Lecture 19
Manipulation II

Prof. Katie DC

Modern Robotics Ch 12.1

Administrivia

- Exam 3 next week
  - Review on Tuesday
  - No class on Thursday
- 5/5 Extra Credit Day
  - Open note quiz
  - ~2% extra credit for overall grade + ~%5 extra credit for exam grade

Multiple points of contact

- The possible twists of a body A with multiple contacts, can be written:

$$V = \{\mathcal{V}_A \mid \mathcal{F}_i^\top (\mathcal{V}_A - \mathcal{V}_i) \geq 0 \; \forall i\}$$

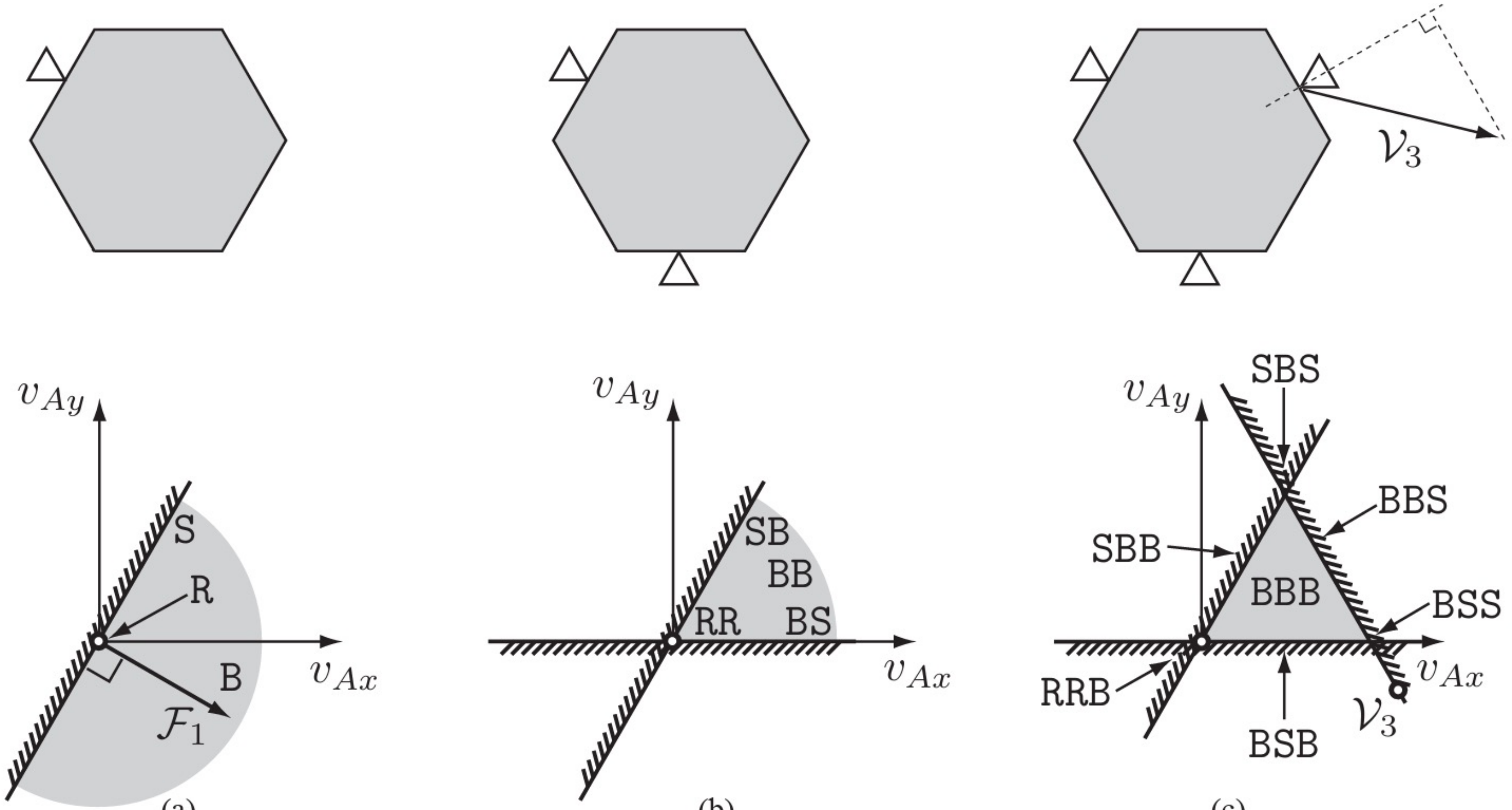
Suppose our figures are not moving ($\mathcal{V}_i = 0$)

The above set becomes a polyhedral convex cone:

$$V = \{\mathcal{V}_A \mid \mathcal{F}_i^\top \mathcal{V}_A \geq 0 \; \forall i\}$$

- If the $\mathcal{F}_i$ positively span the wrench space or the convex span of the $\mathcal{F}_i$ contains the origin then the contacts **constrain the motion of the body**
- We call this **form closure**

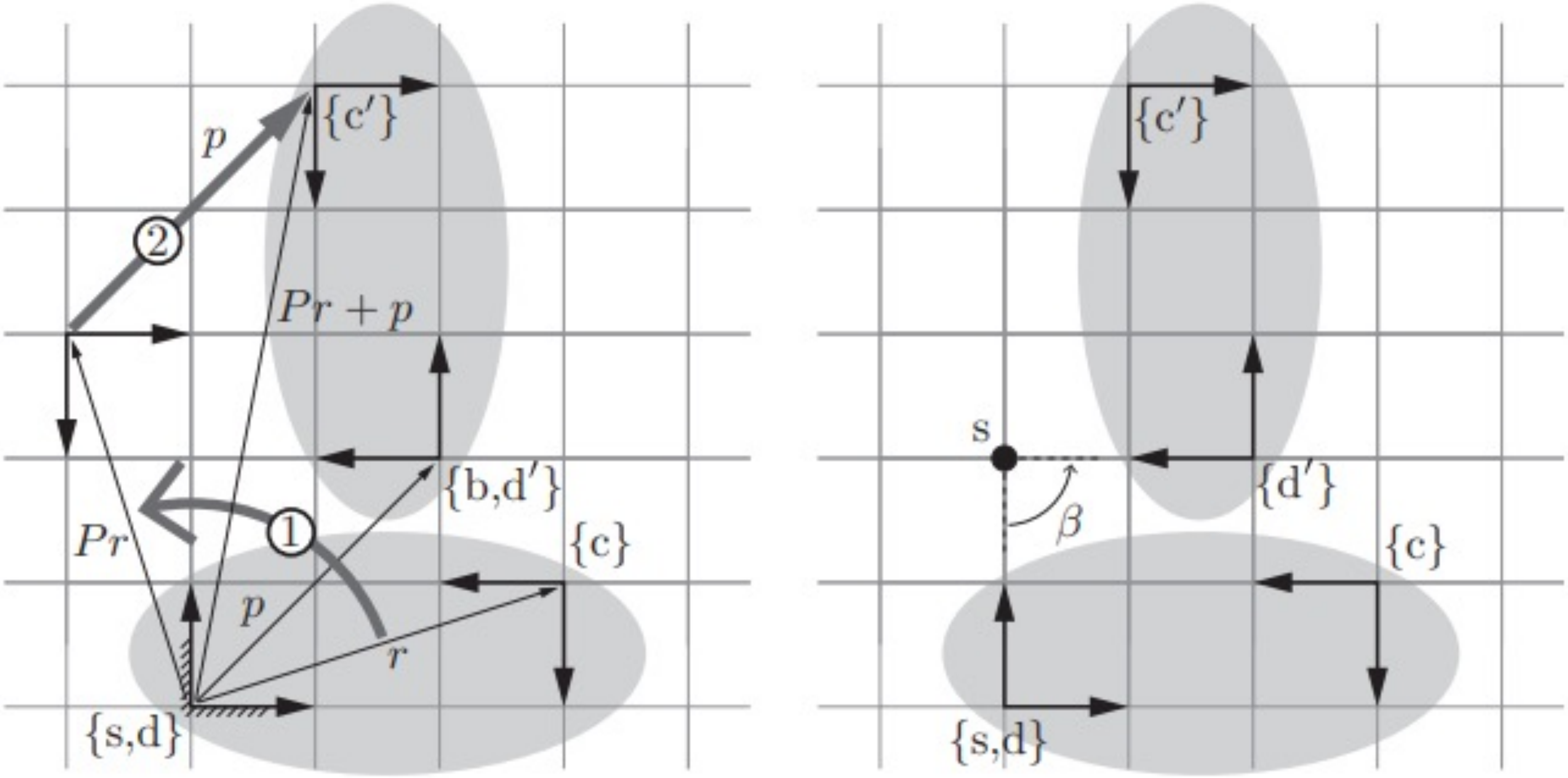
Each contact point restricts motion of a body



(a) (b) (c)

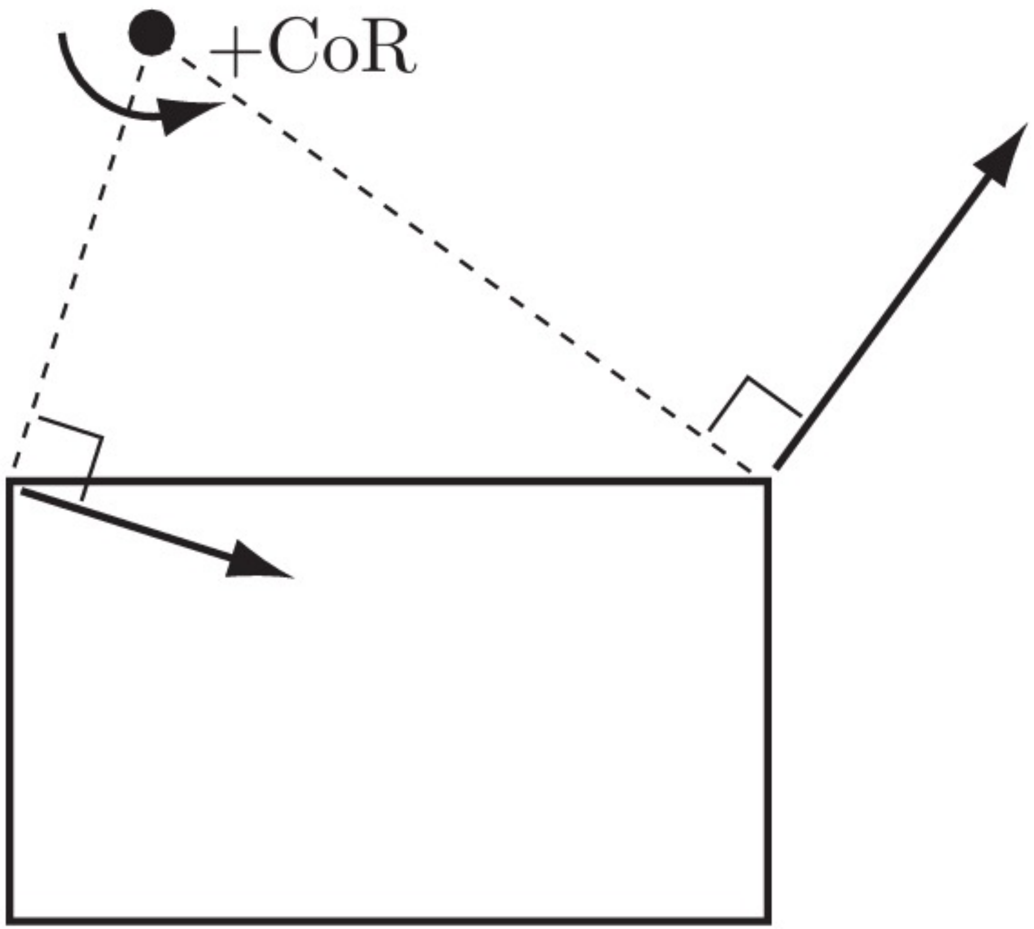
Visualizing Motions of Planar Twists

Twists capture a screw motion. In the planar case for general screw motions, what is the screw axis?



Visualizing Planar Twists

What is the screw axis for these types of motions? The **center of rotation!**

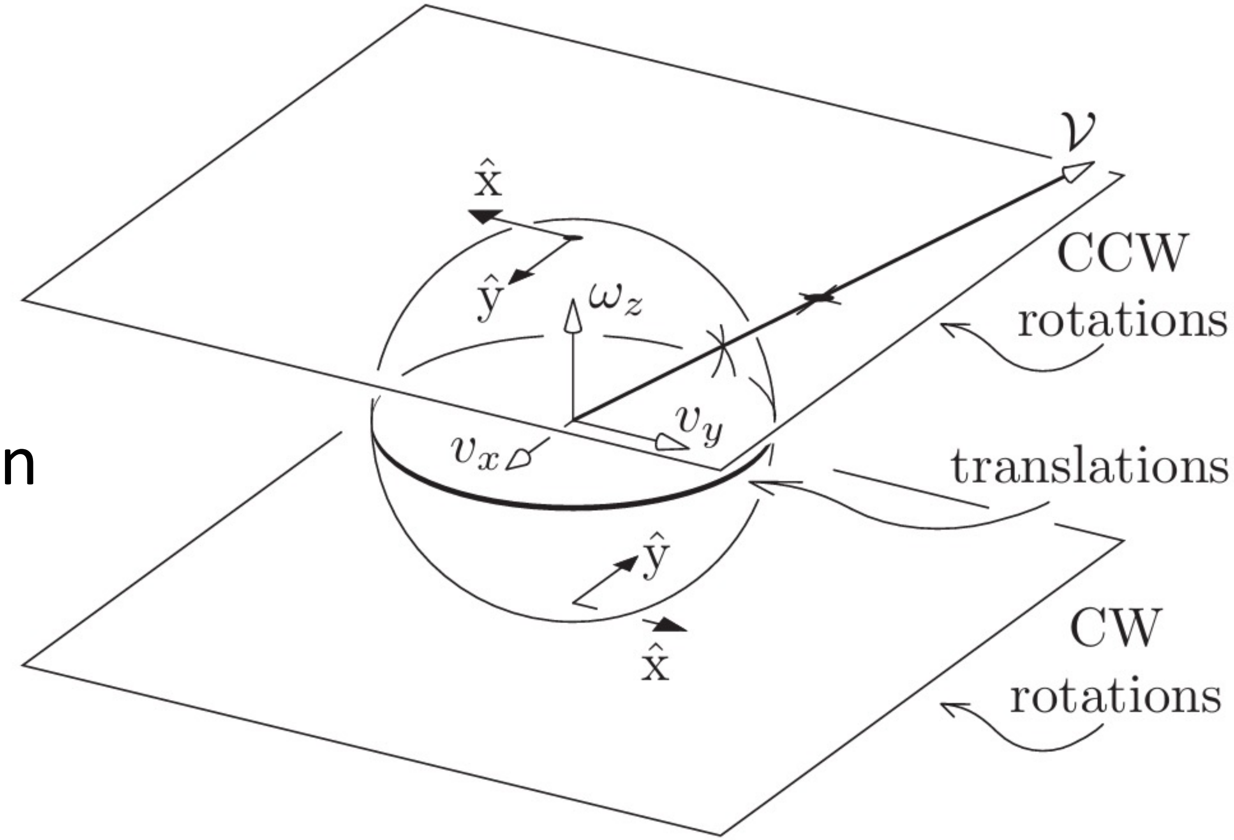


Visualizing Motions of Planar Twists

Abandoned tracks in Urbana

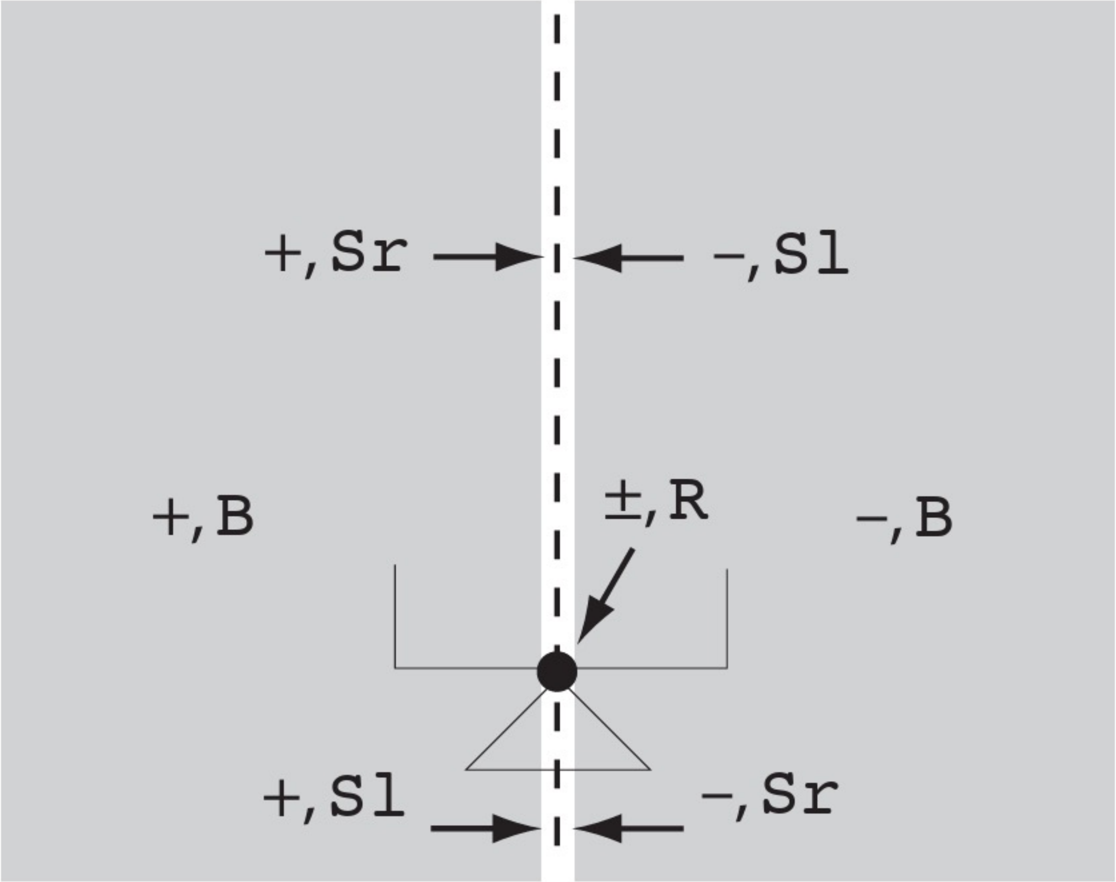
- A planar twist is represented by:

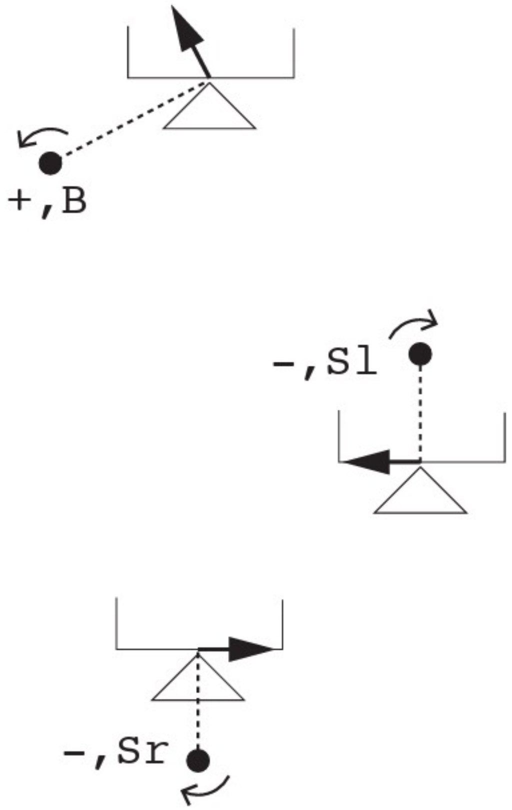
$$\mathcal{V} = \begin{bmatrix} \omega_z \\ v_x \\ v_y \end{bmatrix}$$



- The **center of rotation** is the point in the *projective plane* that stays stationary under the motion
- We compute the CoR for a given twist as: $\left(\frac{-v_y}{\omega_z}, \frac{v_x}{\omega_z}, \omega_z\right)$

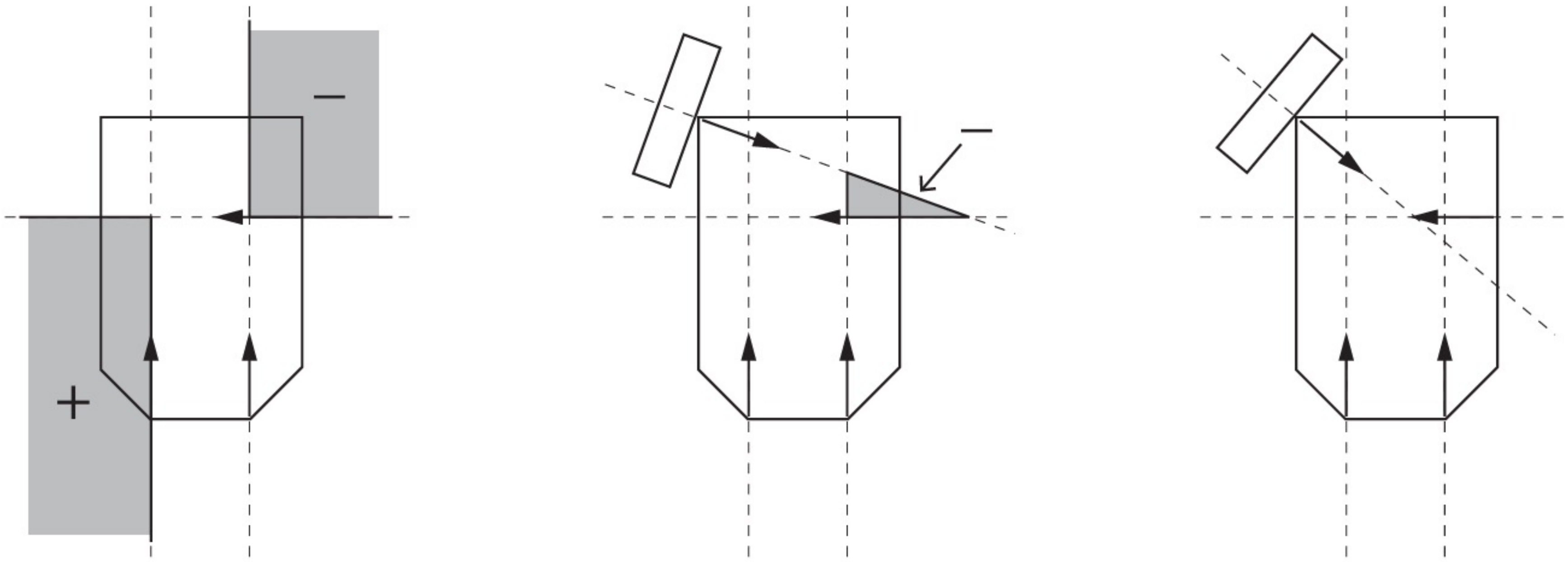
Graphical Methods for Visualizing Motions





- Twist and Wrench cones help us visualize the effect of grasp on motion, but are challenging in higher dimensions
- **Form closure** checkers are more generalizable

What if we want no motion?



Form Closure Grasps

- **Form closure** of a body is when a set of stationary constraints prevent *all* motions
- Recall our stationary contact conditions:

$$\mathcal{F}^\top \mathcal{V} \geq 0$$

- If the only twist $\mathcal{V}$ that satisfies this constraint is a zero vector, we are in form closure. Said in different way:

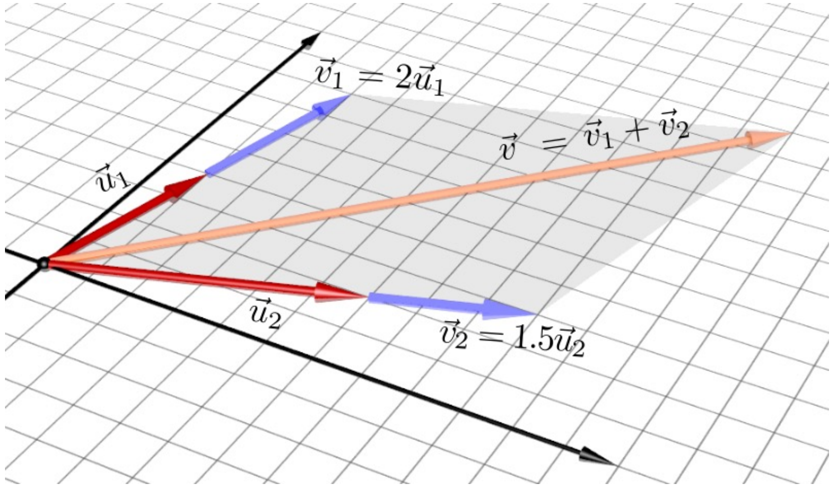
$$\text{positive span}\left(\{\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_j\}\right) = \mathbb{R}^6$$

Positive Span?

- The positive span of vectors is defined as:

$$\text{positive span}(\mathcal{A}) = \left\{ \sum_{i=1}^{j} k_i a_i \mid k_i \geq 0 \right\}$$

- The space $\mathbb{R}^n$ can be spanned by $n + 1$ vectors, but no fewer



Contacts needed for Form Closure

- For a planar body, at least four point contacts are needed for first-order form closure
- For a spatial body, at least seven point contacts are needed for first-order form closure
- Fine print:
  - First order analysis is a simplified estimate! If a body is in form closure by first-order analysis, it is in form closure for higher-order analysis. If it is not in form closure, it could be with higher-order reasoning.
  - **Exceptional objects** are objects that can always spin about their center, for any number of contacts

Form Closure Checker

- Recall our stationary contact condition: $\mathcal{F}^\top \mathcal{V} \geq 0$
  - For cases where $\mathcal{V} \neq 0$, need to check if our contact wrenches meet this criteria
- Given our wrenches, build a matrix:

$$F = [\mathcal{F}_1 \quad \mathcal{F}_2 \quad \cdots \quad \mathcal{F}_j] \in \mathbb{R}^{n \times j}$$

- First, we can rewrite our condition as follows:

$$\exists k > 0 \text{ such that } Fk + \mathcal{F}_{ext} = 0 \;\; \forall \mathcal{F}_{ext}$$

- Second, if F is full rank, an equivalent condition is the existence of $k > 0$ such that $Fk = 0$

Form Closure Checker -- Linear Program

We can formulate this as a linear optimization program:

$$\begin{array}{ll} \underset{k}{\text{minimize}} & \mathbf{1}^\top k \\ \text{subject to} & Fk = 0 \\ & k_i \geq 1, i = 1, \dots, j \end{array}$$

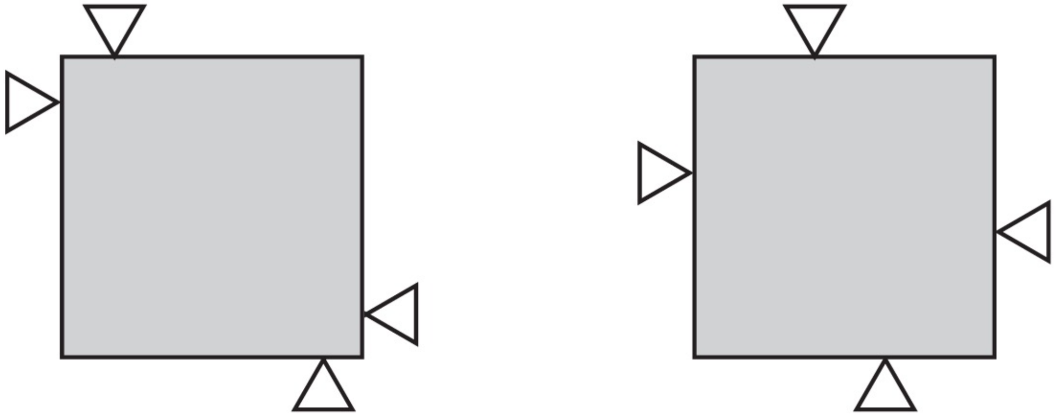
What's up with k_i?

Linear program solvers need to take in constraints in a particular form: $Ax \leq b$, so we re-write our constraints:

$$-\mathbf{I}k \leq -\mathbf{1}$$

For solving this, use a linear program solver:
numpy `np.linalg.solve`, scipy `scipy.optimize.linprog`, or matlab `linprog`

A note on grasp quality and planning



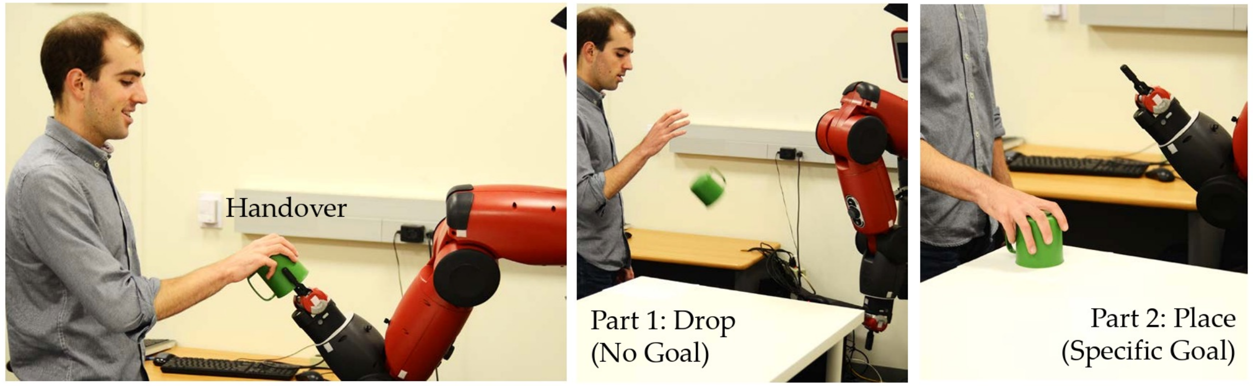


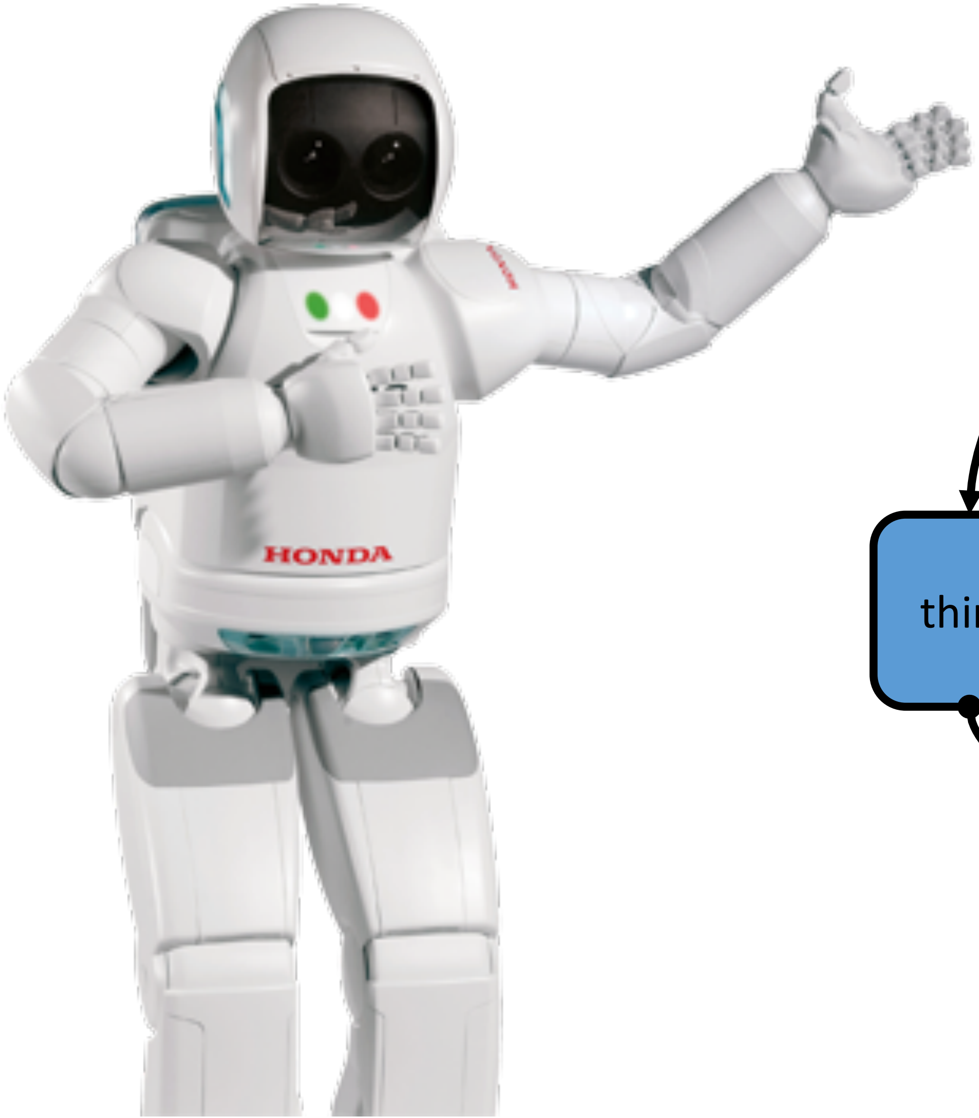
Image Credit: Aaron Bestick

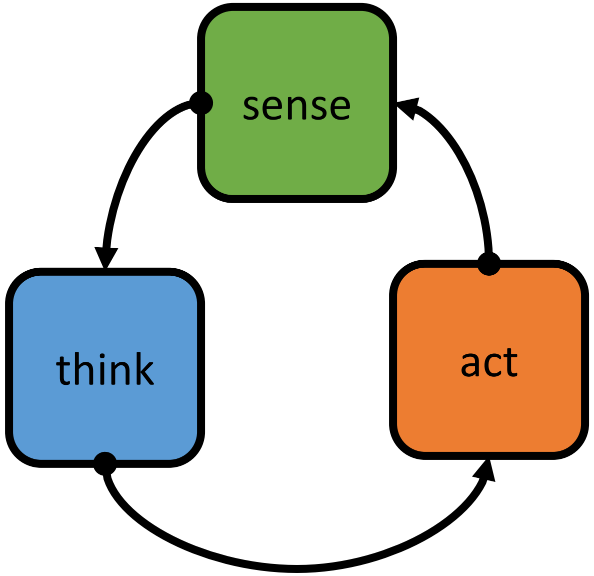
- Both grasps are in form closure, but which is better?
  - $\mathrm{Qual}(\{\mathcal{F}_i\}) < 0$ not in form/force closure
  - $\mathrm{Qual}(\{\mathcal{F}_i\}) > 0$ good grasp
  - $\mathrm{Qual}(\{\mathcal{F}_i\}) \gg 0$ great grasp
- Many resources to help with grasp quality and planning:
  Dex-Net from AutoLab

Summary

- Introduced methods to visualize and assess feasible motions (twists) of bodies
- Formalized **form closure** to determine if we are effectively grasping an object (restricting motion)
- We constructed criteria for checking form closure with a simple **linear program**

Course Recap

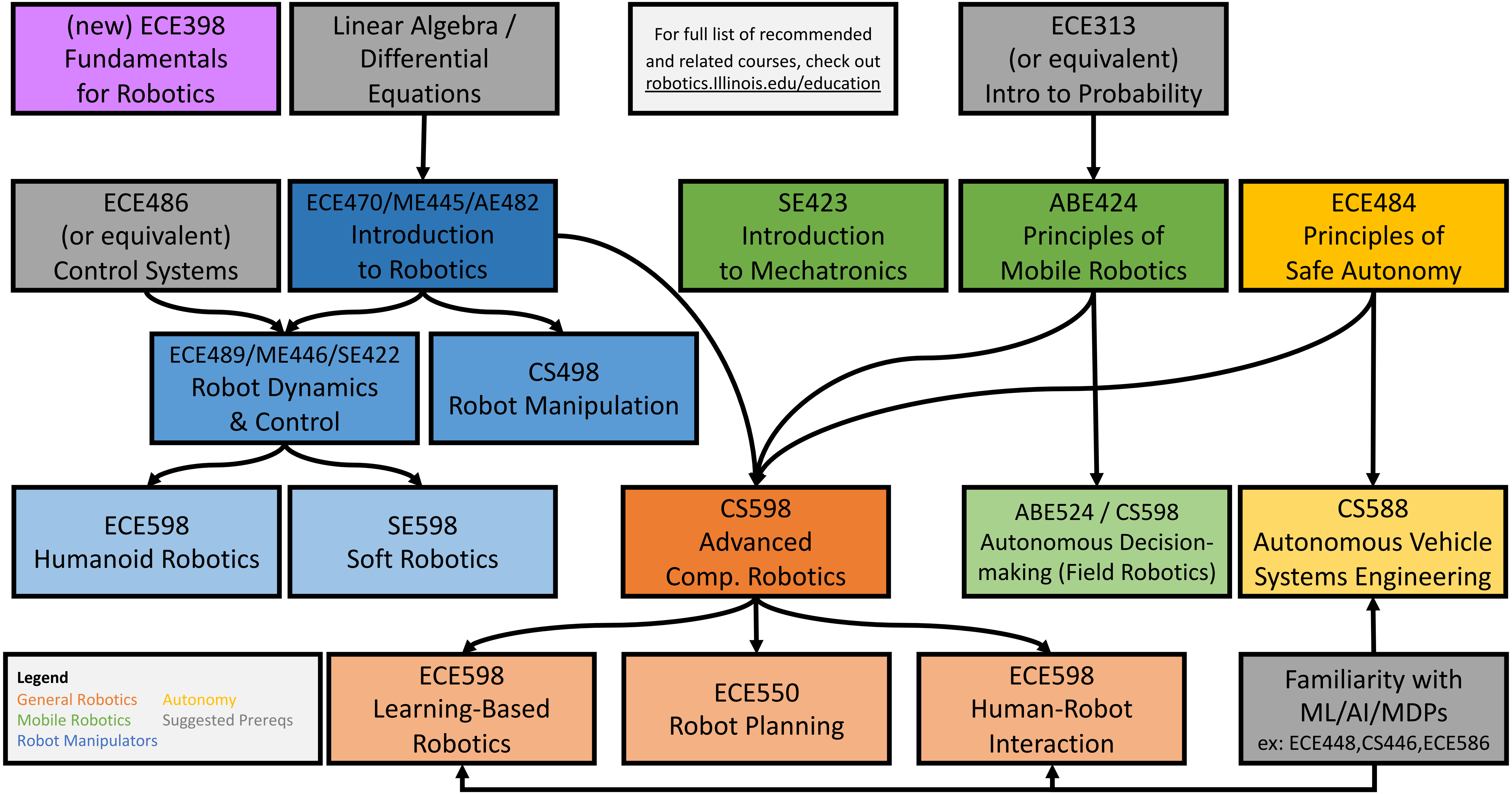




1. Linear algebra and diff. eq. review
2. DoF, configuration space
3. **Rigid body motion & transformations**
4. Screw theory
5. **Forward Kinematics**
6. Velocity Kinematics
7. Inverse Kinematics
8. Motion Planning
9. Manipulation

What are the key takeaways from this course?

- Understanding spaces of variables and frames and how to interact with them
- Hands-on experience working with manipulators (i.e., cool labs)
- Some insights on advanced topics (planning, manipulation, safety)


(new) ECE398 Fundamentals for Robotics
Linear Algebra / Differential Equations
For full list of recommended and related courses, check out robotics.Illinois.edu/education
ECE313 (or equivalent) Intro to Probability
ECE486 (or equivalent) Control Systems
ECE470/ME445/AE482 Introduction to Robotics
SE423 Introduction to Mechatronics
ABE424 Principles of Mobile Robotics
ECE484 Principles of Safe Autonomy
ECE489/ME446/SE422 Robot Dynamics & Control
CS498 Robot Manipulation
ECE598 Humanoid Robotics
SE598 Soft Robotics
CS598 Advanced Comp. Robotics
ABE524 / CS598 Autonomous Decision-making (Field Robotics)
CS588 Autonomous Vehicle Systems Engineering
Legend
General Robotics
Autonomy
Mobile Robotics
Suggested Prereqs
Robot Manipulators
ECE598 Learning-Based Robotics
ECE550 Robot Planning
ECE598 Human-Robot Interaction
Familiarity with ML/AI/MDPs
ex: ECE448,CS446,ECE586