

Screening, Monitoring, and Sorting across Occupations

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Abstract Why are some occupations subject to wage competition for talent, while others are not? In this paper I explore the relationship between the tasks performed in an occupation and how much the employer values ex ante signals of talent. For occupations in which employers can easily observe a worker's talent on the job, the employer is less willing to pay a premium for high-expected-ability workers, regardless of the productivity of the occupation. I develop a frictionless matching equilibrium in which firms hire from a common pool of workers, and I show that matching is negative assortative between worker expected ability and the observability of talent on the job. In the second part of the paper, I develop an efficiency wage model, and show that occupations that are able to provide better information on worker effort optimally match with higher ability workers and pay higher wages. This mechanism indicates that in a unified labor market, the workers with high expected ability will match with jobs that have high productivity, poor on-the-job learning about worker ability, and strong information about worker effort. Workers with poor ex ante signals about ability become stuck in low-wage occupations, regardless of these workers' actual productivity.

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1 Introduction

Some occupational labor markets operate like auctions for rare art, with firms bidding to woo the most desirable candidates. Other occupational labor markets work more like commodity markets, in which firms focus instead on attracting competent labor at the lowest

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cost. For instance, in a 2013 Business Week interview ([Kimes, 2013](#)), Caterpillar CEO Doug Oberhelman explained that the company was fighting with the unions to keep labor costs down because "we have to be competitive if we're going to win." Later, when explaining his own rising compensation, he explained "My salary [has] to be competitive. [...] If the CEO of a company is paid \$10,000 a year, chances are he or she is gonna go somewhere else to make more money." Business leaders often discuss competitive labor markets to justify both high management salaries and low production-worker wages. What leads to such different recruitment behaviors within the same firm? Why might market forces lead to such different outcomes for different types of jobs?

Talent markets are often modeled as assignment problems. If workers and firms are heterogeneous in ability and productivity, efficient assignment leads to positive assortative matching between workers and firms. A competitive labor market decentralizes this assignment through wages, leading to potentially large variation in pay. This was the conclusion of [Terviö \(2008\)](#), which showed that CEO compensation was consistent with an assignment model.

More difficult to explain is the *lack* of talent competition further down the occupational hierarchy. All jobs include variation in worker ability and productivity. Case studies have shown significant variation in productivity between employees; for instance, [Hamilton, Nickerson, and Owan \(2003\)](#) found 48% variation in productivity between the 75th and 25th percentile of productivity in garment workers in their study firm. In the famous [Lazear \(2000\)](#) study of performance incentives for windshield installers, under hourly wages the average worker installed 2.7 windshields-per-day, with a standard deviation of 1.4. Thus, just because the a job is lower-skill or lower-pay, there are still gains to be made from populating a workforce with the most talented workers.

In this paper, I argue that information and uncertainty play a crucial role in how much a firm is willing to pay for talent. As evidenced by the Oberhelman quote, there is a sense that the cost of hiring a low-quality CEO is extremely high. No firm would be willing to

give an unknown worker the helm for a day just to see how it goes. However for many other jobs, this is exactly how hiring is conducted. For instance, consider a dishwasher. The best way to evaluate if someone is able to perform the job is typically to give them a shift and judge the results.

Jobs are heterogeneous in how difficult it is to learn about a worker's ability on the job. Some jobs, like a concert pianist or a janitor, provide clear information quickly about a worker's talents. Other jobs, such as a night watchman or a film director, may provide infrequent and costly signals about the worker's ability. If you only learn your watchman is sleeping on the job when something is stolen, the risk of trying a worker out on the job is high.

When it is costly to decide ex post if a hire is a good fit, ex ante signals take on additional value. Firms are willing to compete for the most talented applicants to reduce the chance they will hire a poor fit. This bids up the wages for high-expected-ability individuals, and in extreme cases can lead to talent wars. In a labor market in which workers sort between occupations with different characteristics, these low-information occupations match with higher-quality workers than would be optimal by sorting on productivity alone.

I develop a two-period model in which worker ability is unknown. Jobs vary in how likely it is for the firm to learn the workers type after the first period. Learning is private to the worker-firm pair, allowing firms to capture rents on high-ability workers. When firms can easily learn about a worker's type, the cost of hiring a low-ability worker is lower since the firm can fire the low-ability worker and hire a new worker.¹ This affects how much the firms are willing to pay for a worker with a strong ex ante signal about expected ability. I show that if labor markets are segmented by occupation, occupations in which firms are better able to observe worker ability are characterized by lower wages and less wage inequality. Next, I show that if workers are allowed to sort between occupations, high-expected-ability

¹An alternative to this somewhat draconian framework is to allow firms to provide training to the low-skill worker, but only be able to provide such training if poor performance is observed. This results in isomorphic predictions.

workers sort to occupations with lower observational capacity. This raises their wages, and increases wage inequality.

When firms learn about worker ability, the worker is unable to manipulate the firm's learning. In the second part of the paper, I focus on the feedback between firm learning and worker effort. In a simple efficiency wage model with sorting, workers with high expected ability are more valuable in jobs that provide more information about the worker's actions. This is because better information about effort affects the worker's behavior, which in turn affects total productivity. However, decentralized competitive wages cannot always induce efficient sorting: if the probability of learning about a worker's effort is too low, workers may choose to match with lower-information jobs.

This paper is related to an old literature on assignment in labor markets. [Sattinger \(1993\)](#) provides a nice survey of a literature dating back to [Tinbergen \(1956\)](#) and [Roy \(1951\)](#). These models are characterized by the productivity of a worker-job match depending on worker and firm quality. Each job is discrete and can match with at most one worker. When productivity is complementary, efficient matching implies positive assortative matching. However, there are generally many wage distributions that can decentralize such assignment.

The role of observing a worker's ability has played a role in the discrimination and promotion literature. [Waldman \(1984\)](#) shows if firms have private information about a worker's ability, there is an incentive to hide such information by inefficiently choosing not to promote. [Milgrom and Oster \(1987\)](#) build upon this idea, showing that if there is an "invisible" class of workers for whom there is less public information about their ability, firms are more likely to hide these workers to avoid paying competitive wages. In these papers, a worker's current employer learns perfectly about his ability (either after one period ([Waldman, 1984](#)) or immediately upon hiring ([Milgrom & Oster, 1987](#))); thus there is no dimension for sorting between occupations based upon the ability of the firm to learn. In contrast, in this paper, jobs vary in how well the firm can learn about the worker's ability, which leads to sorting and wage differentiation.

In the second part of the paper, the simple efficiency wage model is related to that of [Shapiro and Stiglitz \(1984\)](#). However, in this two-period model, firms provide bonuses to induce effort, rather than the cost of unemployment serving in this function as in the Shapiro–Stiglitz model. The idea that the ability to monitor a worker is related to the job performed is fundamental to the multi-task literature, including classic papers such as [Holmstrom and Milgrom \(1991\)](#) and [Feltham and Xie \(1994\)](#). I am not aware of any papers that consider the sorting between jobs based on the strength of contracts inherent to the job. Empirically, [Lazear \(2000\)](#) finds positive selection on the margin of selecting into high-powered incentives.

Finally, this article owes an intellectual debt to [Wilson \(1991\)](#), who explores the relationship between observability and management in government agencies. Wilson demonstrates that, in the public sector, many agencies are charged with tasks that are difficult to monitor. For some agencies, it is difficult to observe day-to-day activities. In other agencies, it is difficult to measure the results. Many of the concerns Wilson brings up are common to private sector workers, and in turn affect wages, effort, and sorting.

2 Ability-Monitoring Model

Consider a two-period model. Firms and workers match each period and produce. Labor is supplied inelastically. At the end of the first period, each worker and firm can decide whether to continue the employment relationship for the second period, or to dissolve the match. Each worker i has ability η_i which is initially unknown. η takes on two values, 0 and 1. Workers' true ability is initially unknown. Workers do not have private information about their own ability. All parties have the same information, and believe each worker is high type ($\eta_i = 1$) with probability q_i .

Each job is endowed with a probability x of learning the worker's quality. This can be interpreted in several ways. First, we can think of jobs as problem solving, along the lines of [Garicano \(2000\)](#). Jobs are defined as a sequence of tasks for the worker to perform, of

varying content. For each job, some tasks are easy, and workers of any ability can perform adequately. Other tasks are separating, with only the high-skilled workers able to complete them. In this framework, x is the probability that a separating task arrives in a production period. In some jobs, these separating tasks arrive frequently. An example might be a hairdresser: hair cuts may vary in their difficulty, but over the course of a few shifts, the salon should have an idea of the worker's ability. On the other hand, a night watchman will only have occasion to show his mettle in the case of a theft.

Alternatively, we could think of jobs as performing tasks that vary in duration. Jobs with tasks that are more frequently long duration have a lower probability of completion during the first period of match, making observation of the match productivity less likely. Regardless of the interpretation of x , it is important that it does not directly affect a worker's productivity.

The timing of market is as follows:

1. Workers and firms enter the labor market,
2. firms offer wage contracts such that the labor market clears,
3. workers produce and are paid,
4. each firm learns it's workers ability with probability x ,
5. firms decide whether or not to offer a wage contract for the second period and workers decide whether to accept or separate,
6. firms without a continuing worker make wage offers to workers (both non-employed and employed),
7. continuing employers can make a counter-offer to any outside offer, and finally
8. labor markets clear, wages are paid and workers produce.

Two pieces of information are private to the worker-firm pair: first, whether or not learning has occurred, and second, the result from said learning. Other firms can update their beliefs about the worker's ability based on the firm's behavior to keep the worker or let him go.

There are two types of workers, A and B , who vary in their ex ante probability of being high ability, with $q_A > q_B$, and both probabilities between zero and one. The natural interpretation of A and B are education levels, but this is not essential to the model. The product price is normalized to one, and a firm's profit is based upon the job's technology Y_j and the worker's productivity. Thus a firm's single period profit is given by $Y_j\eta_i - w_i$, for a worker of type i . When η_i is unknown, the firm's expected profit is $Y_jq_i - w_i$.

Wages are determined by a decentralized assignment process, in which firms offer competitive wage offers to induce high expected ability workers to join their firm. A -type workers are scarce, so the number of A -type workers in the labor market, N_A , is exceeded by the total number of jobs M . There are excess B -type workers, so $N_A + N_B > M$. We will assume that there are many surplus B type workers, so even if workers leave the market in the second period, there are still more workers than firms.² Workers will only accept offers that exceed their non-market option w_0 . Since low-ability workers produce nothing, it will always be optimal for a firm to separate with a worker that is known to have $\eta_i = 0$.

In the second period, each hiring firm's expected profit is given by $Y_jq_{2i} - w_{2i}$, where q_{2i} is the firm's belief that the worker is of high-ability, and w_{2i} is the second period competitive wage for the worker of type i . Note that q_i will be 1 if the firm has learned about the worker's type. In the first period, the expected profit from a job with observational capacity x hiring a worker of type i is:

$$\mathbb{E}\Pi_j(i) = Y_jq_i - w_{1i} + \beta[x_jq_i(Y_j - w_{2i}) + x_j(1 - q_i)(Y_jq_k - w_{2k}) + (1 - x)(Y_jq_i - w_{2i})] \quad (1)$$

where k is the worker the firm hires if the firm learns i is low-ability. Using Equation 1, we

²Sufficient is for $N_B + N_A > (1 + (1 - q_B)x_{max})M$, where x_{max} is the largest x in the labor market.

have the following lemma.

Lemma 1 *Holding wages constant, firm profit is increasing in worker's expected ability q_i .*

This is intuitive: the production is complementary between job productivity Y_j and worker ability η_i . The more likely the worker is high ability, the higher expected productivity.

From the worker's perspective, expected compensation from accepting an offer of a firm of type j is:

$$\mathbb{E}W_i(j) = w_{1i} + \beta(1 - x_j + x_j q_i)w_{2i} + \beta x_j(1 - q_i)w_0 \quad (2)$$

where the worker gets his outside option w_0 if the firm learns he is low-ability.

Finally, we can derive the net surplus from matching, which is the total produced by a match in the first period. Let Π_{2j} be expected productivity if the firm rematches in the second period. Then we can write

$$S(i, j) = Y_j q_i(1 + \beta) + \beta x_j(1 - q_i)(\Pi_{2j} + w_0) \quad (3)$$

Under efficient sorting, firms and workers match such that total output in the economy cannot be increased by switching any matches.

Lemma 2 *If all jobs in the labor market have the same x , efficient matching is positive assortative in worker expected ability and job productivity Y_j .*

This is akin to the typical assignment model. The complementarities between job productivity and worker expected ability means that efficient matching assigns the most productive jobs and workers together.

Proposition 1 *If all jobs in the labor market have the same Y , efficient matching is negative assortative in worker expected ability q_{1i} and the firm's probability of learning x_j .*

Proposition 1 shows that the easier it is for a firm to learn about a worker's ability, the less the relative surplus created by the match. From Lemma 1, firm expected profit

is increasing in worker expected ability, holding wages constant. Equivalently, total surplus $S(i, j)$ is increasing in q_i . However, efficient matching depends on the cross-partial: the value of ability increases *faster* for firms with low capacity to learn about the worker's type.

In order to focus on one-dimensional sorting based on x_j , for the rest of the paper we will restrict analysis to markets in which Y is constant across firms.

2.1 Second Period Matching and Wages

Observing ability is only valuable inasmuch as it allows the firm to find a new employee for the next period, thus in the second period x no longer plays a role. When Y is constant across firms, in the second period, all firms value workers the same, unless the firm has private information about its current worker. Since output is multiplicative with worker expected ability, firms prefer to match with the highest expected ability workers possible. This means that firms will preferentially match with workers that are known to be high ability first, then with A -type workers, who are more likely to be high ability, and lastly with B type workers. There are excess B -type workers that are not hired.

Equilibrium wages are constrained by three assumptions: first, that firms make wage offers, second, that firms are profit maximizing (so offer the minimum wage to attract the worker), and third, that a worker's current employer has the opportunity to match any outside offer. The third assumption drive the following lemma.

Lemma 3 *If the current firm is allowed to match any wage offer, outside firms never bid higher than the unknown-ability wage.*

Other firms observe if the first period employer chose to continue the employment contract with a worker, which makes it more likely that this worker is high-skilled. However, since the current firm has the opportunity to counter any wage offer, outside firms are only willing to offer the unknown ability wage. To see why, suppose an outside firm offers the wage based

on its belief about the worker's ability.³ If the current firm did not observe the worker's ability, it does not update its belief about the worker's ability, and is only willing to pay the unknown-ability worker wage. On the other hand, if the current firm does observe the worker's type, it would be willing to pay up to $(1 - q_b)Y - w_0$.⁴ Thus, when the current firm is able to counter any outside offer, the current firm will only do so if it knows the worker is high-type. This leads to a form of the 'winner's curse', in which the outside firm only gets the worker if it overbids. In this case, outside firms are only willing to offer wages according to the minimum expected ability that the current firm could know, which in this case is no additional information beyond the worker's ex ante type. This wage mechanism is also present in [Milgrom and Oster \(1987\)](#), and is one of the reasons these authors find that the labor market discrimination can persist for low expected ability workers, even after these workers are revealed to be high-skilled.

Thus we have the following equilibrium wages and sorting in the second period.

Lemma 4 *In the second period, if a firm observes a worker is high-ability, the firm keeps the worker. If a firm observes a worker is low-ability, the firm separates and hires a surplus B-type worker. Equilibrium wages are w_0 for B-type workers and $(q_A - q_B)Y + w_0$ for A-type workers. Equilibrium expected profits are $q_B Y - w_0$ if the firm does not know the worker's true type, $(1 - q_A + q_B)Y - w_0$ if the firm employs an A-type worker it knows is high ability, and $Y - w_0$ if the firm employs a B-type worker it knows is high-ability.*

Here we see that profit is higher for firms employing a high-ability B-type worker than a high-ability A-type worker.

³By Bayesian updating, outside firm's believe a worker's ability is $\hat{q}_i = \frac{q_i}{1 - x + xq_i}$, which is greater than q_i .

⁴In a market with heterogeneous Y , this can lead to somewhat complicated wages and sorting, if outside firms value a worker of unknown ability more than the current firm.

2.2 Single Occupation Labor Market

Now we consider the full two-period equilibrium for a segmented labor market consisting of a single occupation. All jobs are identical, so there is no sorting between firms. Using the equilibrium wages and profits from Lemma 4, we can re-write the expected surplus as follows:

$$S(i) = \beta x q_B Y + q_i Y (1 + \beta(1 - q_B x)) \quad (4)$$

Since q_B and x are each less than 1, we see that $S(i)$ is increasing with q_i . Thus in the first period, efficient sorting has A type workers matched first and B type workers comprising the balance of jobs.

Lemma 5 *The efficient sorting of workers and jobs is as follows:*

1. *All A-type workers are employed in the first period and $M - N_A$ of B type workers are unemployed.*
2. *In the second period, all workers that were revealed to be high-type or for whom no information was revealed stay with their first period employer.*
3. *Firms that learn their worker is low-ability separate and hire unemployed B-type workers.*

In order to decentralize this matching, wages must satisfy the following sorting and participation constraints. If worker i optimally matches with firm j , then

$$\mathbb{E}W_i(j) \geq \mathbb{E}W_i(k) \text{ for every other firm } k. \text{ (Worker } i \text{ sorting constraint)}$$

$$\mathbb{E}\Pi_j(i) \geq \mathbb{E}\Pi_j(k) \text{ for every other worker } k. \text{ (Firm } j \text{ sorting constraint)}$$

$$\mathbb{E}W_i(j) \geq (1 + \beta)w_0 \text{ (Worker } i \text{ PC constraint)}$$

$$\mathbb{E}\Pi_j(i) \geq 0 \text{ (Firm } j \text{ PC constraint)} \quad (5)$$

These conditions require that each worker prefers the expected compensation from his optimal match to every other expected compensation, including non-employment. Similarly, every firm must prefer its expected profit from the optimal match to every other expected profit, including not producing. Since A -type workers are scarce, firms will bid up A wages until firms' sorting constraint between A and B workers binds. Thus we have the following equilibrium wages.

Proposition 2 *Given the three assumptions about wage setting, there is a unique set of wages that decentralize the efficient sorting from Lemma 5 as follows:*

1. *All employed B -type workers receive their outside option each period, w_0*
2. *All employed A -type workers receive $w_0 + (q_A - q_B)Y - \beta Y x(q_A - q_B)(q_A + q_B - 1)$ in the first period and $w_0 + (q_A - q_B)Y$ in the second period.*

2.2.1 Wage Inequality and Talent

Now we can explore how wages and inequality vary with x . Suppose there are many segmented labor markets, each with a distinct pool of labor and each with a different occupation, with a distinct Y and x . Then we have the following results.

Lemma 6 *Worker expected compensation is (weakly) decreasing with x . Expected compensation (weakly) increases with Y and $q_A - q_B$.*

Since firms do not have to pay a wage premium to workers that are revealed to be high-skilled, workers only have downside risk from the firm learning their true type. B type workers are indifferent, since they are kept to their participation constraint and receive w_0 regardless of if they are employed or not.

Lemma 7 *Firm expected profit increases with x , Y , β and q_B . Expected profits are independent from q_A and decrease with w_0 .*

A-type workers capture all of the surplus that exceeds that from the firm matching with a B-type worker. Thus only parameters that increase profit from a B match will increase firm profit. From the last two lemmas, we have the following proposition.

Proposition 3 *For both types of workers, the worker share of the surplus decreases with x .*

Proposition 3 indicates that the more the firm learns about the worker's type, the larger the share of the match surplus the firm is able to capture. In an economy with several disjoint occupational labor markets with different values of x , there will be heterogeneity in the worker share of production depending on x , with jobs that provide the least information about worker true ability paying the largest share of production. In this case, it is likely that workers would choose to sort between jobs, a case we turn to next.

2.3 Sorting Between Occupations

Now suppose the market is comprised of two types of jobs, where as before, each firm consists of a single job. These jobs differ only in how easy it is to learn about a worker's type, with $\bar{x} > \underline{x}$. Second period wages and profits are unchanged, so the surplus is again given by Equation 4, thus efficient sorting is negative assortative between x_j and q_i . In order to decentralize this matching, we need to derive firm wage offers and workers choices between contracts. Note that since the jobs differ in x , the same wage offer in dollar terms is less valuable at a high x job, since the worker has a higher risk of separation. To reduce multiplicity of cases, suppose the number of low-observability jobs exceeds the number of A-type workers, so $M_{\underline{x}} > N_A$. Further, suppose B-type workers are again in surplus, so $M_{\underline{x}} + M_{\bar{x}} < N_A + N_B$. Thus we have the following for equilibrium sorting:

Lemma 8 *The efficient sorting of workers and firms is as follows:*

1. All A-type workers are employed with \underline{x} firms in the first period, $M_{\underline{x}}N_A$ B-type workers

are employed in \underline{x} firms, $M_{\bar{x}}$ B-type workers are employed by \bar{x} firms, and the balance of B-type workers are unemployed.

2. In the second period, all workers that were revealed to be high-type or for whom no information was revealed stay with their first period employer.
3. Firms that learn their worker is low-ability separate and hire unemployed B-type workers.

Now we can derive equilibrium wages.

Proposition 4 *Equilibrium wages that decentralize efficient sorting from Lemma 8 are as follows:*

1. All employed B-type workers receive their outside option each period, w_0
2. All employed A-type workers receive $w_0 + (q_A - q_B)Y - \beta Y \underline{x}(q_A - q_B)(q_A + q_B - 1)$ in the first period and $w_0 + (q_A - q_B)Y$ in the second period.

Since A-type workers are scarce, wages are bid up until \underline{x} firms are indifferent between A and B type workers.

3 Effort-Monitoring Model

So far, we have seen that the more information a job provides about a worker's ability, the less the firm is willing to pay for talent. Now we want to consider the case of efficiency wages, to see the relationship between monitoring effort, wages, and sorting. Suppose workers are again heterogeneous, but they are of known ability η_i . This is for simplicity, since firms will not be learning about worker ability. Each period, a firm's profit is given by:

$$Y_j \eta_i (1 + e_i) - w_i \tag{6}$$

where Y_j is the job productivity, i is the worker type, e_i is the effort the worker puts forth, and w_i is the wage. Each period, firms compete for workers, so wages each period can be no less than the market wage, however firms can pay additional wages. In particular, we will allow firms to pay a contingent bonus b_i in the second period, to induce workers to put forth effort. Effort is costly to workers, with $c(e) = \frac{1}{2}e^2$. Thus workers will only put forth effort if they believe it will increase their future pay. After the first period, the firm observes whether or not the worker put forth effort with probability p_j . Thus, the firm and worker write a contract before the first period, in which the firm specifies the amount of effort the worker should perform and the wage payments w_{1i}, b_i, w_{2i} . If the firm observes the worker did not put forth effort, the firm separates and hires another worker. However, the firm cannot prevent the worker from accepting another job, so the penalty for shirking is limited to the lost bonus payment. Further, wages are constrained each period by the minimum wage, w_0 . This set-up is akin to standard efficiency wage models, except that firms are allowed to offer (and commit to) different wages in the first and second period.

In the second period, there is no incentive to put forth effort, so output is just a simple assignment problem, based on worker ability η_i and job productivity Y_j . We will derive wages more concretely later, but for now let Π_{2j} be the firm's profit after optimal assignment, and w_{2i} be the worker's wage. Since output is $Y_j\eta_i$, assignment is positive assortative.

In the first period, the firm knows it will receive Π_{2j} in the second period. The firm's problem is as follows:

$$\max_{w_{1i}, b_i, e_i} Y_j\eta_i(1 + e_i) - w_{1i} - \beta b_i + \beta\Pi_{2j} \quad (7)$$

such that

$$w_{1i} + \beta(b_i + w_{2i}) - \frac{1}{2}e_i^2 \geq w_{1i} + (1 - p_j)\beta b_i + \beta w_{2i} \quad (IR_i)$$

$$w_{1i} + \beta(b_i + w_{2i}) - \frac{1}{2}e_i^2 \geq \tilde{w}_{1i} + \beta(\tilde{b}_i - w_{2i}) - \frac{1}{2}\tilde{e}_i^2 \quad (PC_{1i})$$

$$w_{1i} + \beta(b_i + w_{2i}) - \frac{1}{2}e_i^2 \geq (1 + \beta)w_0 \quad (PC_{2i})$$

$$w_{ti} \geq w_0 \text{ for every } t \quad (PC_{3i})$$

Where the \tilde{x} values represent the equilibrium values for other firms in the market. The first constraint is the individual rationality constraint, this requires that the worker prefers to put forth effort rather than risking losing the bonus b_i . This condition will limit the set of contracts that are feasible, since if the required effort is too large the bonus will not be enough to induce the worker to take the effort cost. The second constraint is that the worker must be at least indifferent between this contract and the best contract the worker can get in the outside market. The third constraint ensures that the worker prefers working to non-employment, in which the worker gets his outside option w_o each period. The final constraint is the per-period minimum wage constraint.

The individual rationality constraint pins down the bonus in terms of effort:

$$b_i = \frac{e_i^2}{2\beta p} \quad (8)$$

Since workers can always move to a new firm in the second period, the total incentive effect must be driven by the bonus b_i . The smaller the probability that the firm can observe the worker's effort the larger the bonus must be to induce the same amount of effort.

Now we can solve the firm's problem, holding the first period wage constant.

$$\max_{e_i} \eta_i(1 + e_i) - w_{1i} - \beta \frac{e_i^2}{2\beta p} + \beta \Pi_2 \quad (9)$$

Which leads to the optimal choice of effort:

$$e_i^* = p_j \eta_i \quad (10)$$

Thus, the optimal choice of effort for the firm to induce is the product of the probability the firm observe the worker's ability and the worker's ability. If the likelihood of observation is low, the firm will induce less effort because it is too costly. On the other hand, the firm will also choose to induce less effort from lower ability workers. This relationship will be key to

the positive assortative matching on p .

Now we can derive the joint surplus.

$$S(i, j) = (1 + \beta)Y_j\eta_i + Y_j^2\eta_i^2\frac{p_j(1 - p_j)}{2} \quad (11)$$

which provides the conditions for efficient assignment.

Proposition 5 *Holding Y constant across jobs, efficient assignment of jobs to workers is positive assortative between the probability of monitoring p_j and worker ability η_i . Holding p constant across jobs, efficient assignment is positive assortative between job productivity Y_j and worker ability.*

Under firm learning about ability, efficient assignment was negative assortative with the information parameter x . In contrast, we see here that efficient assignment is positive assortative with learning about worker effort. This is because workers respond to the probability of monitoring, and adjust their effort. The more likely that a worker is observed, the more effort the firm can induce. Since output is complementary between effort and ability, we have positive assortative matching between ability and incentives.

3.1 Single Occupation Labor Market

Now we can return to the specific case of two types of workers, A and B , with $\eta_A > \eta_B$. As in the first part of the paper, A -type workers are scarce. Thus, firms will compete to hire A -type workers, bidding up their wages until the firms are indifferent between the two types of workers. There are surplus B workers, so they will be held to their participation constraint each period.

We can now express efficient assignment and wages.

Proposition 6 *Given the assumptions about wage setting and contracts, the optimal sorting is as follows:*

1. All *A*-type workers match with firms, workers never shirk.
2. The rest of the jobs are filled with *B*-type workers, who never shirk.
3. Off the equilibrium path, if a worker shirked, he would be denied the bonus payment and separate from the firm. If he is of type *A* he will be re-employed by another firm at the market wage for *A*-types. If he is type *B* he will either be re-employed or non-employed, both of which yield outside option w_0 .

and the optimal wage contracts are:

1. All employed *B*-type workers receive first period wage w_0 , bonus $\frac{p\eta_B^2}{2\beta}$, and second period wage w_0 .
2. All employed *A*-type workers receive first period wage $w_0 + (\eta_A - \eta_B)Y + (\eta_A^2 - \eta_B^2)\frac{pY^2}{2}$, bonus $\frac{p\eta_A^2}{2\beta}$, and second period wage $w_0 + Y(\eta_A - \eta_B)$.

Now we can consider comparative statics. First, consider each worker's expected compensation, net disutility from effort. This is the sum of first period wage, the bonus, the cost of effort, and the second period wage. On the equilibrium path, the worker always follows the contract, so effort is positive. We can write this expected net compensation as follows:

$$\begin{aligned} \mathbb{E}W_A(p) &= (1 + \beta)(Y(\eta_A - \eta_B) + w_0 + \frac{p(1-p)}{2}Y^2\eta_A^2 - \frac{p}{2}Y^2\eta_B^2) \\ \mathbb{E}W_B(p) &= (1 + \beta)w_0 + \frac{p(1-p)}{2}Y^2\eta_B^2 \end{aligned} \quad (12)$$

We can also express each firm's expected profits as follows:

$$\mathbb{E}\Pi(p) = (1 + \beta)(Y\eta_B - w_0) + \frac{p}{2}Y^2\eta_B^2$$

where firms are indifferent between hiring an *A*-type worker and *B*-type worker. Thus, we can derive the following comparative statics.

Lemma 9 *For both types of workers, expected net compensation is increasing with β , w_0 and Y . A-type workers' expected net compensation is increasing with η_A and decreasing with η_B . B-type workers' expected net compensation is increasing with η_B and independent from η_A .*

Lemma 10 *Firm expected profit is increasing with β , η_B , and Y , and decreasing with w_0 .*

Comparative statics with respect to the probability of observing the worker's effort are more nuanced. Let \hat{p} be defined as the p such that $\eta_A^2/\eta_B^2 > 1/2(1-p)$ for all $p > \hat{p}$.

Proposition 7 *Firm expected profit and B-type workers' expected net compensation are both rising with p . For A type workers, expected compensation is rising with p if $p > \hat{p}$. Otherwise, A-type workers' expected compensation is falling with p .*

Since firms only observe a worker's effort with probability p , when p is smaller firms must pay a larger bonus to induce effort. Even though this results in less effort in equilibrium, when p is sufficiently small, worker total net compensation may be decreasing with p for small values of p . B-type workers are held to their participation constraint so they are not able to capture enough to distort their incentives, leading to a positive relationship between expected net compensation and p .

Whether p is above or below \hat{p} will affect whether or not any wages can support the efficient equilibrium in a market with more than one type of job. We turn to this case next.

3.2 Sorting and Efficiency Wages

Now suppose there are two types of jobs, with probability of observing effort $\bar{p} > p$. Suppose there are more \bar{p} jobs than A-type workers and also more p jobs than A-type workers. Then we have the following proposition.

Proposition 8 *The efficient allocation to workers and firms is positive assortative in p :*

1. All A-type workers are employed with \bar{p} firms in the first period, $M_{\bar{p}} - N_A$ B-type workers are employed in \bar{p} firms, M_p B-type workers are employed by p firms, and the balance of B-type workers are unemployed.
2. In the second period, all workers stay with their current employer.
3. Off the equilibrium path, if a worker shirked, he would be denied the bonus payment and separate from the firm. If he is of type A he will be re-employed by another \bar{p} firm at the market wage for A types. If he is type B he will either be re-employed or non-employed, both of which yield outside option w_0 .

From Proposition 7, we know that the market may not be able to support the efficient allocation if the probability of observing the worker's type is too small. If the \bar{p} is small, A-type workers will prefer the wage offer from p firms, even though \bar{p} firms are willing to pay a larger wage, and efficient sorting is positive assortative.

Thus we have the following equilibrium.

Proposition 9 *If $\frac{\eta_A^2}{\eta_B^2} \geq \frac{\bar{p}-p}{\bar{p}(2-\bar{p}-p(2-p))}$, competitive wages can support the efficient allocation from Proposition 8. In particular, the wages are as follows: and the optimal wage contracts are:*

1. B-type workers employed in p jobs receive first period wage w_0 , bonus $\frac{p\eta_b^2}{2\beta}$, and second period wage w_0 .
2. B-type workers employed in \bar{p} jobs receive first period wage w_0 , bonus $\frac{\bar{p}\eta_b^2}{2\beta}$, and second period wage w_0 .
3. All employed A-type workers receive first period wage $w_0 + (\eta_A - \eta_B)Y + (\eta_A^2 - \eta_B^2)\frac{\bar{p}Y^2}{2}$, bonus $\frac{\bar{p}\eta_A^2}{2\beta}$, and second period wage $w_0 + Y(\eta_A - \eta_B)$.

If $\frac{\eta_A^2}{\eta_B^2} < \frac{\bar{p}-p}{\bar{p}(2-\bar{p}-p(2-p))}$, the competitive equilibrium is distorted. Instead, A-type workers match with p firms, B-type workers match with the rest of the p jobs and the \bar{p} jobs. Wages are as follows:

1. *B-type workers employed in p jobs receive first period wage w_0 , bonus $\frac{p\eta_b^2}{2\beta}$, and second period wage w_0 .*
2. *B-type workers employed in \bar{p} jobs receive first period wage w_0 , bonus $\frac{\bar{p}\eta_b^2}{2\beta}$, and second period wage w_0 .*
3. *All employed A-type workers receive first period wage $w_0 + (\eta_A - \eta_B)Y + (\eta_A^2 - \eta_B^2)\frac{pY^2}{2}$, bonus $\frac{p\eta_A^2}{2\beta}$, and second period wage $w_0 + Y(\eta_A - \eta_B)$.*

Thus, if \bar{p} is too small, sorting is no longer positive assortative with p . This is a deadweight loss, as total surplus would be higher under the efficient allocation.

4 Conclusions

In this paper, I show how talent is allocated across jobs, when firm-learning is heterogeneous between jobs. When workers cannot manipulate the signal, efficient assignment is negative assortative with job informativeness, and jobs that provide the least information offer workers the largest share of joint surplus. When workers can manipulate the signal, efficient assignment is positive assortative with job informativeness, but this allocation can only be decentralized if the probability of learning is large enough.

These results have implications for wage inequality and the distribution of workers between jobs. In the full labor market, jobs differ based on information as well as productivity. When workers sort freely between occupations, in equilibrium, high-ability information jobs will pay lower wages and match with lower expected ability workers, even if the true productivity of the jobs are high. This allows firms to capture a larger share of the joint surplus, potentially exacerbating wage inequality.

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