the flux density solution in the core and the material datasheet and calculated **off-line**.
- FEA calculation treats the machine as a whole object. It can neither directly calculate the values of reactances and resistances in the equivalent circuit, nor calculate the individual components of leakage inductances (slot leakage, differential leakage, etc.)
- Designer calculates **efficiency and power factor** off-line based on FEA torque and current.
- In 2D FEA, the end effect is approximated by equivalent circuit comprised of resistances and reactances, which is an input from the designer. 3D FEA can include the end effect in its calculation.
- FEA is time consuming. 2D FEA takes **hours** for simulation the performance of a design. 3 D FEA takes **days.**

Calculate performance

20. Several text books show how to compute rotor bar and end ring currents, resistances, and conductor losses. From this find rotor resistance of an equivalent rotor phase. Now the equivalent circuit is complete.

21. Use FEA to **check for any flux density** violations.

22. Calculate **all iron losses (off-line)** approximately from material data sheets of losses in W/kg depending on flux density and frequency.

23. **Assume friction and windage** as typically 1% of input power.

24. All the **elements of the equivalent circuit** have now been determined. Use this to compute **efficiency and power factor** at full load. If these do not agree closely with assumed values in step (1), then **return to step** (1) and repeat all the steps (2) to (17)

Traditional induction motor design steps
(continued)

25. Calculate motor performance data from equivalent circuit and compare with results from FEA:
    - Slip at full load
    - Starting current and torque
    - Torque-speed curve **(if not acceptable then change rotor slot geometry and return to step 12)**
    - Torque ripple if fed from converter

26. Mechanical design

27. Thermal design. If temp rises are too high, either increase cooling by adding heat sink fins for example, or return to step (1), adjust choice of **magnetic loadings** and/or **electric loading**, and repeat design.

28. Calculate weight and volume.

Approaches to modify designs

Problems	Causes	Solutions
Small T_{start}	Large $X_{lr,s}$	1. Modify rotor and stator slot shape (decrease slot height or increase slot width) 2. Decrease stator turns or coil pitch 3. Use less skew 4. Choose proper N_s/N_r combination 5. Review values of leakage components
	Small R_r	1. Modify rotor slot shape to increase skin effect 2. Decrease rotor slot area
Large I_{start}	Small $X_{lr,s}$	1.Increase $X_{lr,s}$ 2.Modify rotor slot shape, use deep slot or double squirrel cage 3. Increase stator turns or coil pitch 4. Avoid too small number of rotor or stator slots to prevent too much saturation

Approaches to modify designs[contd]

Problems	Causes	Solutions
Difficult to start	Large torque from harmonics	1. Choose proper N_s/N_r combination 2. Skew the rotor 3. Increase airgap
Small power factor	Large $X_{lr,s}$	Decrease $X_{lr,s}$
	Small X_{mg}	1.Decrease airgap 2. Increase stator turns or coil pitch

Approaches to modify designs [contd]

Problems	Causes	Solutions
Low efficiency	Large stator copper loss	1. Increase wire diameter 2. Decrease stator turn or coil pitch 3. Increase power factor
	Large core loss	1. Decrease flux density by increase stator turn or coil pitch, and increase length 2. Use better steel
	Large stray loss	1. Modify Ns/Nr combination 2. Increase airgap 3. Modify rotor skew
	Large rotor copper loss	1. Increase rotor slot area 2. Decrease stator turn or coil pitch to decrease rotor current

Approaches to modify designs[contd]

Problems	Causes	Solutions
High temperature	Large losses	1. Decrease losses 2. Modify design for proper loss distribution
	Poor cooling	1. Increase cooling gas flow 2. Increase surface heat rejection capability, like fins 3. Increase heat conductivity from winding to core 4. Improve contact between core and frame
	Large thermal load	1. Decrease current density 2. Increase axial length, decrease stator turn 3. Increase insulation level

Missing steps

- **Automating** the optimizing process to remove the need for repeating the many steps and choices to arrive at so-called optimized solutions by trial and error.
- What are best **materials** to use at higher frequencies?
- How to make **initial choices** to satisfy specific requirements such as high starting torque?
- More accurate **cooling** calculations.
- 2nd order effects: end winding effects, harmonics, inverter interactions, ripple losses, etc.

Induction machine (IM) vs PM machine

- A comparison study of IM and PM machine (M. J. Melfi, S. Evon and R.Mcelveen, “Indution vs Permanent Magnet Motors”, *IEEE Industry Applications Magnazine*, pp. 28-35, Nov-Dec 2009)
  - Comparison of performance test results of three machines: Induction Machine, Surface Mount PM machine, and Interior PM machine
  - Operating condition: 75 HP, 1800 rpm, similar voltage(459 V-395 V), same stator laminations, different windings, no information on rotor

	IM (459 V)	Surface Mount PM (405 V)	Interior PM (395 V)
Base frequency	60 Hz	120 Hz	60 Hz
Full load current	92.3 A	85.5 A	90.2 A
Full load efficiency	93.6 %	96.2 %	96.8 %

  - Comparison results appear to show PM machines are better, but comparison is not fair.
    - IM is probably an off-the-shelf machine, while PM machines are specially designed
    - Whether the three machines are optimized, and the optimization objective, are unknown
    - NEMA design type of IM is unknown.
    - Comparison from two machines at different frequencies is unfair
  - Further comparison study needed

References

- [1] I. Boldea and S. A. Nasar, *The induction machine handbook*, 1 ed.: CRC express, 2001.
- [2] T. A. Lipo, *Introduction to AC machine design*, 2 ed.: University of Wisconsin-Madison, 2004.
- [3] M. G. Say, *Performance and design of AC machines*: Pitman, London, 1970.
- [4] F. Fu and X. Tang, *Induction machine design handbook*: China Machine Press, 2002.