Work-in-Progress: How Do Engineering Students Misunderstand Number Representations?

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Abstract – **It is often taken for granted that students who pursue engineering in college are "numbers people" who excel at mathematics. We believe that there is reason to doubt this assumption based on our studies of student misconceptions in first and second year engineering students. We interviewed and tested undergraduates in electrical and computer engineering or in computer science at the University of Illinois at Urbana-Champaign who had just completed a first course in digital logic. These interviews and tests required students to solve problems relating to binary arithmetic and number base conversions. We present our early findings from these interviews and assessment tools.**

Index Terms – Learning models, number representations, digital logic, mathematics misconceptions

INTRODUCTION AND BACKGROUND

New methods of instruction are continually proposed, but it is difficult to compare the merits of these methods without the proper assessment tools to objectively or empirically compare them. Developing reliable, validated assessment tools is time consuming and difficult, but their development creates the possibility of accelerating later improvements in instruction and education research [1].

This study is part of an ongoing research project to develop standardized multiple-choice conceptual assessment tools, called concept inventories (CIs) for introductory courses in computer science and engineering. Creating CIs will provide one way for instructors and researchers to directly compare new teaching methods and decide which methods to adopt. The Force Concept Inventory (FCI) provided much of the impetus for the adoption of interactive engagement pedagogies in physics [2].

To provide similar stimulus to computing education, we are creating the Digital Logic Concept Inventory (DLCI). The DLCI tests students' conceptual understanding of four topics in digital logic. One of these topics is number representations and binary arithmetic [3]. This topic was previously rated by a panel of experts to be the easiest topic covered in a digital logic course [4]. To check the validity of this rating, we are searching for student misconceptions about number representations. These misconceptions will be used to create wrong answer choices for multiple choice questions on the DLCI.

In previous work, researchers have found that even freshmen engineering students and professional engineers struggle with common mathematics misconceptions [5], [6]. Other research has demonstrated the difficulty of gaining an accurate intuitive sense of the nature of decimals and fractions [7]. These studies have shown that misconceptions surface frequently when subjects perform specific tasks. These tasks include comparing two numbers to determine which is greater or manipulating symbolic notation.

In this study, we asked engineering students to complete number comparison tasks and other conceptual questions to discover what misconceptions they still possess. We present the misconceptions that we have found so far.

METHODOLOGY

To learn how students think about number representations, we interviewed 27 students, administered the DLCI to more than 350 students, and studied the final exam papers of 20 students. We interviewed undergraduates in electrical and computer engineering and in computer science at the University of Illinois at Urbana-Champaign who had just completed a first course in digital logic. During interviews, students solved traditional number representation problems while vocalizing their thought processes. Final exams were collected from the same pool of students with their consent and IRB approval after the semester had ended.

To analyze our interviews and student exams, we used a research paradigm called *grounded theory*. Grounded theory is a rigorous qualitative research methodology that should be used when there are no specific theories about how people think in a given context. This structured analysis method helps new theories to emerge from generated data through an iterative, open-ended inquiry process [8]. Our methodology is described in detail in our previous work [9].

The DLCI was administered to all students in four courses over two semesters shortly before their final exams. The DLCI contains six questions (out of 24) that probe a student's grasp of number conceptions. We are performing statistical analyses on students' responses to these questions to supplement the findings from our interviews.

QUESTIONS AND EARLY FINDINGS

During interviews we asked students to complete a worksheet that required them to select which of two numbers expressed in different bases was greater or to decide whether the numbers were equal. Students solved these types of problems as a warm-up to accustom them to thinking-aloud while solving problems. Students were also asked to convert numbers between two bases as well as add and subtract two binary numbers, two numbers represented in two's complement or two numbers in signed magnitude.

We asked students several conceptual questions about why we use different number systems (such as binary, hexadecimal, or two's complement) in digital systems. For example, we asked students to answer, "What are some advantages of using two's complement number representation?" or "Why do computers use binary representations of numbers?"

The final exams were written by the course instructors (not the authors). The exams included number base conversion problems and bit-wise manipulation of numbers. Bit-wise manipulation of numbers is the process by which a computer performs logical or arithmetic operations to binary numbers. These manipulations include bit shifts and changing individual bits of a number.

The exams and interviews showed that students are generally adept at converting numbers between common bases. While students could convert numbers well, they struggled to explain why conversion techniques worked. Several students said "I don't know" or "This is just the way we were taught to do it," when asked for an explanation. A couple of these students forgot one part of the conversion procedure during interviews. None of the students who forgot the procedure was able to figure out a technique to convert the numbers. The majority of students could perform the procedures we asked them to complete, but many could not describe the conceptual underpinnings that gave these procedures meaning.

In a similar manner, when students were asked to explain how two's complement number representation works, many would simply cite the operation of converting a positive number into a negative number in two's complement representation (i.e., complement the bits and add one). When asked to explain why the conversion works, students were again at a loss. We believe that much of student knowledge of numbers and bases is grounded much more in operations than in conceptions. Students do not *understand* numbers despite their ability to manipulate them.

Our suspicions were confirmed when we discovered that students fail at basic number comparison tasks more than expected. On the DLCI, students are asked to order three numbers, $(1.100)_2$, $(1.4)_{10}$, and $(1.5)_{16}$, from least to greatest. Only 45% of DLCI examinees have successfully completed this task. This success rate is much lower than the success rate on any other DLCI questions.

During interviews, we observed that students incorrectly order the above numbers according to the size of the base and reason that since 16 is greater than 10, then $(1.5)_{16}$ is greater than $(1.4)_{10}$. The reasoning of these students is reversed as 1/16 is less than 1/10.

We have observed that students struggle to articulate why we use specific bases in different situations. Students have argued that computers must use binary because binary "makes truth tables easier." Others have asserted that two's complement number representation allows us to represent more numbers than any other representation when using the same number of bits. These comments are either false or are overly-broad generalizations.

CONCLUSIONS AND FUTURE WORK

Our interviews and assessments of engineering students have demonstrated that our students do not necessarily have a strong conceptual understanding of numbers. Conceptual weaknesses can be exposed by simply changing the way that numbers are represented.

We have presented a few findings in the paper, but we have also observed many other conceptual and operational difficulties that students possess when manipulating numbers that we still do not currently understand. For example, we still need to discover why students struggle to comprehend bit-wise manipulations of binary numbers. We hope that further interviews and administrations of the DLCI will yield more information about what conceptual difficulties are prevalent among our engineering students.

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