Lecture 18
Manipulation I

Prof. Katie DC

Modern Robotics Ch 3.4 and Ch 12.0

Administrivia

- Exam 2 grades have been updated
- Exam 3 is next week!
  - Review is on Tuesday
- 5/5 will be an in-class extra credit “activity”

Spatial Forces as Wrenches

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→ Can we do the same with forces?

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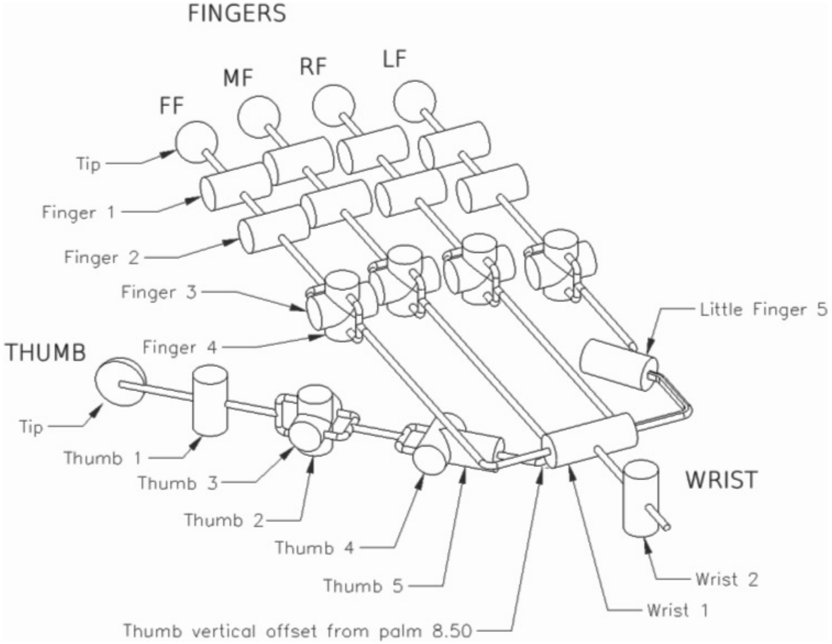
Got more than one wrench?
The total wrench on the body is the vector sum of the individual wrenches (in the same frame).

Robot Hands are Kinematic Chains!

Parallel Jaw

RobotIQ

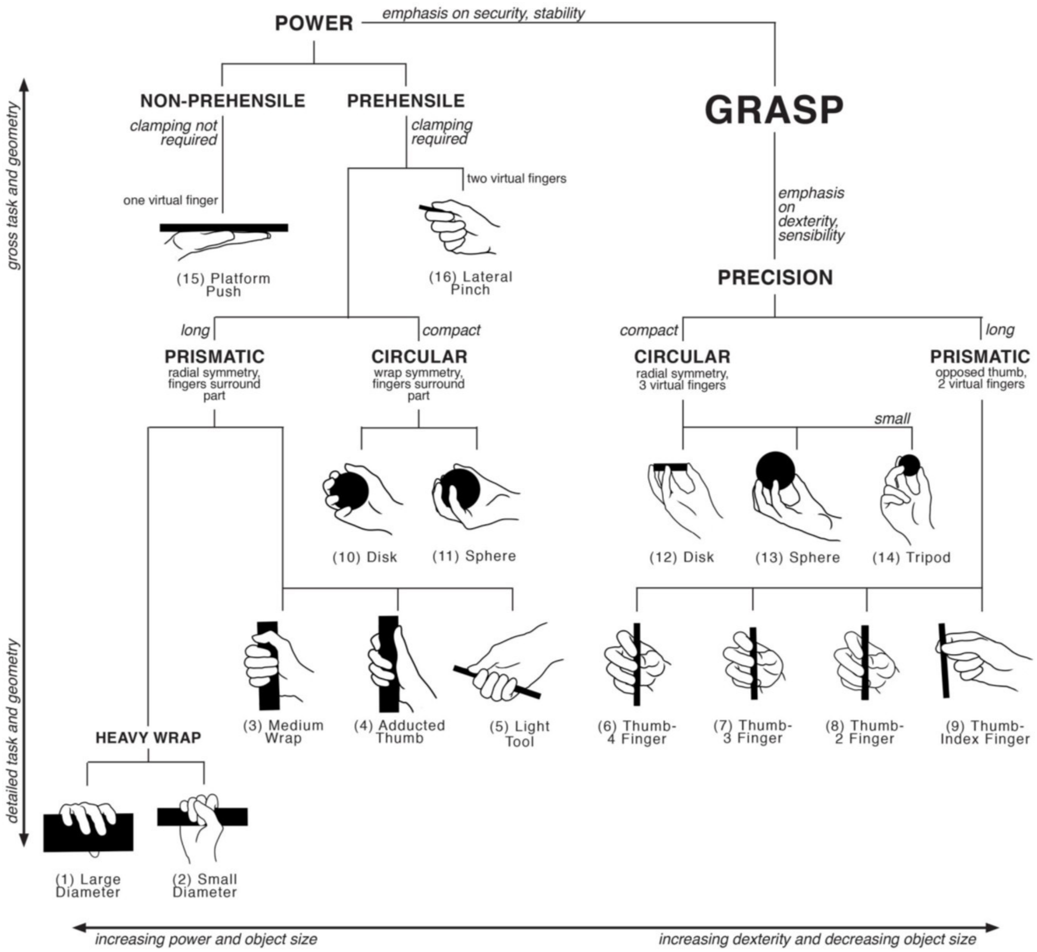
Five Fingered Hand



iCub hand

Manipulation and Grasping

Cutkosky Grasp Taxonomy



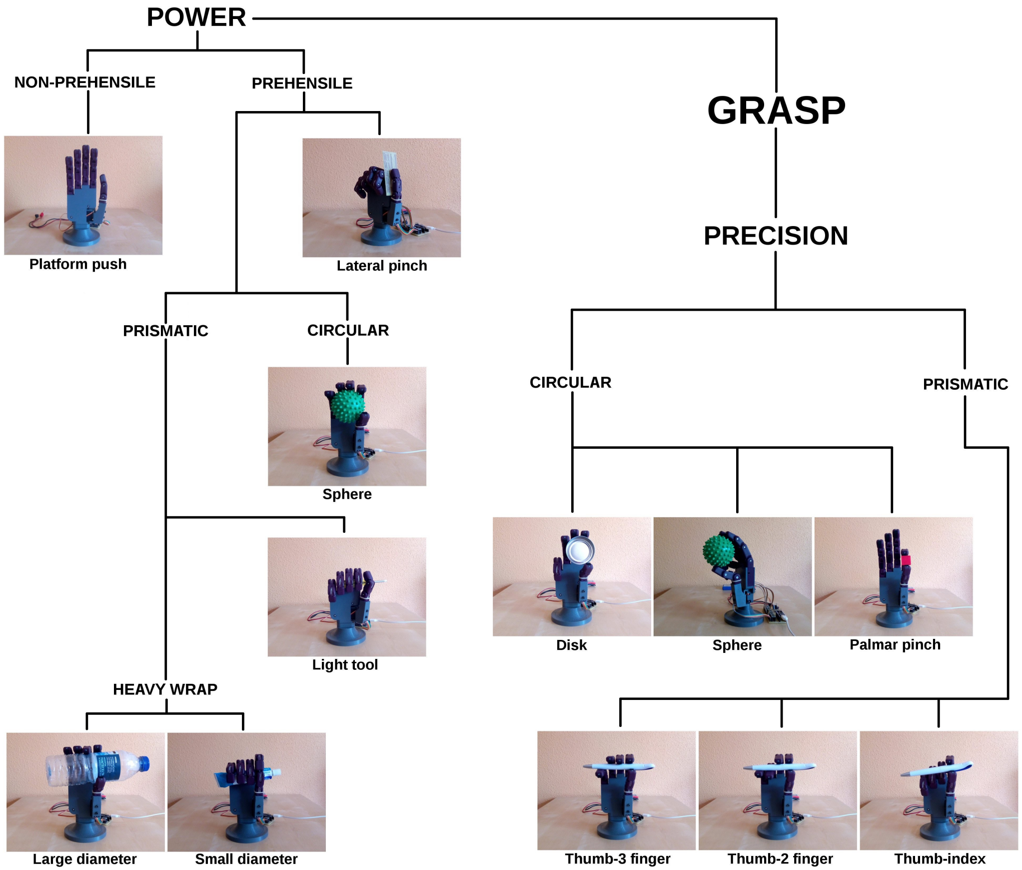
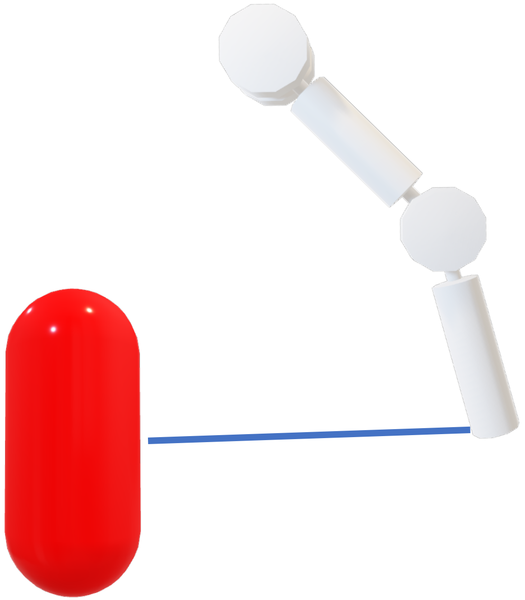


Image credit: Alvaro Billoslada, Project log for Dextra

Contact Dynamics



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$$d(q = (q_1, q_2))$$

Contact Dynamics

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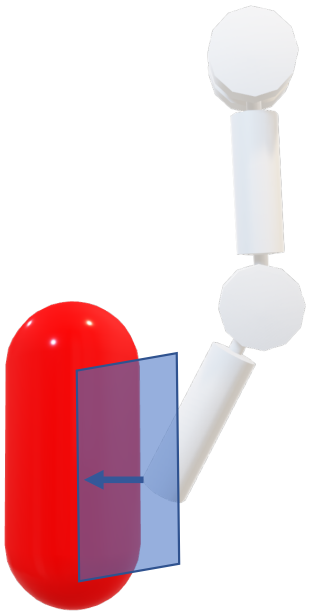
Consider a first order analysis:

d	$\dot{d}$	situation
> 0		No contact
< 0		Infeasible
$= 0$	> 0	Contact, but **breaking free**
$= 0$	< 0	Contact, but infeasible
$= 0$	$= 0$	In contact

Contact Dynamics



Write distance between two points of contact:

$$d(q = (q_1, q_2))$$

Consider a first order analysis:

$$\dot{d}(q) = \frac{\partial d}{\partial q}\dot{q}$$

This derivative gives the **contact normal** $\hat{n}$

Contact Normal

- For contact, we write the velocity condition as a constraint:

$$\dot{d}(q) = \frac{\partial d}{\partial q}\dot{q} \geq 0$$

- We can rewrite this equation as follows:

$$\hat{n}^{\top}(\dot{p}_A - \dot{p}_B) \geq 0$$

where p_A is a point on body A and p_B is a point on body B

Back to Twists!

- What is $\dot{p}_A$? Recall twist derivation from previous lectures:

$$\dot{p}_A = v_A + \omega_A \times p_A = v_A + [\omega_A] p_A$$
$$\dot{p}_B = v_B + \omega_B \times p_B = v_B + [\omega_B] p_B$$

- Can we bring in a wrench $\mathcal{F} = (m, f)$?
- If we apply a unit force to along the contact normal:

$$\mathcal{F} = \begin{bmatrix} p_A \times \hat{n} \\ \hat{n} \end{bmatrix} = \begin{bmatrix} [p_A]\hat{n} \\ \hat{n} \end{bmatrix}$$

Contact Constraints with Twists and Wrenches

Rewrite: $\hat{n}^{\top}(\dot{p}_A - \dot{p}_B)$ as two constraints:

Impenetrability constraint: $\mathcal{F}^{\top}(\mathcal{V}_A - \cancel{\mathcal{V}_B}) \geq 0$

Active contact: $\mathcal{F}^{\top}(\mathcal{V}_A - \cancel{\mathcal{V}_B}) = 0$

If body B is a static object, we drop $\mathcal{V}_B$

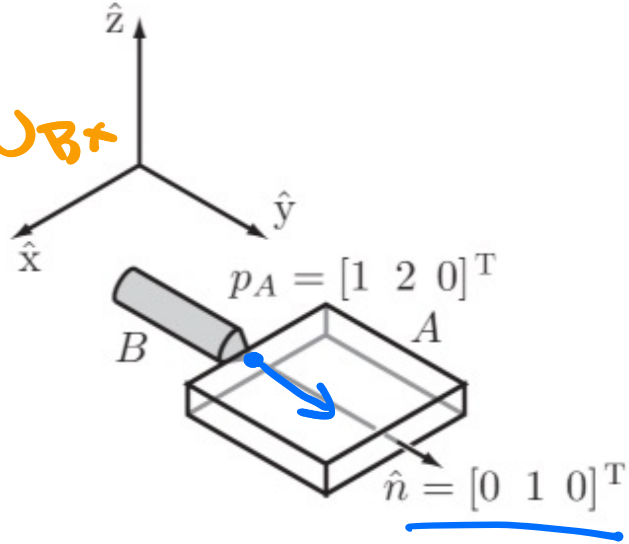
Roll-slide contacts satisfy the active contact constraint

- Rolling contacts: no motion relative to each other $\dot{p}_A = \dot{p}_B$
- Sliding contacts: all other satisfying solutions

Example (1) $p_A = p_B,\ \hat{n}$

$$F^T(v_A - v_B) \geq 0$$

$$\Delta\omega_x = \omega_{Ax} - \omega_{Bx}$$



$$\begin{bmatrix} [p_A]\hat{n}^T & \hat{n}^T \end{bmatrix} \begin{bmatrix} \omega_A - \omega_B \\ v_A - v_B \end{bmatrix} \geq 0$$

$$[0\ 0\ 1\ 0\ 1\ 0] \begin{bmatrix} \Delta\omega_x \\ \Delta\omega_y \\ \Delta\omega_z \\ \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{bmatrix} = \underbrace{\Delta\omega_z + \Delta v_y = \omega_{Az} - \omega_{Bz} + v_{Ay} - v_{By}}_{(*)} \geq 0$$

for active contact $(*) = 0$

Example (2)

rolling constraint : $\dot{p}_A = \dot{p}_B$

$v_A + [\omega_A] p_A = v_B + [\omega_B] p_B \rightarrow$ expand + write as eq

$$\begin{bmatrix} v_{Ax} - \omega_{Az} p_{Ay} + \omega_{Ay} p_{Az} \\ v_{Ay} - \omega_{Az} p_{Ax} - \omega_{Ay} p_{Az} \\ v_{Az} + \omega_{Ax} p_{Ay} - \omega_{Ay} p_{Ax} \end{bmatrix} =$$

B follows similarly

plug in p_A + p_B =

$$v_{Ax} - 2\omega_{Az} = v_{Bx} - 2\omega_{Bz}$$

$$v_{Ay} + \omega_{Az} = v_{By} + \omega_{Bz}$$

$$v_{Az} + 2\omega_{Ax} - \omega_{Ay} = v_{Bz} + 2\omega_{Bx} - \omega_{By}$$

Multiple points of contact

- The possible twists of a body A with multiple contacts, can be written:

$$V = \left\{ \mathcal{V}_A \mid \mathcal{F}_i^{\top} (\mathcal{V}_A - \mathcal{V}_i) \geq 0 \ \forall i \right\}$$

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- If the $\mathcal{F}_i$ positively span the wrench space or the convex span of the $\mathcal{F}_i$ contains the origin then the contacts **constrain the motion of the body**
- We call this **form closure**

Summary

- Introduced **wrenches** to capture forces applied at various points
- Robot hands / fingers are more kinematic chains!
- Formalized **contact dynamics** to understand relationships between bodies in space
- **Next Time:** We will introduce methods to check feasible motions as well as **form closure** to determine if we are effectively grasping an object