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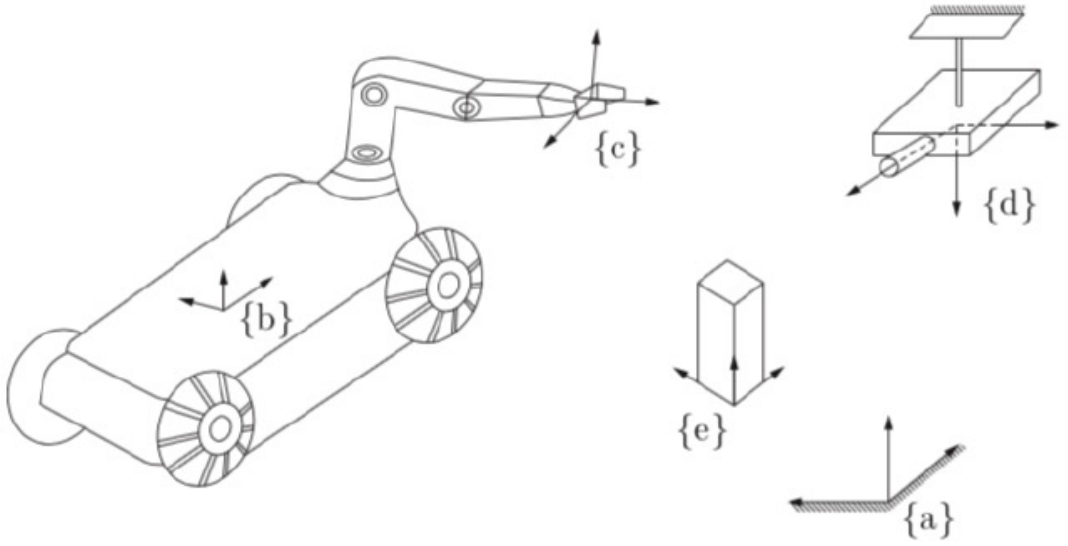
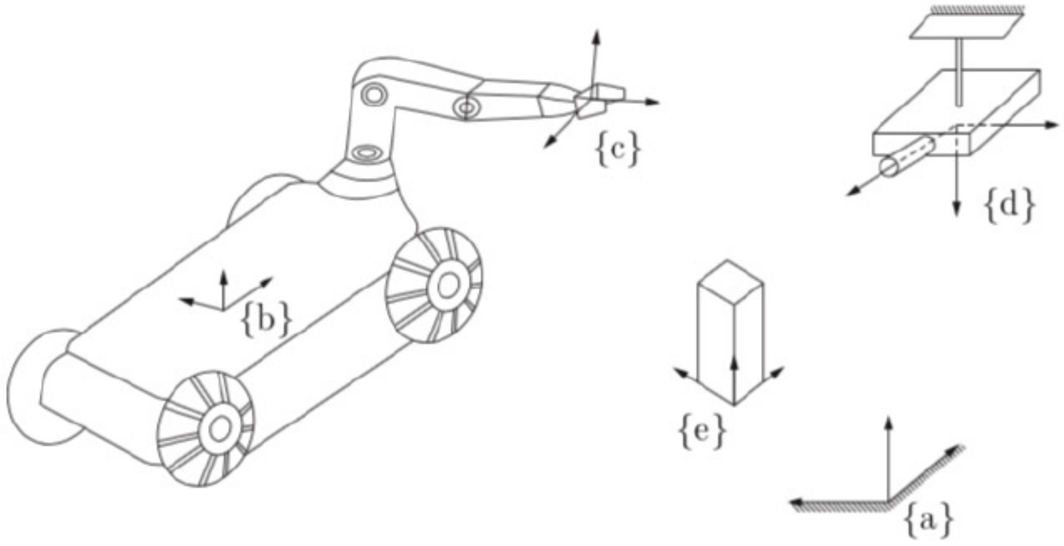
  - Rodrigues did not use it and elected to go to La Sorbonne
- The proof that a spatial displacement can be decomposed into a rotation and slide around and along a line is attributed to the astronomer and mathematician Giulio Mozzi (1763),
  - In Italy, the screw axis is traditionally called *asse di Mozzi*
- However, most textbooks refer to a subsequent similar work by Michel Chasles dating 1830
- Several other scholars/contemporaries of M. Chasles obtained the same or similar results around that time, including G. Giorgini, Cauchy, Poinsot, Poisson, and Rodrigues

Mobile arm example



- Robot arm mounted on wheeled platform. Camera fixed to ceiling.
- {b} is body frame, {c} end-effector frame, {e} frame of object, and {a} is fixed frame.
- We assume the camera position and orientation in {a} is given.
- From camera measurements, you can evaluate the position and orientation of the body and the object in the camera frame.
- Since we designed our robot and have joint-angle estimates, we can obtain the end-effector position and orientation in the body frame.
- To pick up the object, we need the object position and orientation in the frame of our end-effector.


{c}
{d}
{b}
{e}
{a}

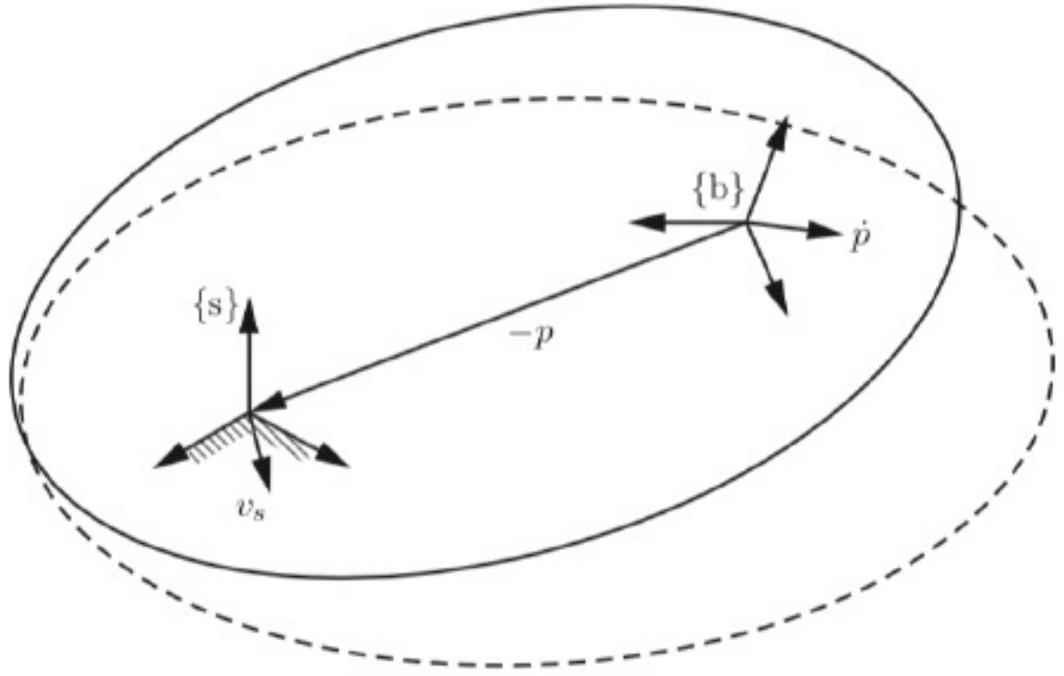
Moving Frames: linear and angular velocity (1)

Moving Frames: linear and angular velocity (2)

Linear and angular velocity in reference frame

Physical interpretation of v_s

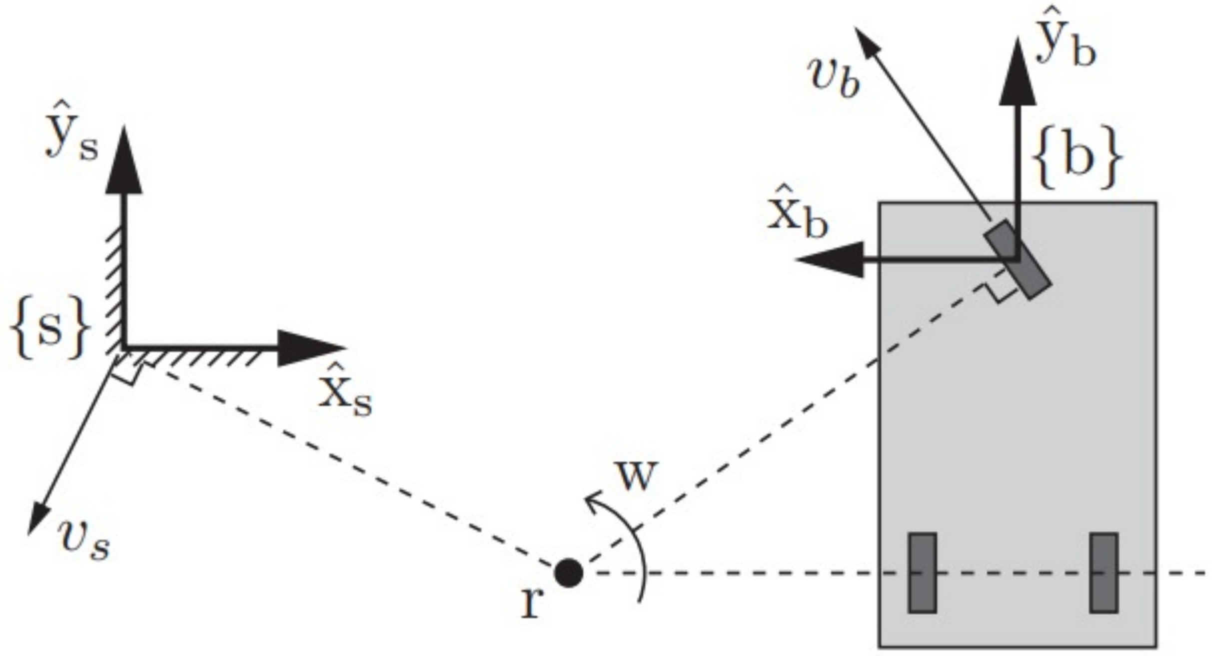
- Assume that a very large rigid body is attached to the frame {b}, and is large enough to contain the origin of {s}
- What is the velocity of the point of this body as the origin of {s}?
- There are two components:
  - $\dot{p}$: the motion of the body
  - $\omega_s \times (-p)$: the rotation of the body



Twists

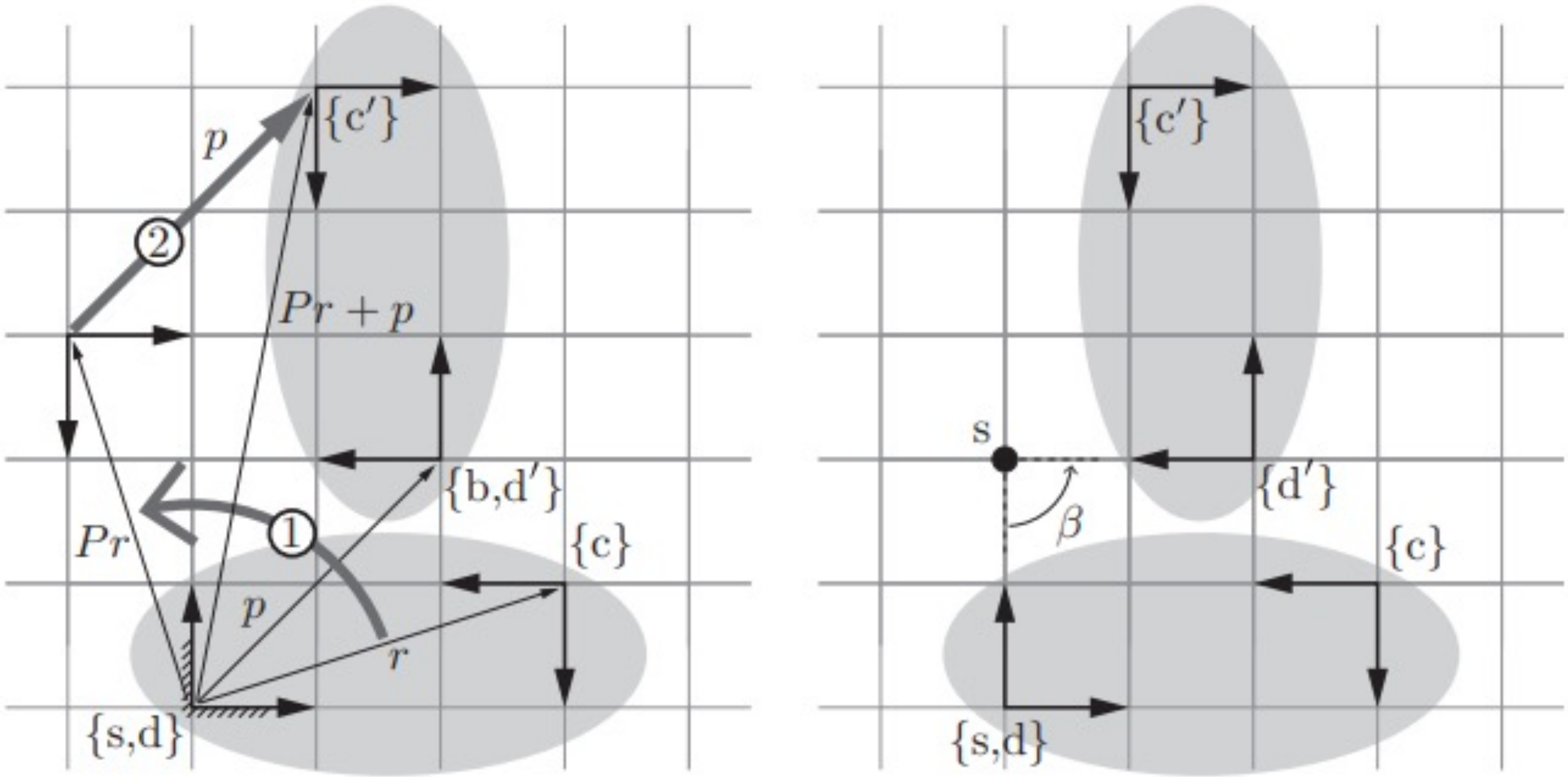
Adjoint Map associated with T

Twist Visualization



The twist corresponding to the instantaneous motion of the chassis of a three-wheeled vehicle can be visualized as an angular velocity w about the point r.

Screw Motions



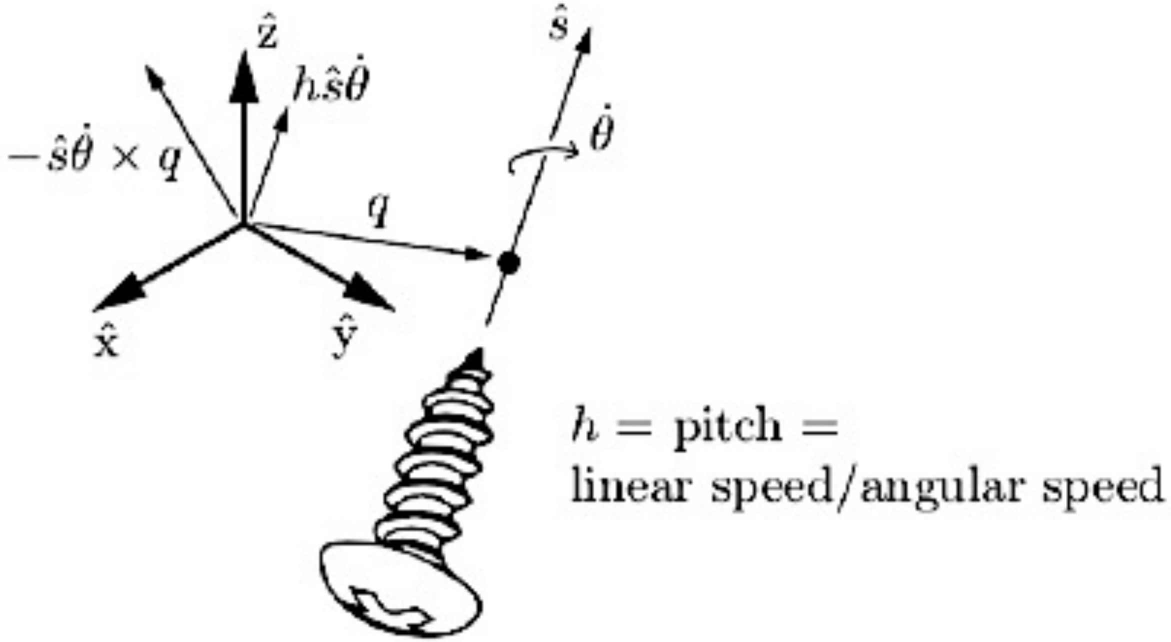
2D Screw Motions

- Any rigid motion in the plane can be represented by a rotation around a well-chosen center
- We can encode it with (β, s_x, s_y), where (s_x, s_y) is the position of the center of the rotation, and β the angle

- Recall that the angular velocity ω can be viewed as $\hat{\omega}\dot{\theta}$, where $\hat{\omega}$ is the unit rotation axis and $\dot{\theta}$ is the rate of rotation
- A twist $\mathcal{V}$ can be interpreted in terms of a screw axis $\mathcal{S}$ and a velocity $\dot{\theta}$ about that axis

Screw motions in 3D

Chasles-Mozzi Theorem:
Any displacement in 3D can be represented by a rotation and translation about the same axis, referred to as a screw motion



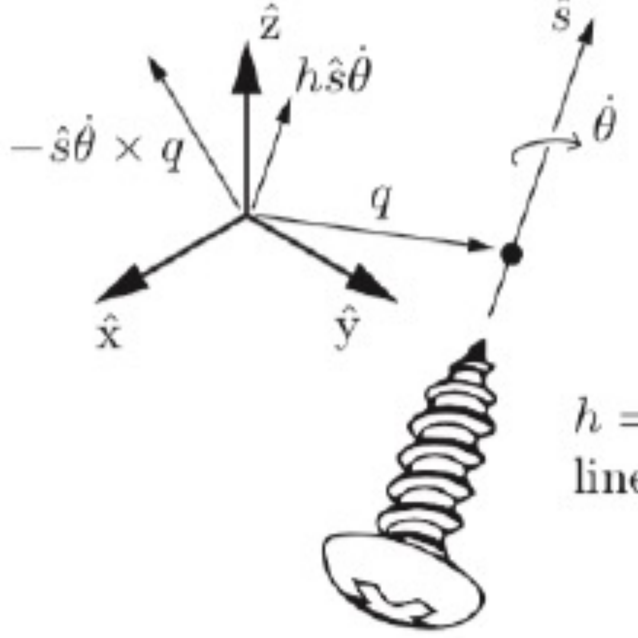
$q \in \mathbb{R}^3$ is any point along the axis

$\hat{s}$ is a unit vector in the direction of the axis

h is the **screw pitch**, which is the ratio between the linear and angular speed along axis

We write this as $\mathcal{S} = \{q, \hat{s}, h\}$

Screw motions and Twists



Screw axis

Exponential Coordinates for Rigid Body Motions

Logarithm of rigid body motions

Summary

- Used transformations to define spatial velocities as **body and spatial twists**
- Introduced **screw motions** and the screw interpretation of a twist
- The **screw axis $\mathcal{S}$** was defined and used to define the **exponential coordinates of homogeneous transformations**
- Connections between last lecture and this lecture:
  - Transformation matrix T is analogous to rotation matrix R
  - A screw axis $\mathcal{S}$ is analogous to rotation axis $\widehat{\omega}$
  - A twist $\mathcal{V}$ can be expressed as $\mathcal{S}\dot{\theta}$ and is analogous to angular velocity $\omega = -\widehat{\omega}\dot{\theta}$
  - Exponential coordinates $\mathcal{S}\theta \in \mathbb{R}^6$ for rigid body motions are analogous to exponential coordinates $\widehat{\omega}\theta \in \mathbb{R}^3$