CW code: 3696

Lecture 02: Mechanisms and Robot Basics

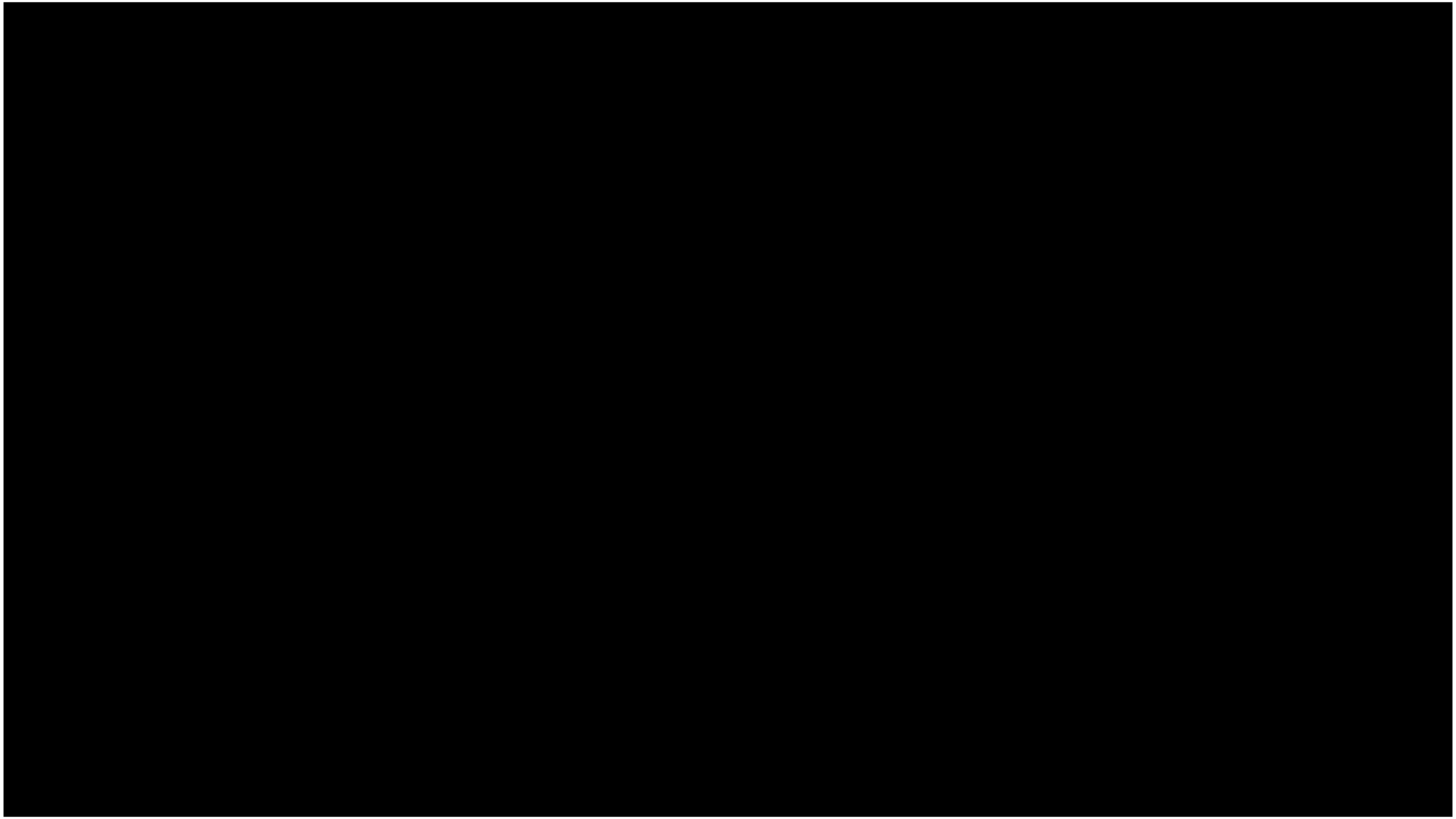
Prof. Katie Driggs-Campbell

Notes from Modern Robotics Ch 2

Slides adapted from Prof. Wenzhen Yuan

January 22, 2026

Robot Spotlight: ASIMO by Honda in 2000



https://www.youtube.com/watch?v=Bmglbk_Op64

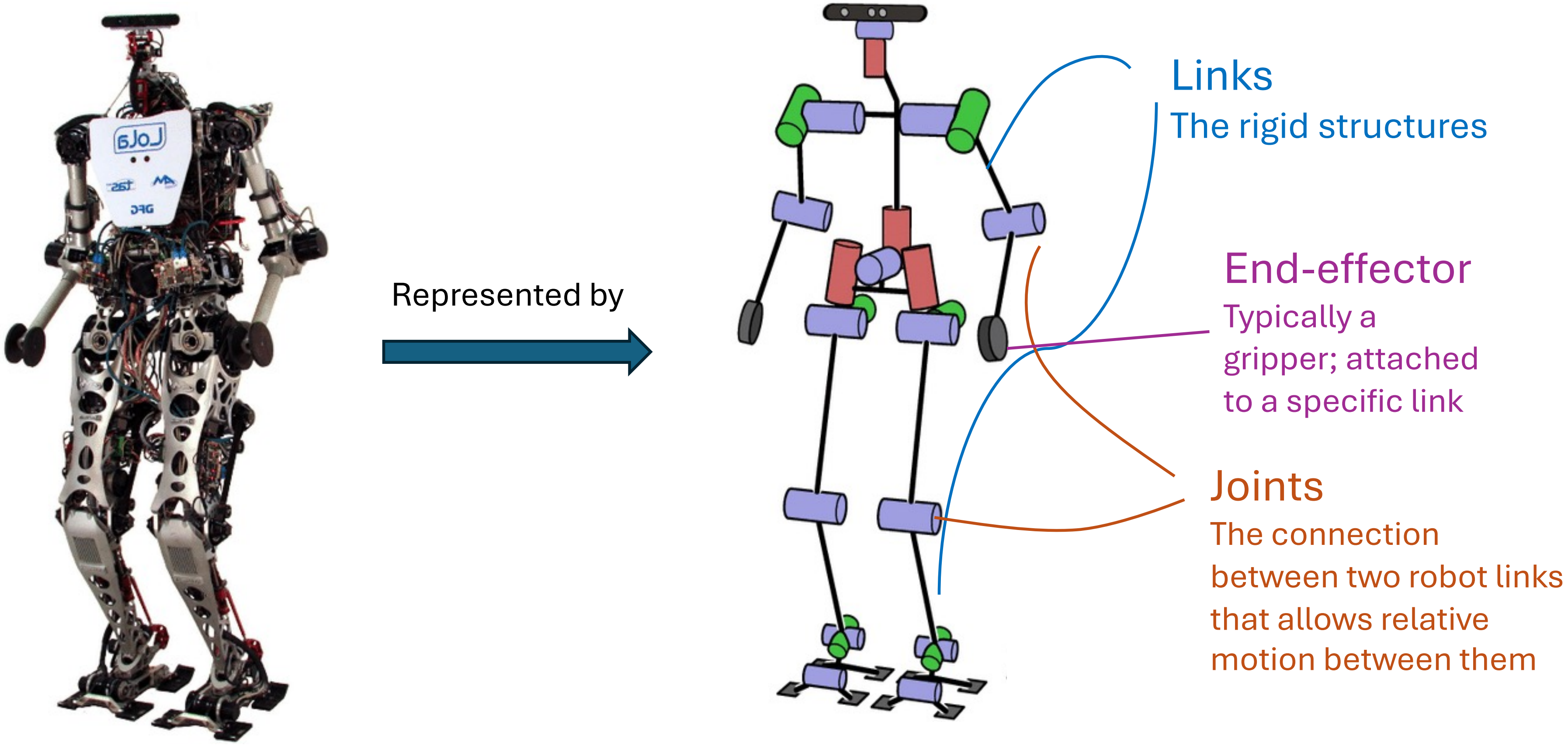
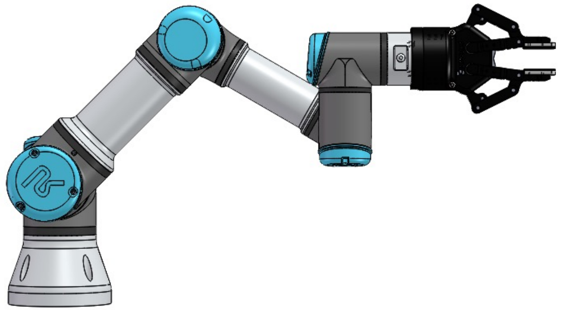
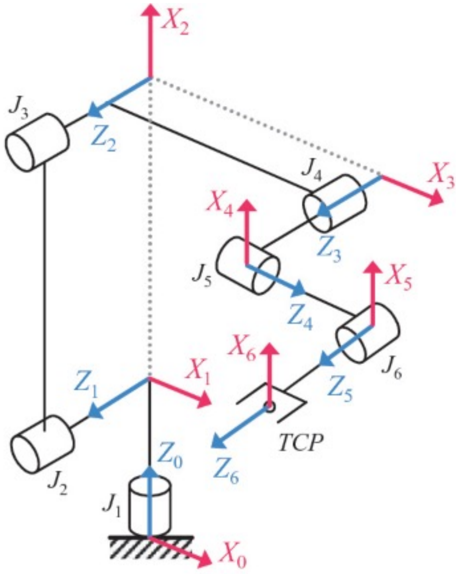


Figure from “Kinematic optimization for bipedal robots: a framework for real-time collision avoidance” by Hildebrandt, et al. 2019

Simpler Robot Arms

Universal Robot



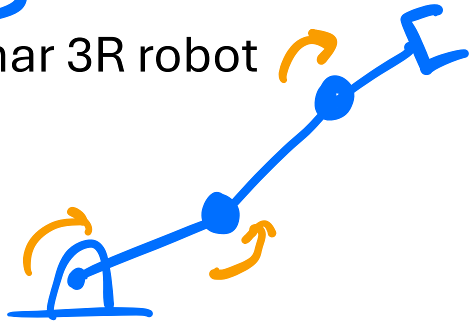


This robot:

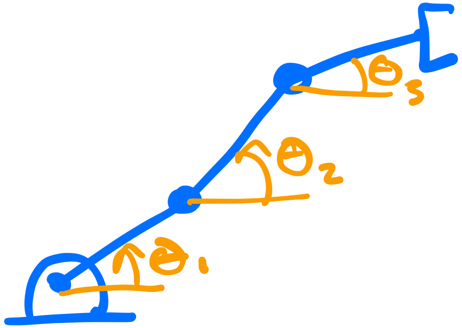
- 7 **links**
- 6 revolving **joints**
- **end-effector** (parallel-jaw gripper)

2D

Planar 3R robot

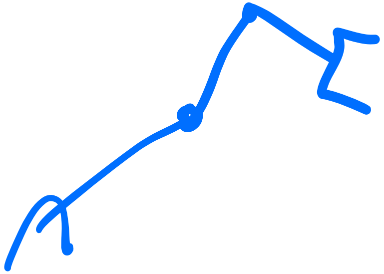


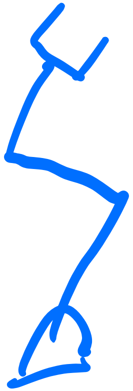
Configurations and DoF



Robot **configuration:** a complete specification of the positions of every point of the robot

Degrees of freedom (DoF): the smallest number of real-valued coordinates needed to represent its configurations

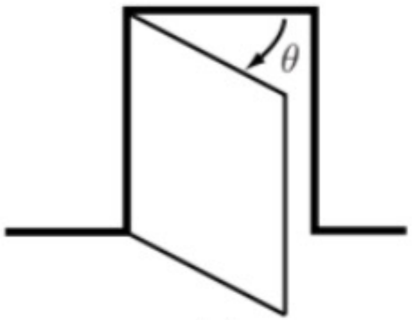


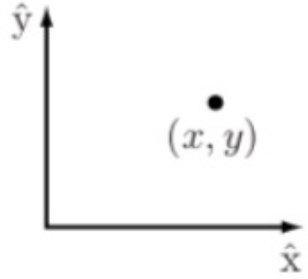


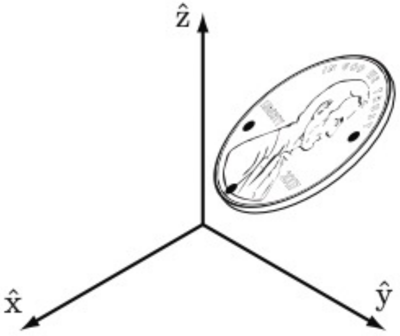
Other examples of DoF

How to represent the configuration?

How many DoFs are there?

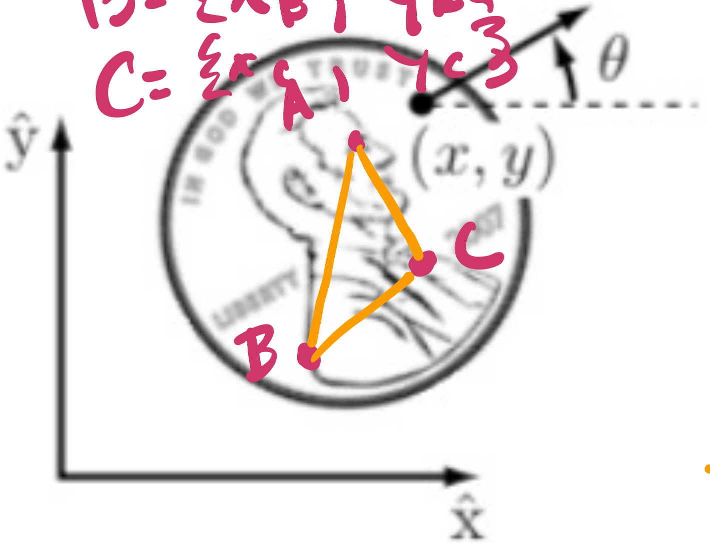






Can we use another form of configuration?

$A = \{x_A, y_A\}$
$B = \{x_B, y_B\}$
$C = \{x_C, y_C\}$



DoF = {Number of variables} – {Number of independent constraints}

① variables: 6

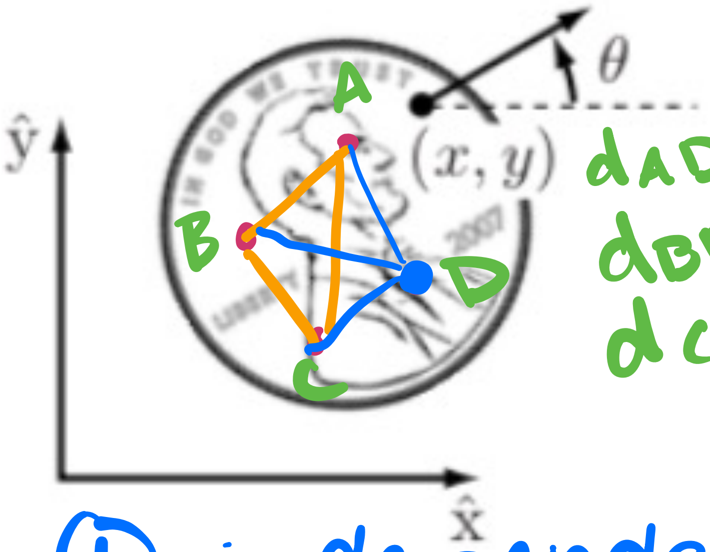
② constraints: 3

$DoF = 6 - 3 = 3$

What if we use four points to represent the configuration?

variables: 8

constraints: 5



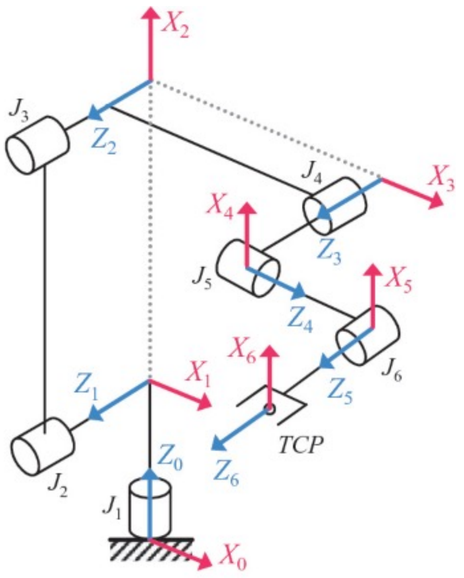
d_{AD}
d_{BD}
d_{CD}

DoF: 8-5=3

① independent constraints

② DoF is independent from config. representation

Degrees of Freedom for a Robot



- A robot can be viewed as the combination of multiple rigid bodies
  - How many rigid bodies are there in this robot?
  - What are the constraints?
- The joints ~~posed~~ impose constraints of the rigid bodies

Grübler's Formula

$$\text{dof} = \underbrace{m(N-1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^{J} c_i}_{\text{joint constraints}}$$

$$= m(N-1) - \sum_{i=1}^{J}(m - f_i)$$

$$= m(N-1-J) + \sum_{i=1}^{J} f_i.$$

N : number of links +1 (ground)

J : number of joints

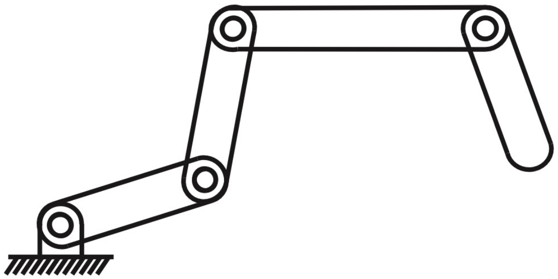
m : number of degrees of freedom of a rigid body (3 for planar mechanisms and 6 for spatial mechanisms)

f_i : number of freedoms provided by joint i

c_i : number of constraints provided by joint i

Grübler's Formula – examples

$$m(N-1-J)+\sum_{i=1}^{J} f_i.$$



$m=3$

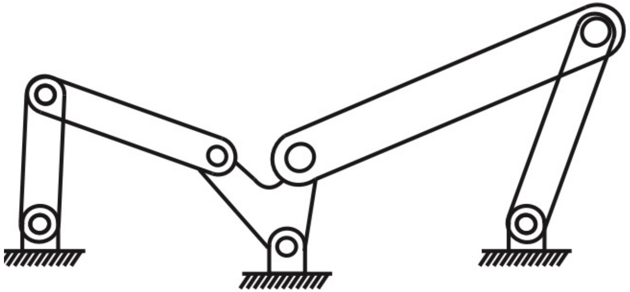
$N=4+1=5$

$J=4$

$f_i=1 \ \forall i$

$DoF = 3(5-1-4)+4\cdot 1$

$= 4$



$m=3$

$N=5+1=6$

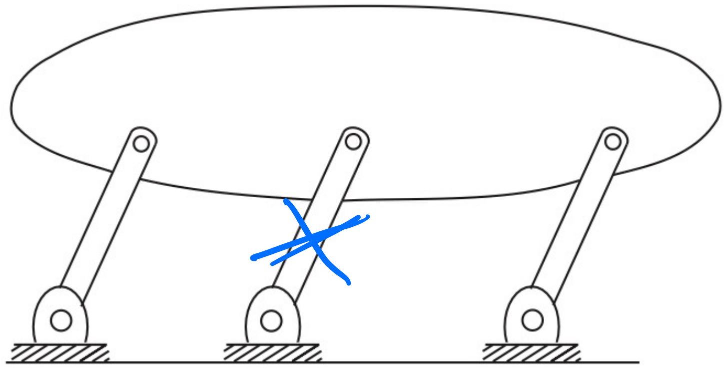
$J=7$

$f_i=1 \ \forall i$

$DoF = 3(6-1-7)+7$

$= 1$

Redundant constraints



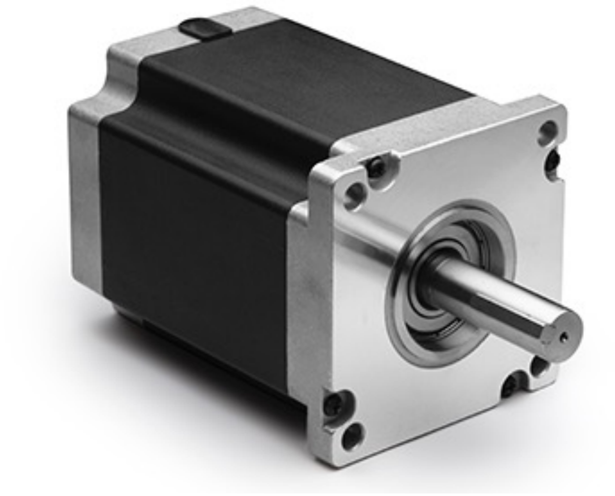
$$m(N-1-J)+\sum_{i=1}^{J} f_i.$$

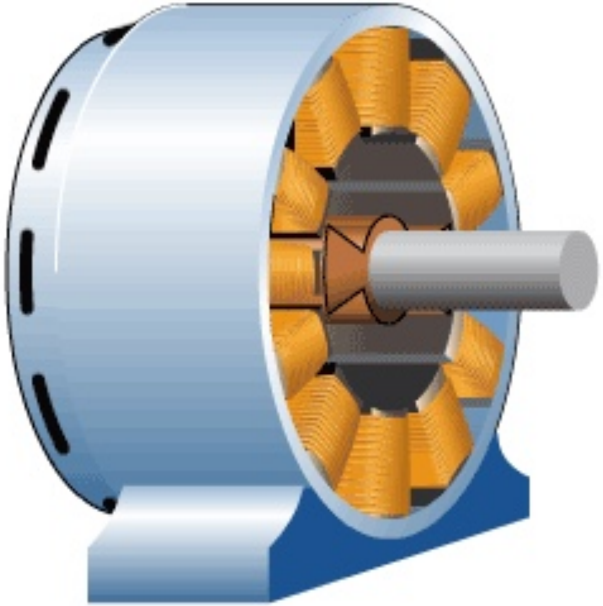
$3(5-1-6)+6=0$

$3(4-1-4)+4=1$

Why do we keep talking about rotating joints?

Most robots use motors as actuators

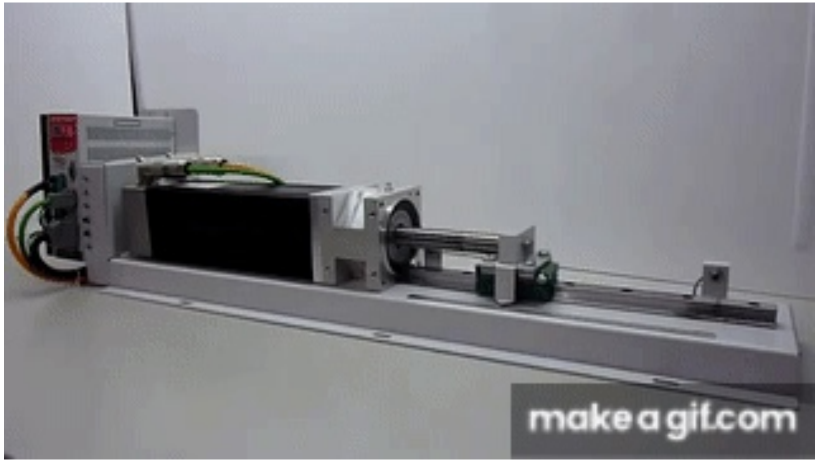


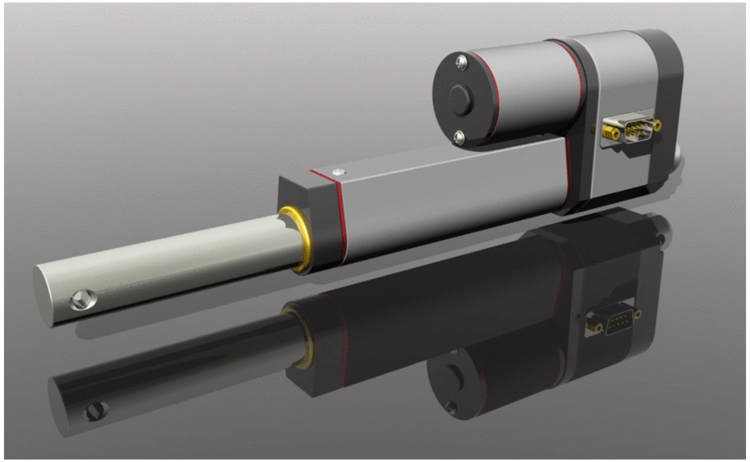


Actuators supply the motive power for robots, provide force and torque

Other kinds of joints?

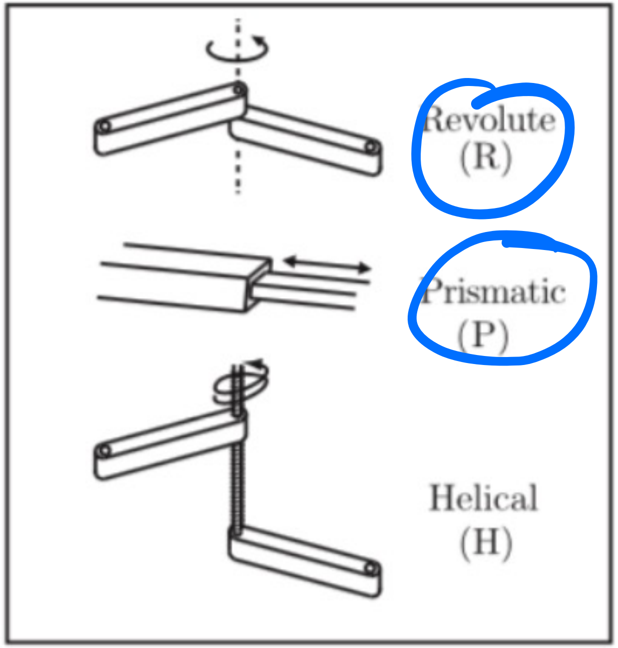
Linear motor / Prismatic Joint

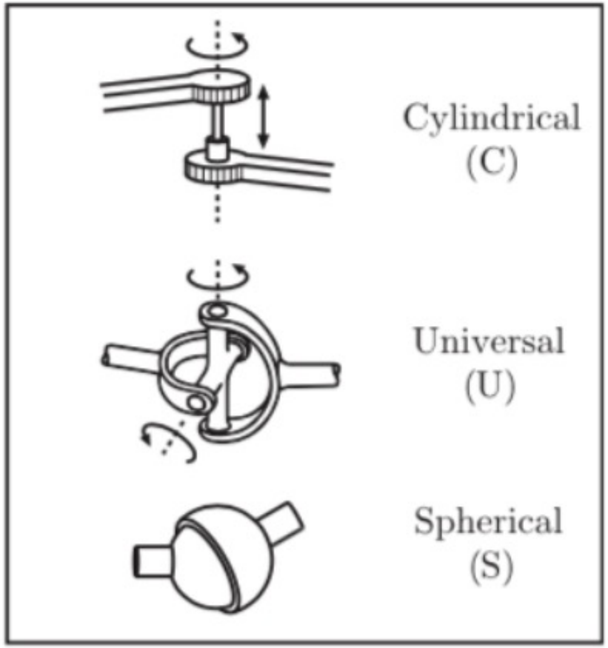




Other kinds of joints?

DoFs	Constraints 2D/3D
1	2/5
1	2/5
1	5

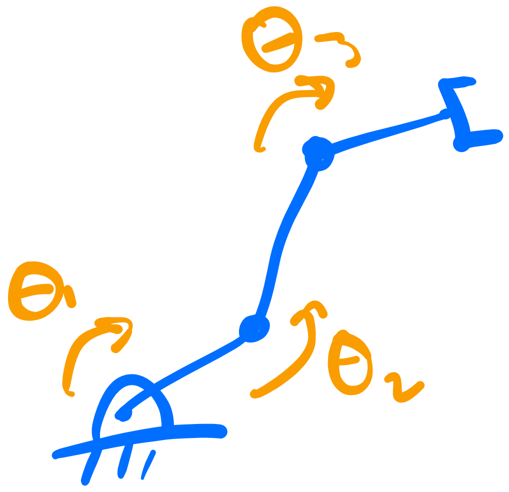




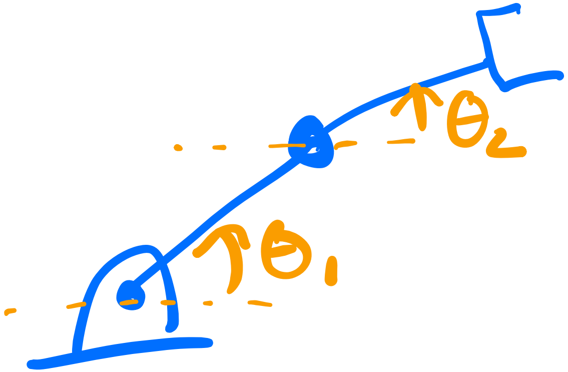
DoFs	Constraints
2	4
2	4
3	3

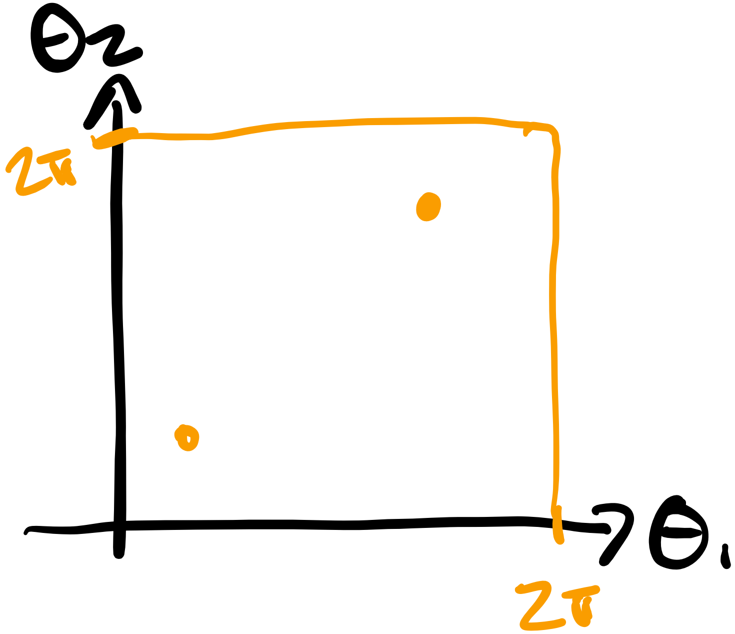
Configuration Space (C-space)

- The **configuration space (C-space)** is the n-dimensional space containing all possible configurations of the robot
- The configuration of a robot is represented by a point in its C-space.



How to represent the configuration space of a simple 2R robot?



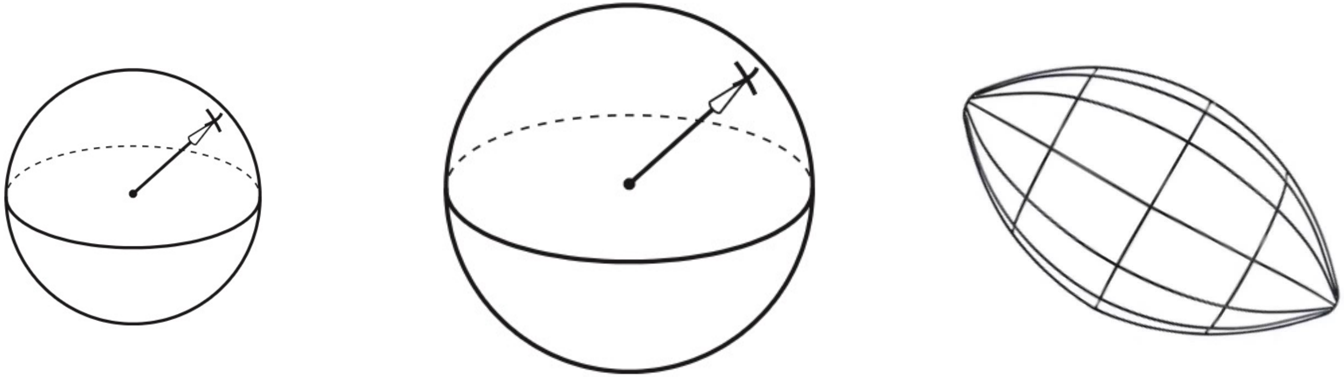


2R robot configuration space:

$T^2 = S^1 \times S^1$

Shape of the C-space

- "Shape" vs "kind of shape"
- Topology

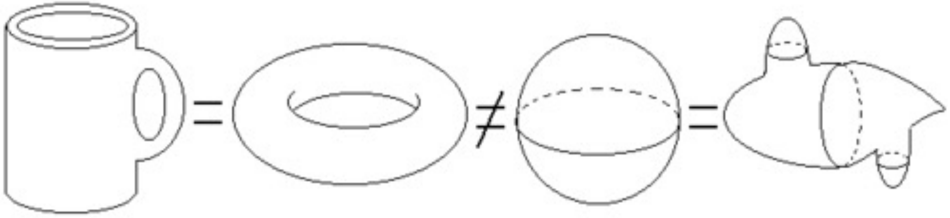


They are considered the same "kind of shape" Or, the same topology

Two spaces are topologically equivalent if one can be continuously deformed into the other without cutting or gluing

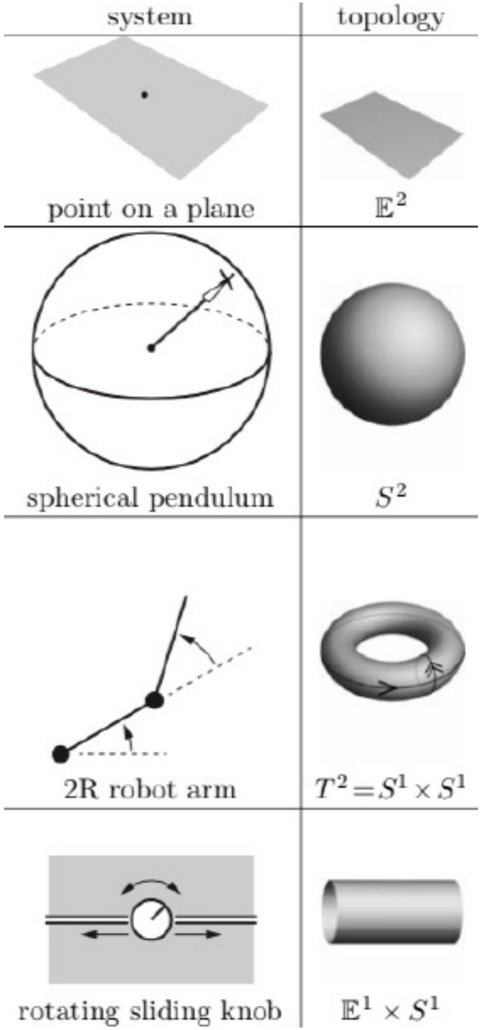
Topology

- Two spaces are topologically equivalent if one can be continuously deformed into the other without cutting or gluing



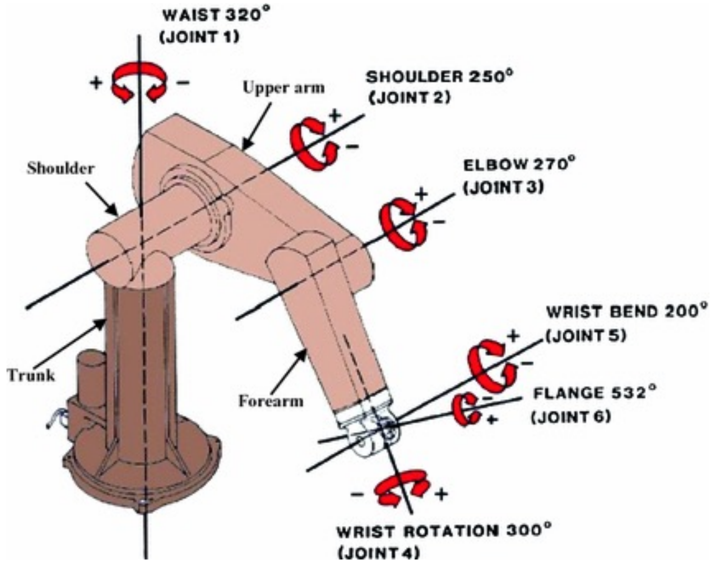
C-space representation with topology

- A line can be written as $\mathbb{E}$ or $\mathbb{E}^1$ or $\mathbb{R}$
- A plane can be written as $\mathbb{E}^2$ or $\mathbb{R}^2$
- A circle is written as a S or S^1
- A sphere is written as S^2



Workspace

- The **workspace** is the specification of the configurations that the end-effector of the robot can reach
  - Driven by the robot structure, independent of the task



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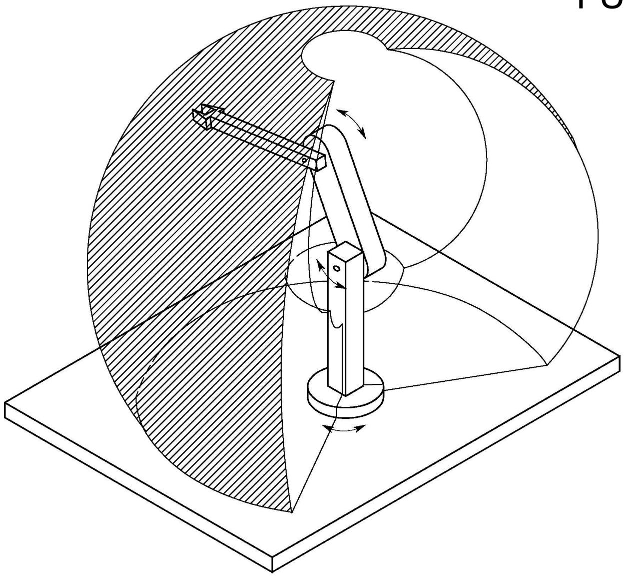
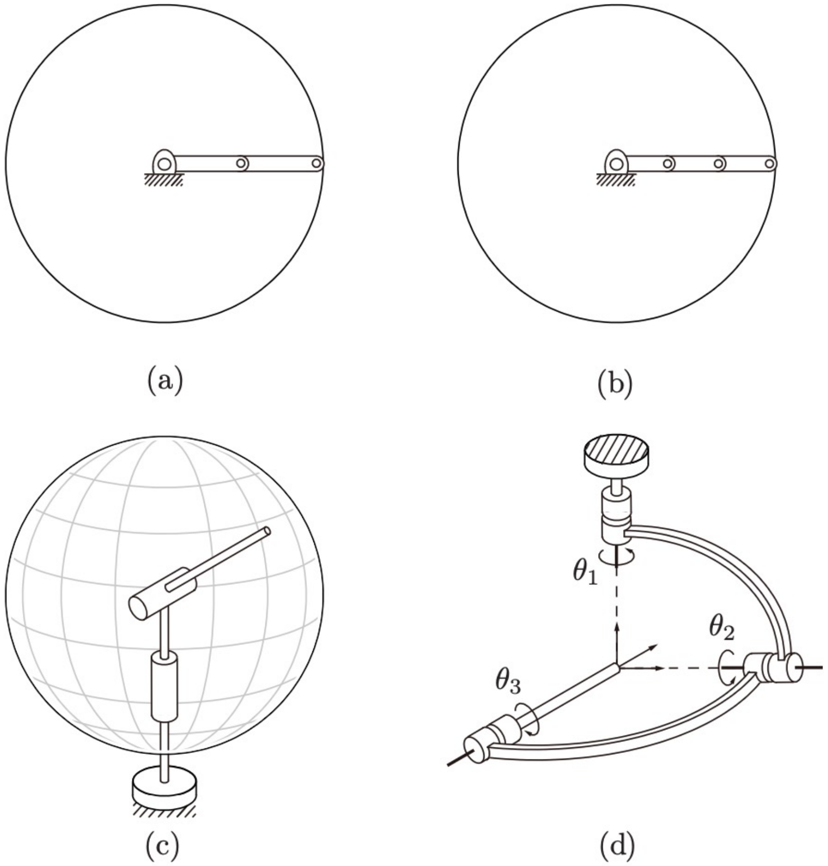
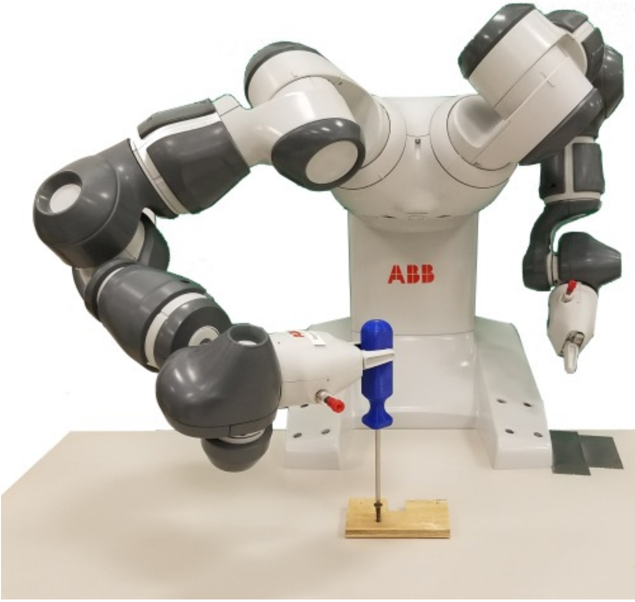


Examples of workspaces



Task Space

- The task space is the space in which the robot's task can be naturally expressed
  - For manipulation, a natural representation is the C-space of the robot's end-effector
  - Driven by the task, independent of the robot



Summary

- Robot constructions:
  - Links
  - Joints
  - Actuators
  - End-effectors
- DoF: the smallest number of coordinates needed to represent a robot configuration
  - Grübler's Formula

$$m(N-1-J)+\sum_{i=1}^{J} f_i.$$

- Configuration:
  - The configuration of a rigid body is a specification of the location of all its points.
  - The configuration of a robot is a specification of the configuration of all its links.
- C-space: the set of all possible robot configurations
- Workspace: the configuration space of the end-effector
- Task space: a space in which the robot's task can be naturally expressed