Students' Conception and Application of Mechanical Equilibrium Through Their Sketches

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1. Introduction and Relevant Literature

Sketching is central to engineering practice, especially design[1]–[4]. When constructing sketches, a student/engineer must synthesize various pieces of knowledge and reasoning into an ideally self-consistent graph or set of graphs. University educators have attempted to enhance students use of the material by introducing sketching in digital environments (e.g., tablets)[5], [6]. Without knowing what knowledge and reasoning students use when constructing their sketches by hand, educators cannot make fully informed pedagogical decisions. In this paper, we present descriptions of the range and nuances of student knowledge comprising the conception and application of mechanical equilibrium when sketching shear force diagrams.

Statics is a core course in most engineering curricula and is usually among the first engineering courses engineering majors take. Statics courses go one step above introductory mechanics, focusing on analysis of the shear force within beams when the system is not accelerating. During instruction, shear force diagrams are introduced as exercises via in-class examples and then as sketching activities on homework and exams. Sketching in general is an integral part of engineering instruction as well as engineering practice. Literature on sketching behavior has covered a wide range of concepts from digital logic [7] to the broader cognitive processes behind sketching in various fields[8]. There is also work that covers the quality of students’ sketches of concepts common in statics courses, however these studies focus on developing sketch recognition and analysis for assessment [9]–[11].

Statics education literature has focused on classroom intervention strategies [12]–[16] and implementing assessments such as the concept assessment tool for statics CATS[17], [18]. However, how students learn statics concepts from a cognitive science perspective is relatively unexplored. Those who have studied the cognition space include investigations into how students draw FBDs for multi-body problems[19], understand external forces on single body problems[20], and general problem solving strategies such as the inconsistent nature of when students apply force equilibrium versus moment equilibrium[20]. For the most part, studies focus on students’ use of external forces. On the more advanced end of mechanical properties, there is a small body of work that looks at mechanics of materials concepts such as Montfort et al who look at students’ conceptions of shear stress[21]. Our work is situated at the interface of these two bodies of literature where students are expected to understand force and moment equilibrium externally and internally. By internally, we mean students recognize that the material plays a role in maintaining equilibrium (e.g., shear force). Additionally, our work is unique in investigating student cognition through think-aloud interviews concurrent with sketching rather than retrospective interviews after the student has completed an assessment.

Our study fills the above gap in the statics education literature and contributes to the current debate in the conceptual change literature regarding the content and structure of students’ engineering knowledge. At the heart of the debate is whether students’ understanding is coherent or fragmented across different contexts. Coherent means that students use one coherent concept or idea to guide their reasoning when solving problems such as believing that the earth is flat or hollow earth [22]. Alternatively, the model proposed by diSessa known as “knowledge-in-pieces" characterizes students' mental models as composed of sub-conceptual pieces. The students' reasoning then emerges ad hoc from the interaction of these sub-conceptual pieces. These interactions are context dependent, meaning conceptual change occurs by addressing the individual pieces and how they are organized in the context of the problem. This contrasts with Posner's overarching naïve theories, which are context independent and organizes the student's knowledge framework.

The context-dependent nature of cognition in engineering has been recognized in several areas of the literature. The work of Herman et al. has supported the fragmentation perspective, demonstrating that engineering students' misconceptions of computing concepts are chaotic and context-dependent [7], [23], [24]. Similarly, in their materials conceptual understanding studies, Krause et al. engaged students in multi-modal conceptual assessment tools [25]–[27]. These tools elicited students' conceptual understanding by asking them to draw sketches of materials concepts and verbally describe those concepts [25]–[27]. These studies revealed that students' misconceptions were revealed differentially across modalities; students could verbally describe atomic bonding, but they could not produce accurate sketches of these same atomic bonds [25]–[27]. Our work contributes to the literature by providing commentary on the contextual dependency of applying internal equilibrium in students sketches versus multiple choice measures focused on external equilibrium like those in Newcomer and Steif[20].

Our research question for this study was 1) how do students apply the concept of equilibrium during problem solving? In this case, problem solving refers to the general processes that students use to sketch shear force diagrams for a given problem context. During the interviews, students sketched diagrams for both the shear force and bending moment. For the sake of brevity, this paper focuses only on their shear force diagrams.

2. Methodology

We chose to use a grounded theory approach for our qualitative study, deciding to use constant comparative analysis of open coding for three reasons. First, there is a lack of literature concerning how students conceptualize and operationalize mechanical equilibrium in statics problems. While there has been some literature on student knowledge using the Conceptual Assessment Tool for Statics (CATS), CATS does not address knowledge expressed through sketching. From prior work, we hypothesize that modality of testing (multiple-choice selection vs. self-generated sketching) should also change what knowledge students choose to use. This distinction is useful for instructors looking to develop curriculum innovations since the shear force diagrams are usually tested in classrooms via sketching and not multiple choice. Second, part of the broader study is to describe the range of student's knowledge and reasoning. While generalizability was not the goal of the project, we did find saturation of unique knowledge and sketching behaviors after a relatively low number of participants (N=15). This is good news for educators of small and big class sizes who wish to do a similar exploratory analysis since the pool we recruited from was typical of a large research-focused university (>300 per semester). Third, constant comparative analysis allows us to build a robust theory by using broad (problem solving strategies) and narrow (specific knowledge pieces) scopes that complement each other to build our theory.

2.1 Terminology, Concepts, and Diagrams

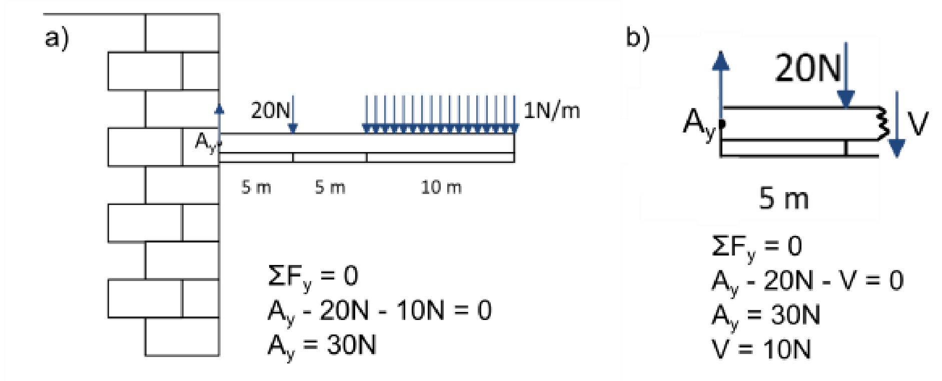

Figure 1: Schematic of a) a rigid beam with a fixed joint and b) a section of the beam after a "cut" was made at x=6m (reaction force A_y was not present in the interview protocol but was added for clarity). Figure 1a represents applying equilibrium over the whole beam while 1b represents applying equilibrium for a specific section of a beam after making an imaginary cut.

Statics coursework involves analysis of beams (see Fig. 1a) that are affixed to surfaces via joints (fixed, pin, or roller). Students use schematics like in Fig. 1a to sketch what the shear force looks like within the beam. Static problems refer to situations where the system is not accelerating either in translation or rotation. Therefore, the sum of external forces and moments must equal zero over the whole system in accordance with Newton's second law of motion. Common exercises in statics coursework ask students to sketch diagrams of the shear force within a rigid body at equilibrium. Shear force represents a material's internal reaction to external loading to restore mechanical equilibrium. To construct these diagrams effectively, students must incorporate both domain knowledge and knowledge of diagram conventions that are tacitly encoded within problem schematics. For example, the lone arrow labeled 20N in Fig. 1 should be interpreted as a point load, however, the cluster of arrows at the end of the diagram should not be interpreted as 17 point loads.

Three faculty who have taught or currently teach statics at the University of Illinois at Urbana-Champaign expressed that students are expected to follow three general steps when constructing shear force diagrams (see Figure 2). First, use sum of external forces and moments to calculate values for reaction forces and moments. To follow this step, students must recognize when a specific joint does or does induce a reaction force/moment. Second, use the schematic to create a free body diagram. Free body diagrams are one way for students to abstract the situation and condense all relevant information into one diagram. Third, use the method of sections to translate the free body diagram to the shear force diagram. In this step, students make imaginary cuts in the beam (see Fig. 1b) and use sum of the forces in that section to calculate the internal reaction force necessary to maintain mechanical equilibrium. This step should be the student's first indication that the material itself plays a role in establishing equilibrium.

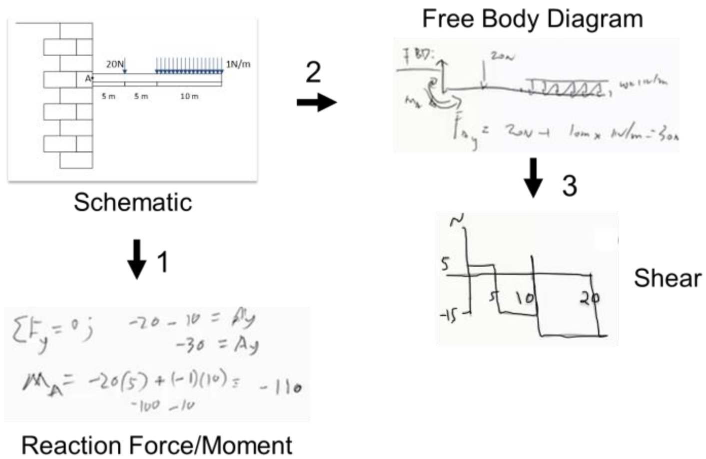

Figure 2: The general steps statics instructors identified as necessary when constructing shear force diagrams. First, calculate the reaction forces. Second, use the schematic to construct a free body diagram, abstracting all relevant information. Third, use the free body diagram to construct the shear diagram. Each step is composed of screen shots from various students' solutions.

2.2 Sampling

We recruited students from a large-enrollment statics course taken by multiple engineering majors typically in their sophomore year. The course was lecture-based, administered weekly homework assignments, and administered three midterm exams plus one final exam. All interviewed students were traditional age undergraduates (18-22 years old) spread across a variety of engineering majors and had passed the statics course the semester before being interviewed. These criteria ensure that students have seen the conceptual content needed to solve the problems in the interview protocol but will have had limited time and experience to develop expertise.

We decided to collect interviews until we reached saturation of the data. We define saturation as seeing no new unique knowledge, strategies, or sketching behaviors. From the first round of recruitment, we interviewed 15 students at Midwestern University in the spring of 2016. We decided to collect more interviews only if we did not observe saturation. During the analysis of these first interviews, we observed saturation of our coding scheme after the eighth interview. Between the ninth and fifteenth interviews, we primarily observed nuances of the core observations but no new behavior.

2.3 Data Collection

To understand students' knowledge and reasoning of equilibrium during sketching, we conducted think-aloud interviews with screen captures of the students' sketches. Think-aloud interviews have been used to provide rich descriptions of novices' and experts' mental models of concepts in other fields such as chemistry[28], physics[29], and circuits[30]. During the think-aloud interviews, we instructed participants to verbalize their thought process as they sketched their shear force diagrams for various beams and loads (see Figure 3 for sample problems from interview protocol). Students received no feedback on the correctness of their answers during the interview but were frequently asked to explain what they were doing and why they answered the way they did. Before the interview, we briefed the participants on the goals of the study and allowed a debrief session at the end to discuss any of their answers. All participants were paid for their participation in one hour interviews and gave written consent to be interviewed under IRB approval (Midwestern University).

2.4 Interview Questions
All students were given the same interview protocol, which included training tasks and sketching tasks. The training tasks were designed to get students familiar with using the stylus (and speaking aloud) and included drawing shapes, writing the alphabet, and writing numbers 1-10. The students were also asked to reproduce a picture of a partially peeled banana. The sketching tasks were broken into two parts with the same four problems in each part. In all cases the students were asked to sketch the shear force within the rigid body. Problems 1-3 and 4-6 had massless beams while problem 4 and 8 had beams with mass but no other applied loads. Problems 1-4 asked students to sketch these diagrams without performing any calculations while problems 5-8 asked the students to first calculate the reaction forces and then sketch their shear force diagrams. All problems were designed to be solvable within the interview per procedures detailed in section 2.1 and were analogous to problems participants solved during class.

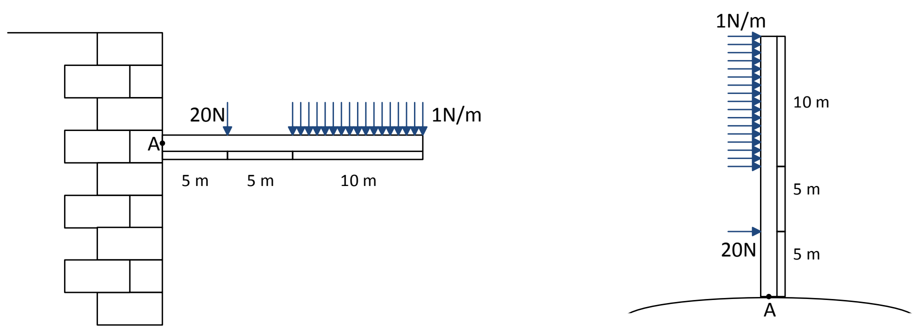

Figure 3: Problems 1 and 3 from the interview protocol. The beams are conceptually analogous but contextually different (orientation is different).

2.5 Thematic and Constant Comparative Analysis
The competing theories for students' knowledge structure have focused on the knowledge revealed from fixed diagrams or set visualizations. Knowledge revealed through the students co-creating the display via sketching is not as well explored. We therefore chose a data-driven approach to investigate the interplay between students' sketching and their conceptual knowledge. Consequently, we used constant comparative analysis, relying primarily on the data to inform our understanding of how existing theories may apply in this context. This technique has been used in the literature for inductive theory building though "categorizing, coding, delineating categories and connecting them"[22]. Initially as the data for each student is coded, emerging hypothesis are entered into an audit trail. Then as each new student's interview is coded and sorted, the codes will be compared to the old data as well as against the emerging hypothesis in the audit trail. By the time we reached saturation, this process should ensure a theory that is robustly based in the data. This method of analysis has been shown to be both internally and externally valid. We established the granularities of analysis (categories) based on pilot studies and analysis [32]. The four categories used in this study are as follows:

Subject classifies each subject as uninformed (n=3), novice (n=9), or advanced novice (n=3) based on the accuracy of their solutions over the course of the interview. Uninformed students failed to correctly solve most problems and demonstrated numerous misconceptions. Novices solved some or most problems correctly and demonstrated some misconceptions. Advanced novices solved most or all problems correctly and did not reveal misconceptions. The designation was made by consensus among the researchers.

Problem includes a description of the overall strategies participants used when sketching their shear force diagrams. For example, students either applied equilibrium to the whole beam but not in sections vs expressing no equilibrium reasoning.

Translation includes when a student uses information from one representation to construct another. In this work, hybrid translations refer to when a student uses multiple representations to construct another. For example, a student could use the schematic plus numeric calculations to construct their shear force diagram.

Statement includes correct and incorrect concepts students used either during their sketching behavior or their auditory explanation of their sketches. These effectively document a student's understanding, or misunderstanding, in the moment. For example, a student might use knowledge of specific joint types to determine start and end points for their shear force diagrams.

Constant comparative analysis allows us to gather evidence for or against emergent hypotheses that will ultimately compose our theory of how students interact with statics concepts during sketching. Rather than constructing a coding scheme from the data, the goal of this analysis technique was to provide a rich description of the various observations we observed and compare them across different levels of granularity. For example, comparing strategies used by uninformed students vs. the advanced novices. Emergent themes from this work are holistic descriptions of knowledge and reasoning that hold over multiple granularities and participants. This paper presents one piece of what will later be a theory concerning students' statics knowledge and reasoning revealed through their sketches.

2.6 Trustworthiness and Reliability

We used multiple coders, an audit trail, and negotiation to maintain trustworthiness and reliability in our analysis. Two researchers initially coded the interviews to get familiarity with the data and to keep track of any emergent hypotheses. We negotiated any disagreements in either the name of the code or the timestamps the code encompassed. Only agreements were kept during the comparing granularities and theme building part of the analysis. Without a formal coding scheme, we chose to use an audit trail during the initial individual coding and the negotiation phase to keep track of agreements, disagreements and emerging hypotheses. Once a theme or hypothesis was entered into the audit trail, each successive interview was compared to that new theme/hypothesis to test its validity.

3. Results: Equilibrium *is* Sum of External Forces = 0

The first theme that emerged from the data was *Equilibrium* ***is*** *the Sum of External Forces = 0*. This theme articulates that students invoked the idea of equilibrium as a mathematical exercise for finding external reaction forces at joints and not a tool to reason about a beam's external and internal behaviors. This theme was revealed through the following findings that we describe below. First, students predominantly applied equilibrium when calculating reaction forces, but rarely when analyzing sections of beams (if they did at all). Second, students rarely used cuts and summing of the forces about that cut to find shear forces. Third, advanced novices used sum of external forces reasoning when sketching without calculations whereas increasingly novice students increasingly failed to use sum of forces reasoning when sketching.

3.1 Students predominately applied equilibrium when calculating reaction forces

Table 1: List of codes concerning equilibrium, including their granularity and definitions

Code	Granularity	Definition	Abbreviation
Equilibrium for Full Beam and Sections	Problem	A student applies equilibrium via sum of forces = 0 or equilibrium reasoning over the full beam and within sections of the beam via "imaginary cuts"	Full and Sections
Equilibrium for Sections Not Full Beam	Problem	A student applies equilibrium via sum of forces = 0 or equilibrium reasoning within sections of the beam via "imaginary cuts" but not the full beam	Sections Not Full Beam
Equilibrium for full beam not sections	Problem	A student applies equilibrium via sum of forces = 0 or equilibrium reasoning over the full beam but not within sections of the beam via "imaginary cuts"	Full Not Sections
Reasons from Sum of Forces – Point Based	Problem	A student incorrectly applies equilibrium via sum of forces = 0 on a point-by-point basis rather than over the whole beam or within sections of the beam	Point-based Sum of Forces
Never Discusses Sections	Problem	A student does not make any explicit cut of the beam to analyze the sum of the forces inside the beam	Never Discusses Sections
No Equilibrium Reasoning	Problem	A student does not provide evidence of equilibrium reasoning within the main problem solving strategy	No Equilibrium Reasoning
Correct Sum of External Forces = 0 Reasoning	Statement	A student applies the sum of external forces = 0 procedure or concept correctly	Correct Sum of Force
Incorrect Sum of External Forces = 0 Reasoning	Statement	A student applies the external forces = 0 procedure or concept incorrectly or fails to do so entirely	Incorrect Sum of Force
Equilibrium Reasoning	Statement	A student justifies discussion about a reaction force or shear force by appealing to the lack of acceleration of the beam	Equil Reasoning

Table 1 presents a sub-section of our full codebook that answer this paper's research question: how do students apply the concept of equilibrium during problem solving? These codes reveal the range of students' use of equilibrium reasoning during problem solving. Students appeared to more readily use equilibrium as part of their problem-solving strategy during the computation problems than when sketching without computations. Because not every student solved every problem, we calculated the number of times we applied a code to each problem that a student solved as a percentage of the total problems each student completed. We report these percentage counts rather than raw frequency counts. The percentage counts were then averaged for each expertise level. This averaged percentage per expertise level is shown in Figure 4 for both the sketch and computation problems.

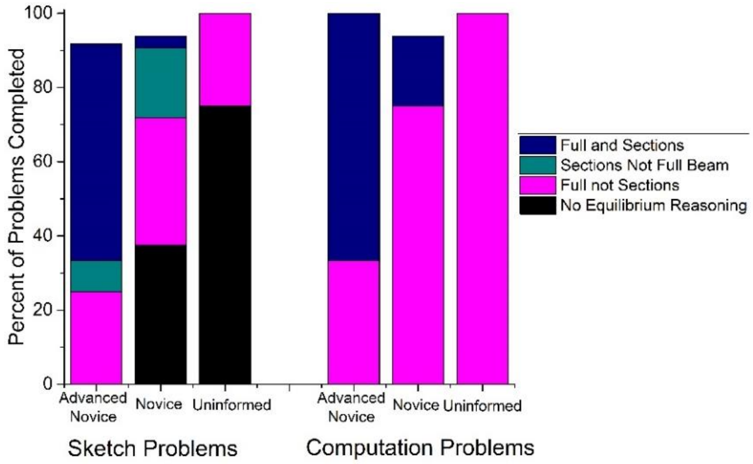

Figure 4: Number of problems where a specific equilibrium code was used was used as a percentage of total problems completed averaged over each expertise level in sketch and computation problems.

From Figure 4, we observed that the novices and uninformed participants applied equilibrium over the full beam less than 40% and 20% of the time respectively during the sketch problems but 100% of the time during the computation problems. This observation is amplified by seeing that the novice and uninformed participants exhibited no equilibrium reasoning during 50% and 75% of the problems respectively during the sketching problems. This is in sharp contrast to the advanced novice students who predominately used equilibrium for full beam and sections (~58% of the sketch problems) and used some form of equilibrium reasoning on almost all sketch problems. In the computation problems, however, every student exhibited some sort of equilibrium reasoning with equilibrium over full beam and not sections as the predominant equilibrium solution strategy. We argue that discouraging students from performing calculations caused students with weaker conceptual understanding of equilibrium to abandon the concept of equilibrium entirely. This observation is corroborated by behaviors such as participants not drawing reaction forces when sketching but drawing them when performing calculations. Asking students to calculate reaction forces explicitly, not surprisingly, prompted students to perform the sum of external forces = 0 over the whole beam.

The disconnect between the sum of forces computation and the concept of equilibrium is further revealed by the disparity between the number of occurrences of the *Correct Sum of External Forces = 0 Reasoning* code (appearing in 14/15 interviews) compared to the *Equilibrium Reasoning* code (appearing in only 10/15 interviews). This disparity shows that participants did not consistently connect the calculation to its physical interpretation.

3.2 Students rarely made cuts or summed the forces about that cut to find shear forces
Figure 5a shows the percentage of problems that students made a cut (as recorded by the code M*akes a Cut*) at any point when solving each problem. From this figure, we find that advanced novices made cuts in more problems (66%) than the novices (10%) and the uninformed students (0%). Making a cut is not strictly necessary to solve these problems correctly, but invoking the analysis tool of cuts reveals what participants thought was important for solving these problems. Without making a cut, the students cannot recognize the concept of equilibrium *inside* the beam, much less perform sum of forces calculations. This point is emphasized in Figure 5b, which shows the percentage of problems where students did not discuss sections as part of their solution strategy. Since shear force is an internal force meant to restore mechanical equilibrium, it is therefore likely that most our participants (and potentially most engineering students in statics courses in general) do not understand that shear force is a reactionary internal force. Without making a cut, students have little or no reason to consider mechanical equilibrium inside the beam. Interestingly, prompting students to first calculate the reaction forces neither prompts nor dissuades students from making cuts or talking about sections as part of their problem solving. Instead, students place more emphasis on equilibrium over the whole beam (Figure 4).

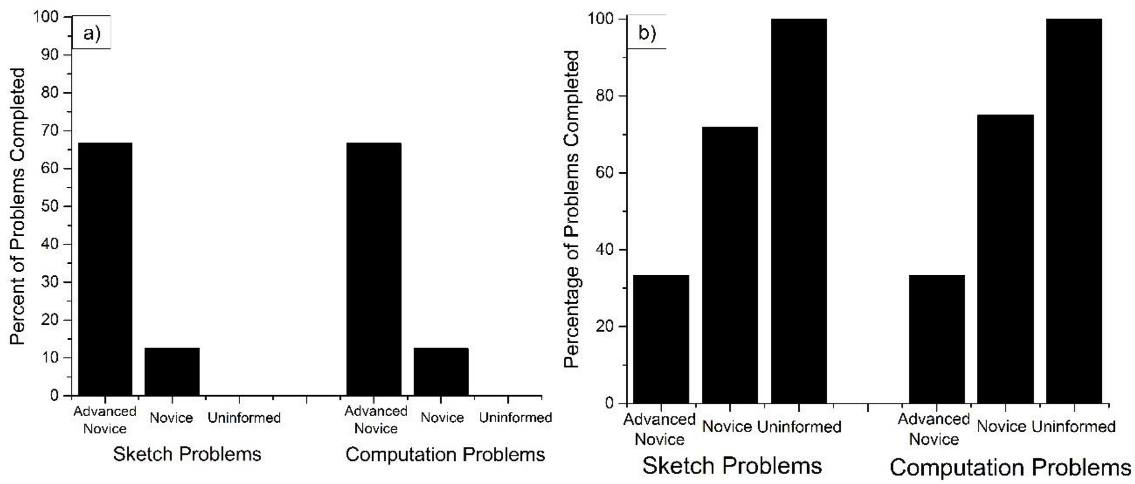

Figure 5: Number of problems where the a) *Makes a Cut* and b) *Never Discusses Sections* code was used as a percentage of total problems completed averaged over each expertise level in sketch and computation problems.

3.3 Increasingly novice students increasingly fail to use sum of external forces = 0 during sketch problems but not computation problems

During analysis, we observed that students tended to use sum of forces = 0 calculations or equilibrium reasoning more readily when calculating reaction forces than when sketching their shear force diagrams. This trend was further revealed when looking at the total instances of both sum of forces = 0 codes as well as equilibrium reasoning, which occurs when students justify having a reaction force or shear force to prevent the beam from moving. Figure 6 shows the percentage of instances belonging to each of those three codes averaged over each expertise level. As can be seen, the percentage of instances for correct sum of external forces = 0 and incorrect sum of external forces = 0 trend opposite of each other with increasing expertise. Students know they need to use sum of forces = 0 but tend to get it wrong unless they are told to first calculate the reaction forces. In the absence of calculating a reaction force, the novice and uninformed students usually began and ended their shear force diagrams at zero. This is interesting since the reaction forces should only dictate where the diagram should start (or end in problems 2, 4, 6 and 8) and not what the rest of the diagram would look like. Advanced novice students were usually able to recognize the sign of the reaction forces and thus where the shear force should approximately begin by using a sum of external forces = 0 reasoning without explicitly doing the calculations.

Another interesting trend is the use of equilibrium reasoning across the type of problem. In the sketch problems, equilibrium reasoning increases with increasing expertise but is not used as much as sum of forces = 0 reasoning. Once the students move onto the computation problems, however, use of equilibrium reasoning decreases among most of the students. This implies to us that students are heavily relying on the mathematical equations of equilibrium to justify their shear force diagrams. The students could either be more comfortable with the math or view equilibrium more as a mathematical exercise. Given the prevalence of the preference for numeric code and the lack of internal equilibrium reasoning, the former is probably more likely for advanced students while the latter is more likely for the novice and uninformed students.

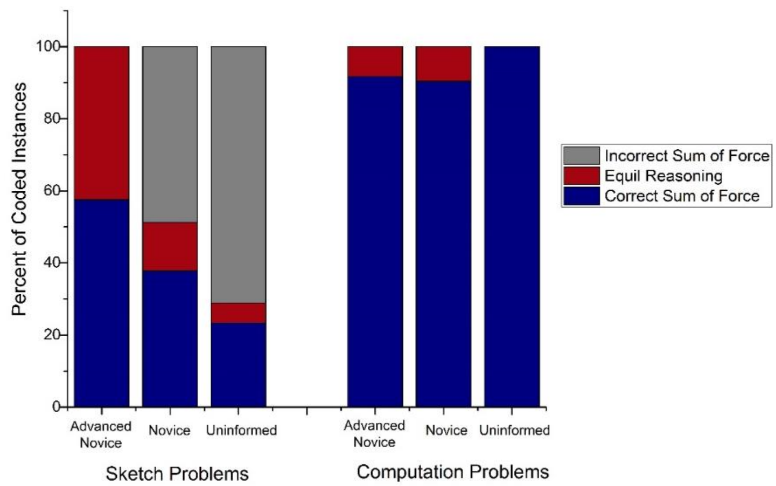

Figure 6: Number of instances where a specific external equilibrium code was used as a percentage of the total instances of external equilibrium codes averaged over each expertise level in sketch and computation problems.

4. Limitations

We believe our study's biggest limitation was not randomizing the order of the problems within each half of the interview. This matters because some of the students could have been learning as they solved the problems, partially explaining the increased use of equilibrium codes during the computation portions of the interview. Additionally, we observed that the way students generally approached the problems emerged within the first two problems and was generally solidified by the third problem. Had we randomized the order, we could more confidently say whether specific affordances of the problem influenced the way students used equilibrium in their problem solving. We did find (see Figure 5) that students' either use, or fail to use, the method of sections over the course of the entire interview. This consistency alleviates some concern about students learning during the interview.

5. Relevance of Results to the Literature

Students' approach to internal equilibrium as a mathematical exercise rather than a conceptual tool to understand internal forces such as shear force is also observed in the literature. Litzinger et. al. found that both strong and weak students tended to rely on procedure in their problem solving (e.g., using memory to determine what reaction forces are present in different joints)[19]. Similarly, Anderson and Taraban[33] identified procedural preference by showing students wrote and solved their equilibrium equations in an order presented in statics courses (forces first, then moments) but opposite to how experts solve the problem (moments first, then forces). Students in their study solved every equilibrium equation they wrote even when some of the equations were not necessary to answer the problem. The preference for procedure was observed more frequently in low performing students, which correlates with our observations. Our work builds on this literature by demonstrating procedural preference for internal equilibrium.

While our presented analysis does not explicitly address the fragmentation or coherence of students' knowledge, there are hints toward both perspectives. When applying equilibrium (Figures 4 and 6), students used different strategies between the sketch and computation problems. This shift is present across all expertise levels, but is most pronounced among novices and uninformed students who used no equilibrium reasoning in sketching contexts, but always used some form of equilibrium reasoning in the computational contexts. In contrast, Figure 5 suggests that some approaches/knowledge are use independent of context. Whether students made cuts or discussed sections depended only on expertise, not context of the task. It is possible that students' reasoning is more consistent with respect to the concept of cuts or that students had simply failed to learn the technique when they were taught. In either case, it is important to explore what clues within the context of the task encourages or discourages a student's approach. Observations presented in this work are insufficient to fully explore the contextual dependence of students' knowledge. Further analysis of the data will more fully explore the relationship between context and students' knowledge.

Both the students' preference for procedure and the context dependence lead us to believe that there could be broader curricular design problems that need to be addressed. For example, there is a push to incorporate computational tools. This work suggests that students' reliance on procedures to solve problems could be disrupted by the increased use of computational tools. Students' learning could decrease as their knowledge becomes tied to procedures in software or their learning could increase as students need to use more conceptual knowledge to grasp what the software is doing and they cannot use mathematical procedural knowledge as a crutch. Future research can explore this dynamic.

6. Conclusions

Students in this study view mechanical equilibrium as 1) only external to the material via sum of external forces = 0 and 2) a mathematical exercise to be solved rather than a tool to understand and make decisions about a static system. The *Equilibrium* ***is*** *the Sum of External Forces = 0* theme applies in varying degrees to all expertise levels, but for novices and uninformed students especially. Because students have reduced the concept of equilibrium to a computational task, they reliably solve sum of forces equations, but fail to discuss the overarching concern of statics – systems that are not accelerating. In contrast, students invoke the concept of equilibrium more when discouraged from performing sum of forces calculations. The contextual dependence of students' problem solving strategies is mixed but does seem to give credence to diSessa's knowledge-in-pieces view. More detailed analysis of the knowledge students use when sketching is needed to fully explore the question of contextuality of student knowledge in statics. Future work could explore interventions that increase students' awareness of this broader application of equilibrium. For example, exercises that focus on analyzing pre-made cuts or explaining the internal reactions of beams during writing exercises may help make this application of equilibrium inside the beam more salient.

7. Acknowledgements

We would like to acknowledge Lance Pittman for helping collect the interviews. We would also like to acknowledge the National Science Foundation for funding this work (Award# EEC 1429348).

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