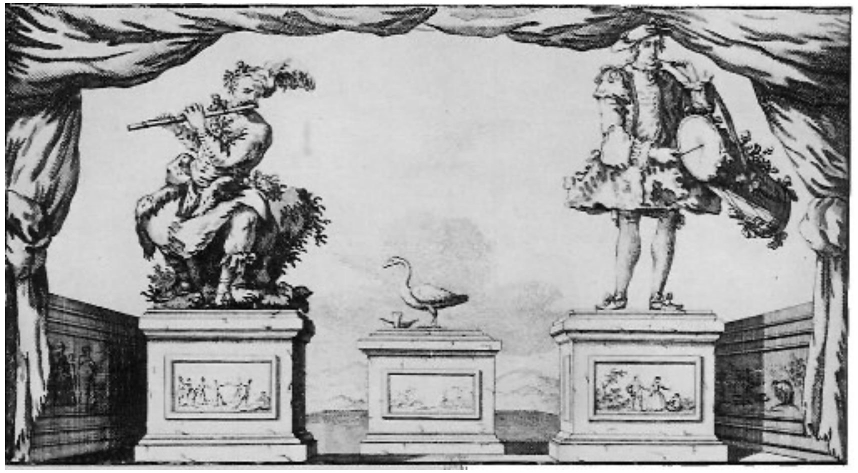
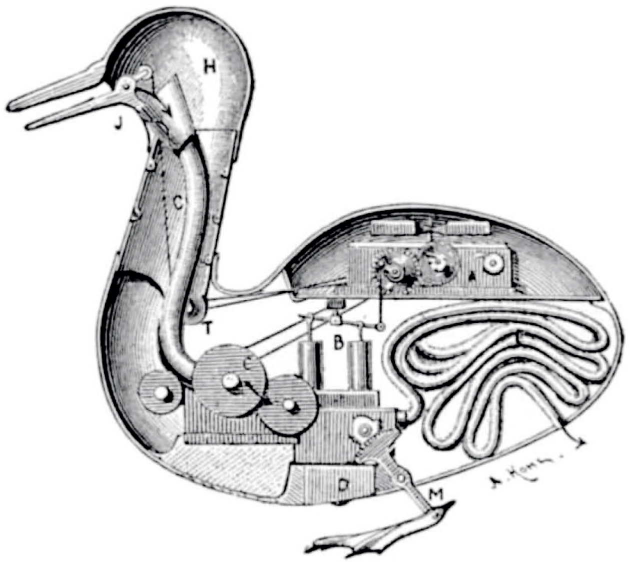
OH: 2PM Tuesday CSL 166.

Vaucanson’s Duck

- On 30 May 1739 in France, Jacques de Vaucanson unveiled a mechanical duck
- The duck had over 400 moving parts in each wing alone, and could flap its wings, drink water, “digest” grain, and seemingly defecate
- Robert-Houdin described this as “a piece of artifice I would happily have incorporated in a conjuring trick”
- Voltaire wrote in 1741 that “Without the voice of le Maure and Vaucanson's duck, you would have nothing to remind you of the glory of France."



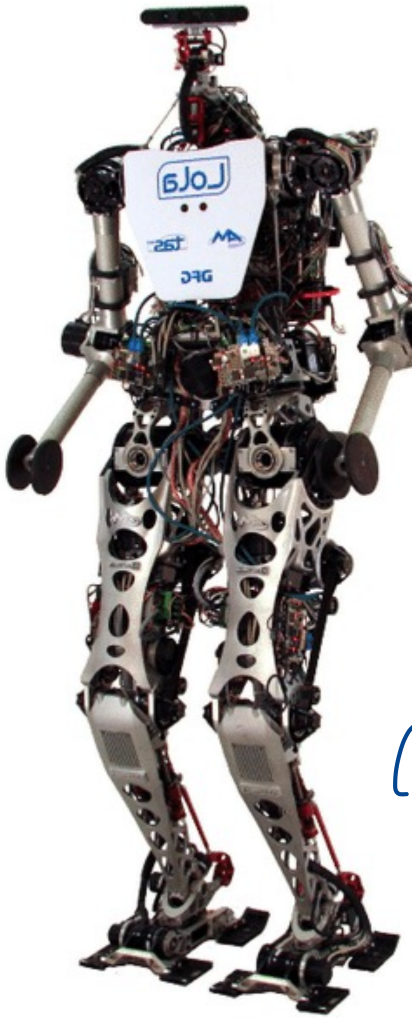


INTERIOR OF VAUCANSON'S AUTOMATIC DUCK.

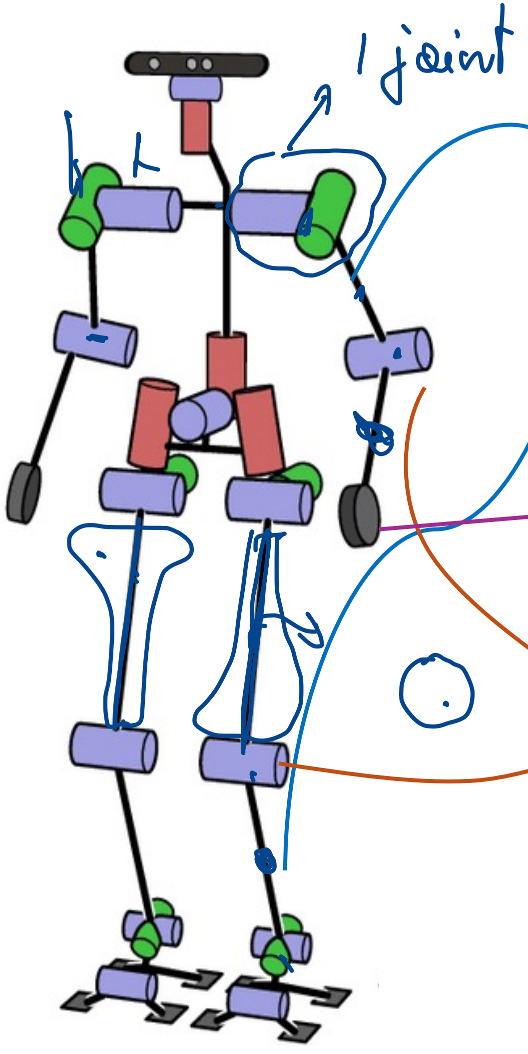
A, clockwork; *B*, pump; *C*, mill for grinding grain; *F*, intestinal tube; *J*, bill; *H*, head; *M*, feet.

Credit: Wikipedia

link – joint – link.



Represented by



Links
The rigid structures

End-effector
Typically a gripper; attached to a specific link

Joints
The connection between two robot links that allows relative motion between them

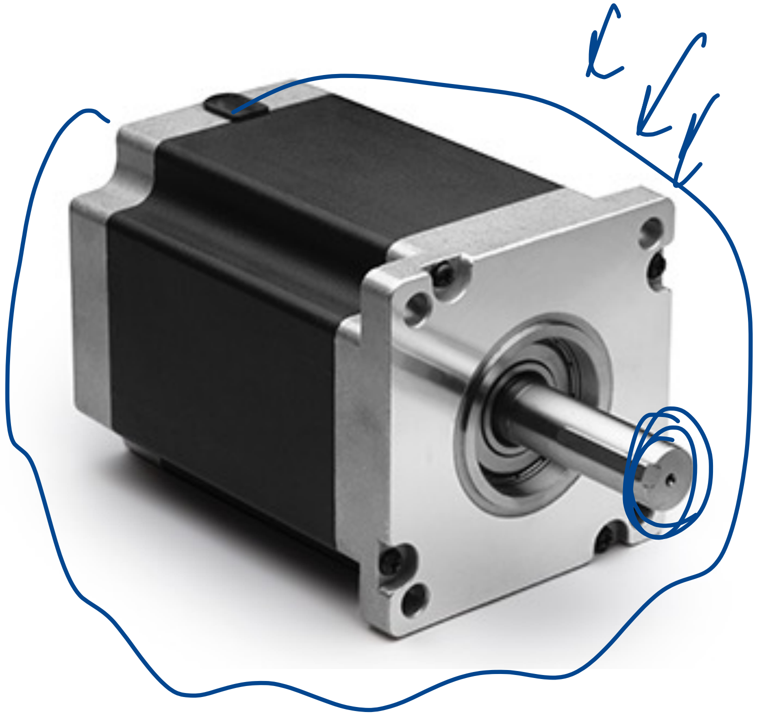
#DOF
(degrees of freedom)
of a robot
mechanism.

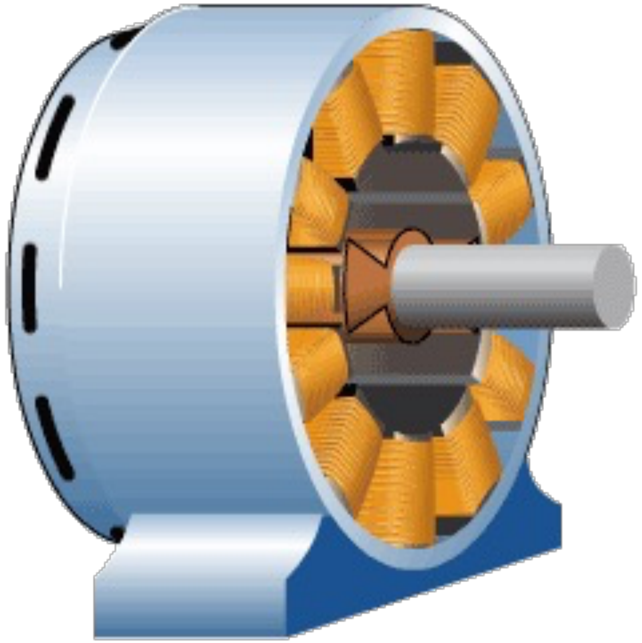
Figure from "Kinematic optimization for bipedal robots: a framework for real-time collision avoidance" by Hildebrandt, et al. 2019

Why "rotating"?

REVOLUTE joints

- Most robots use motors as actuators





Actuators supply the motive power for robots, provide force and torque

Some Definitions (2)

DOFs.

(x_1, y_1) 4 variables

(x_2, y_2)

-1 Constraint

$\parallel$

3 DOFs

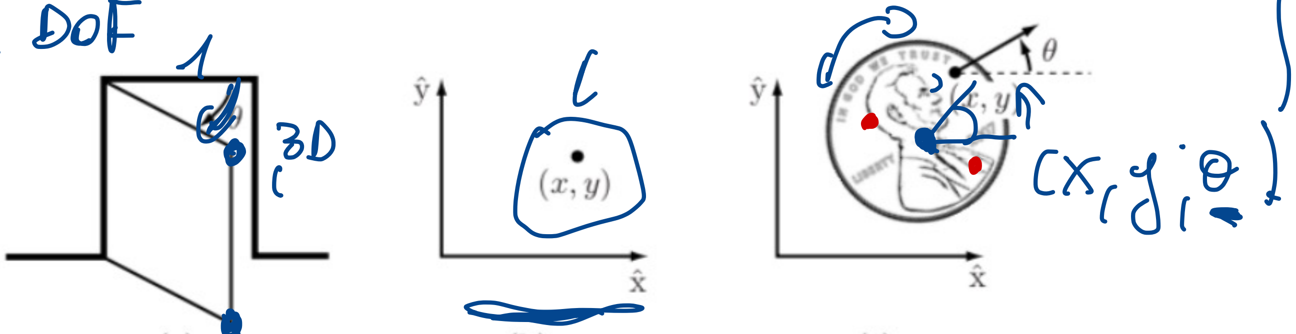
- We use our knowledge of the links, joints, and actuators to represent the **configuration** of the robot

$d_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

- The **configuration space (C-space)** is the n-dimensional space containing all possible configurations of the robot

Rigid object in 2D: 3 DOFs

1 DOF



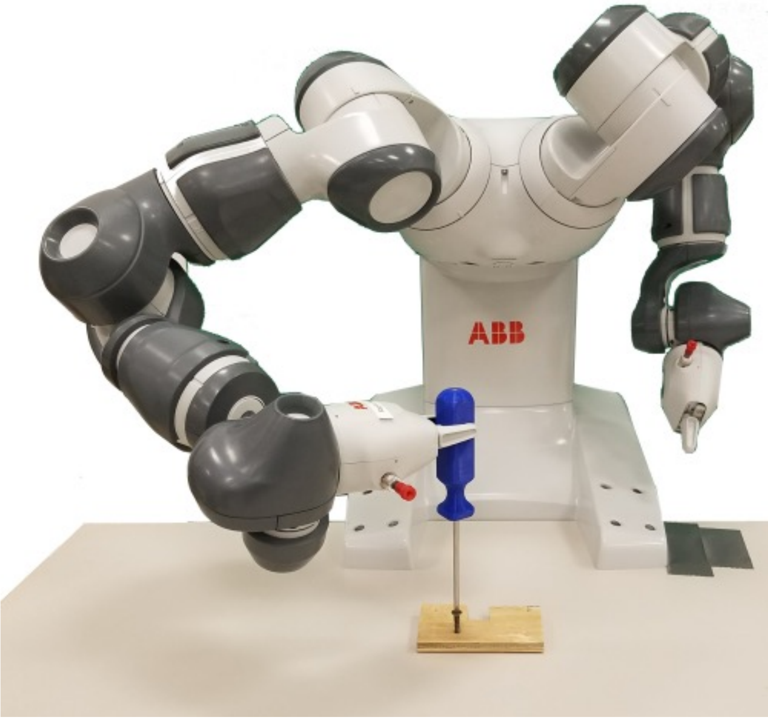
(a) (b) (c)

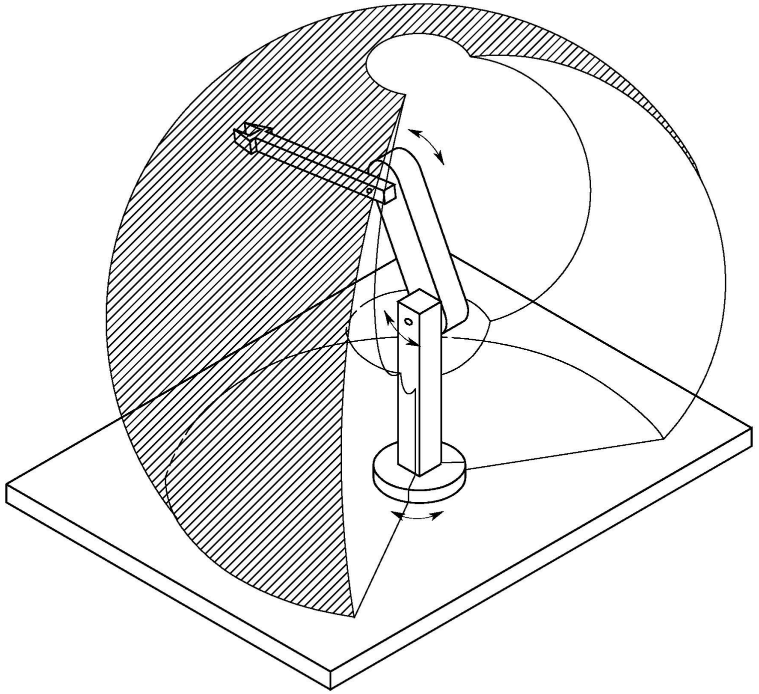
- The number of **degrees of freedom** of the robot is the smallest number of coordinates needed to represent the configuration

More Definitions (3)

- The **task space** is the space in which the robot's task can be naturally expressed
  - For manipulation, a natural representation is the C-space of the robot's end-effector
  - Driven by the task, independent of the robot
- The **workspace** is the specification of the configurations that the end-effector of the robot can reach
  - Driven by the robot structure, independent of the task
- Both involve choice by the user and are distinct from the robot's C-space

dim C-space = # dofs





Degrees of Freedom of a Rigid Body (1)

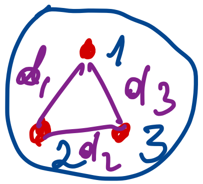
For a rigid body in 2D.

3 - DOFs : (x, y, θ)

= position of a point rigidly attached to the object & orientation.

Explicit : (x, y, θ)

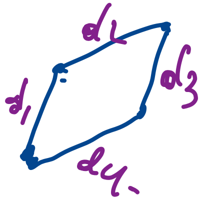
Implicit : (x_1, y_1, x_2, y_2) + 1 constraint.



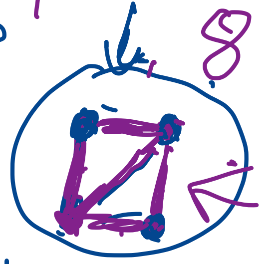
$(x_1, y_1, x_2, y_2, x_3, y_3)$.

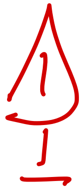
$d_4 = d_1 + d_2 = c_1 + c_2$

$d_1 = c_1$
$d_2 = c_2$
$d_3 = c_3$



$8 - 5$





Constraints need to be independent

When are constraints independent?

$$g_1(x) = 0 \qquad \left(\text{here, } x = \begin{pmatrix} x_1 \\ x_2 \\ \theta \\ y_1 \\ y_2 \end{pmatrix}\right)$$

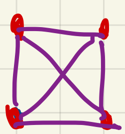
eg: $g_1(x) = \sqrt{(x_1-y_1)^2+(x_2-y_2)^2} - 3 = 0$

↳ constraint says $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ & $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ are at distance 3 of each other.

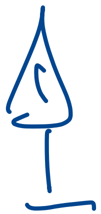
$g_1 \dots g_p$

$$\begin{pmatrix} g_1 \\ \vdots \\ g_p \end{pmatrix} \xrightarrow{\text{Jacobian}} \begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial g_p}{\partial x_1} & \cdots & \frac{\partial g_p}{\partial x_n} \end{pmatrix}$$

Result: Constraints are independent if Jacobian is of full row rank.

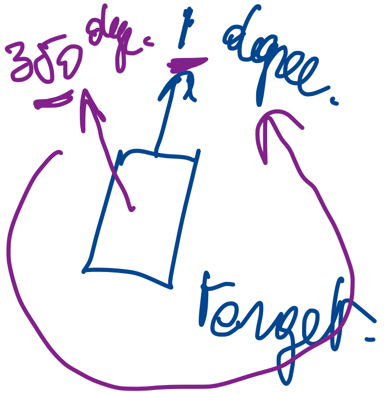


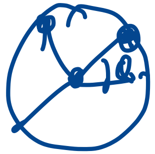
Degrees of Freedom of a Rigid Body (2)

*  θ is not a "real number", it is "periodic".

$$\theta = \theta + 2k\pi \quad \text{for } k \in \mathbb{Z} \rightarrow \text{integers.}$$

Winding phenomenon.





$$\theta \in S^1 = \text{circle}$$

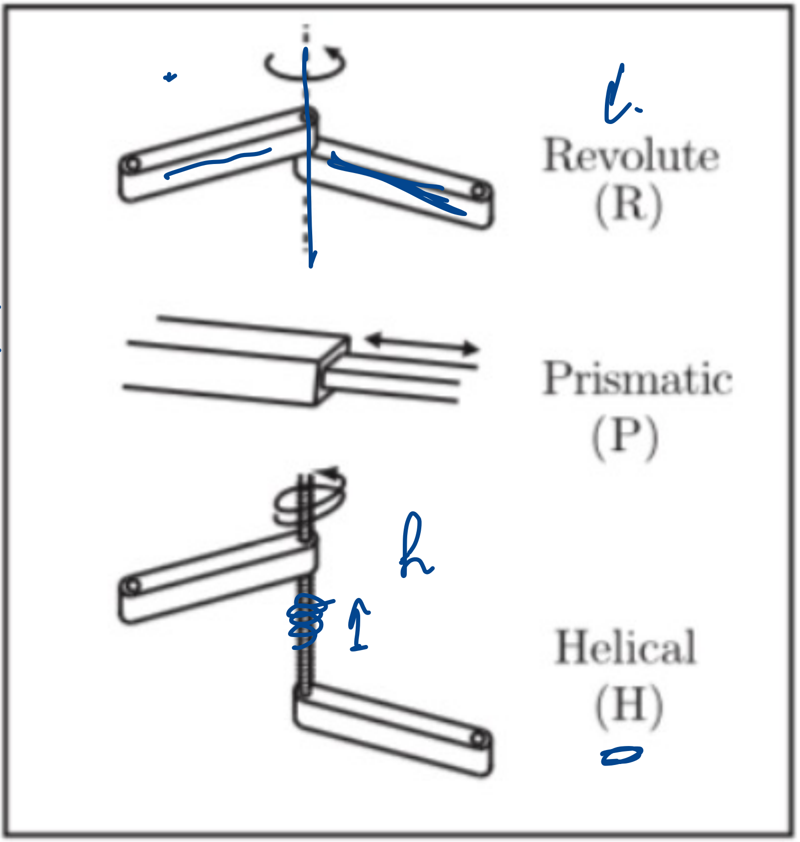
* Temperature. :

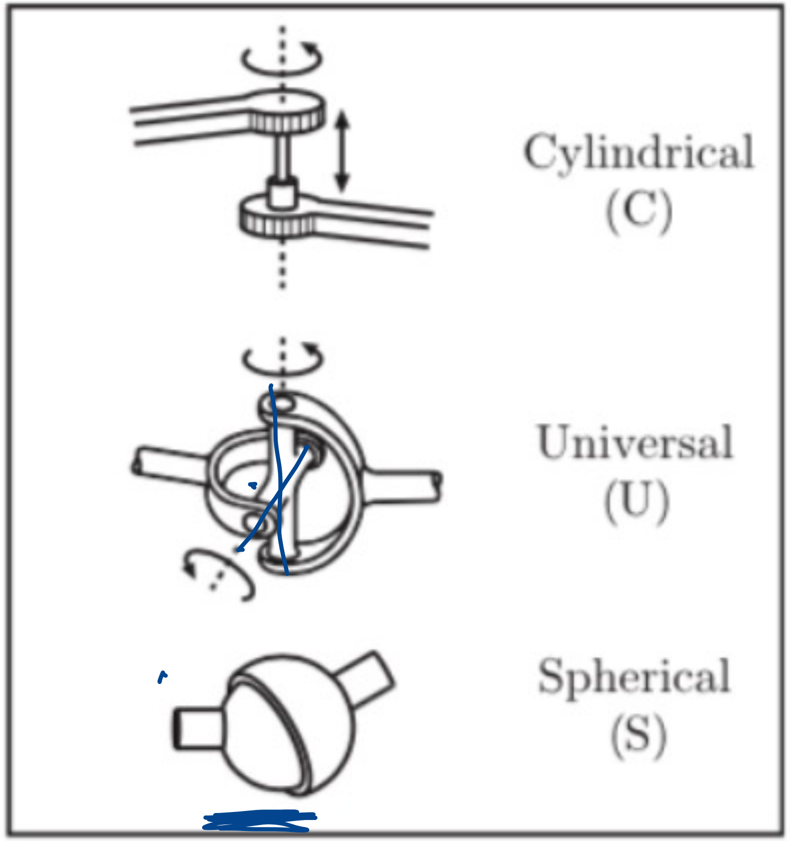
Rigid object in 3D : 6 DOFs
2D 3 DOFs.

# of dofs of rotations in n-space: $\frac{n(n-1)}{2}$

Degrees of Freedom of a Robot

DOF	C_i constraints (3D/2D)
1	5/2
1	5/2
1	5





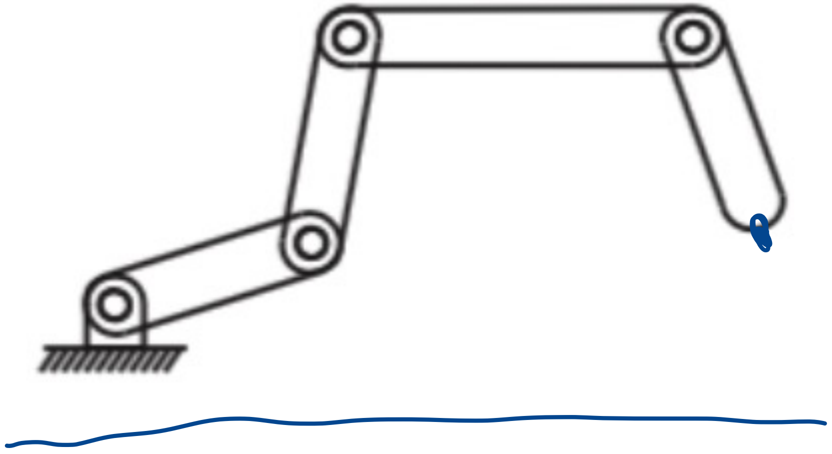
DOF	C_i Const.
2	6-2 = 4
2	4
3	6-3 = 3

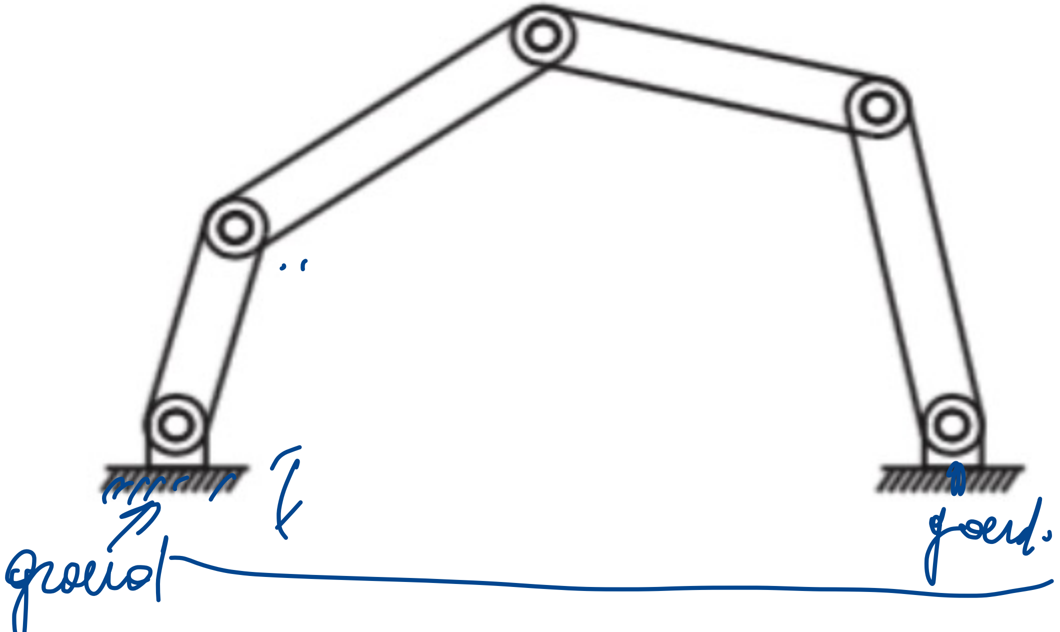
Passive vs active joints

Even More Definitions

hybrid

- **Closed-chain mechanisms** are mechanisms that have a closed loop between the links and the ground
- **Open-chain mechanisms** (serial mechanisms) is any mechanism without a closed loop





Grubler's Formula : # DOFS

robot with N = # links + 1 (ground)

J = # joints

for joint i, f_i = # dofs

$c_i = m - f_i$

$= \underline{\#\text{ constraints}}$

$$m = \begin{cases} 6 & \text{if robot is in 3D spatial} \\ 3 & \text{2D planar.} \end{cases}$$

Then: $\# DOFs = m(N-1) - \sum_{i=1}^{J} c_i$ Grubler formula

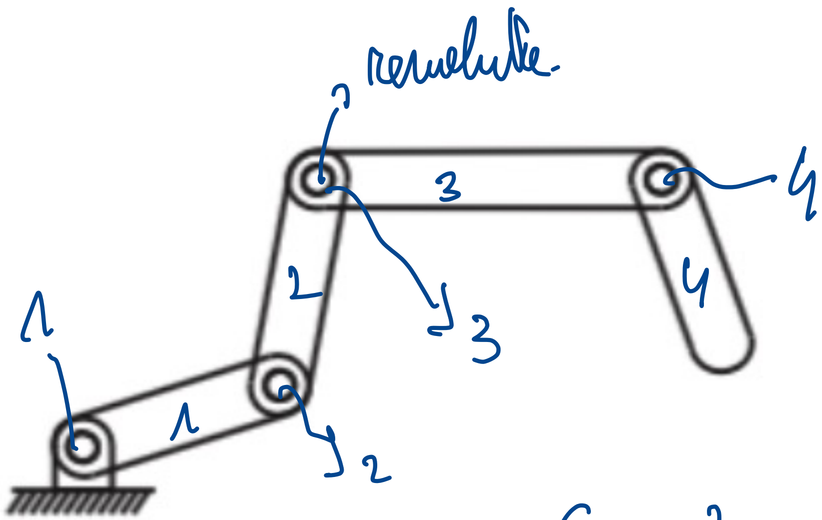
$$= m(N-1) - \sum_{i=1}^{J}(m - f_i) = m(N-J-1) - \sum_{i=1}^{J} f_i$$

Grubler's Formula – Example

$m = 3$ or 6

2D, 3D

#DOFs



$N = 5 \ (4+1)$

$m = 3$ # of DOFs of a link.

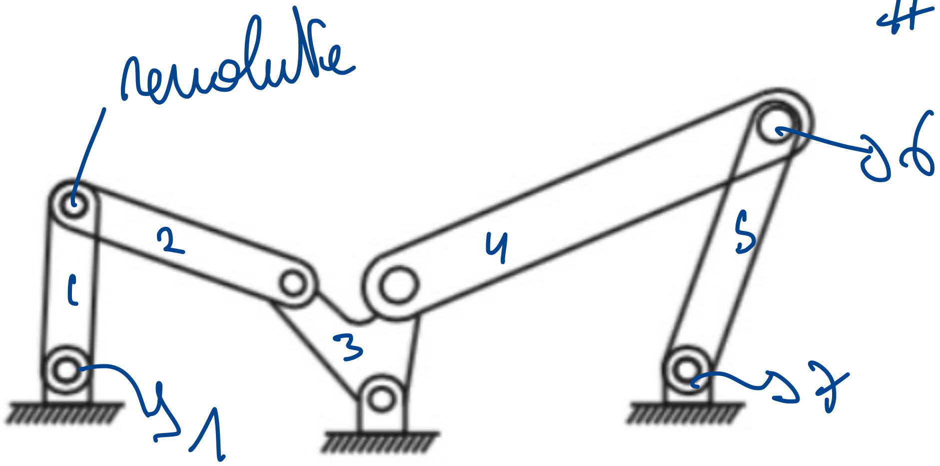
$J = 4$

$$\#DOFs = 3 \cdot 4 - \sum_{i=1}^{4} 2 = 4.$$

$c_1 = 2$

$c_2 = 2.$

#DOFs.



$N = 5 + 1 = 6$

$J = 7, \quad c_i = 2 \quad i = 1, \dots 7.$

$m = 3$

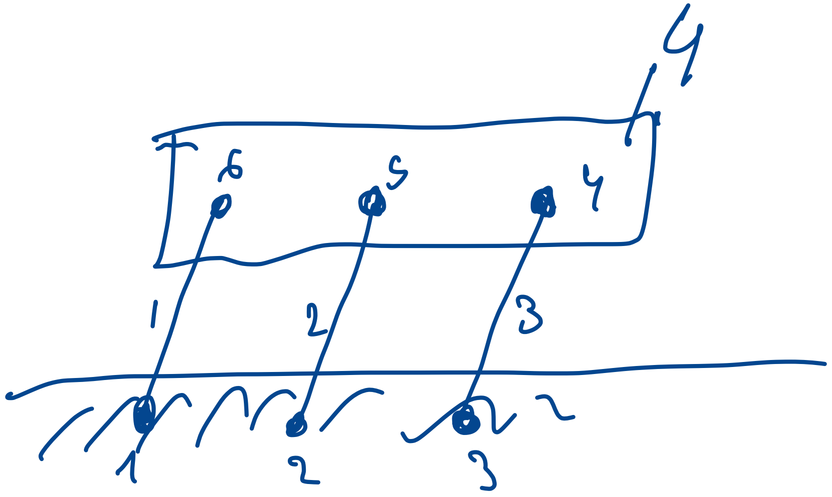
$f_i = m - 2 = 1$

$$\#DOFs = 3 \cdot 5 - \sum_{i=1}^{7} 2 = 1$$

Redundant Constraints

Gruebler formula is a simple count.

FAILS if constraints are dependent.

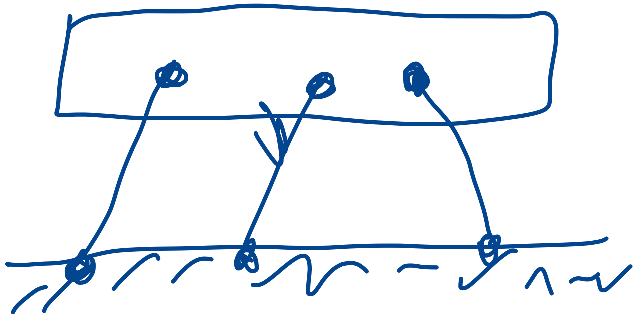


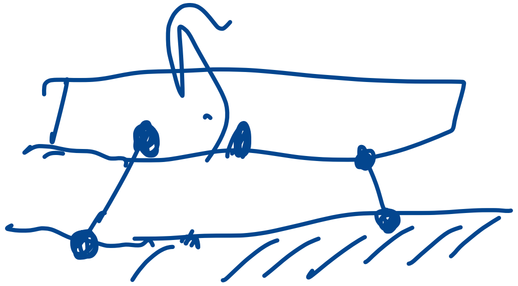
$N = 5$

$c_i = 2$ for $i = 1 \ldots 6 = J$

$m = 3$

$DOF_1 = 3(4) - 6 \cdot 2 = 0$





$(x \quad y)$ 2DOF

1 constr. $x + y = 0$

↳ 1 DOF

$x \rightsquigarrow y = -x$

$(x \quad y)$

$$\boxed{x^2 + y^2 = 0}$$