ECE 470: Intro to Robotics
ME 445
AE 482

HW: - PrairieLearn (sign up link will be shared)

- no HW this week

due dates: Fridays at 11.59PM!

<u>EXAMS</u>: - 3 "in class" exams

↳ CBTF

(No "Final" or no exam during exam period)

· dates of exams TBA.

exams are cumulative w/ emphasis on later material.

- Final project.

ROS (Robot Operating System) (in lab)

HW: due Fridays at 11:59 PM.

Topics

1. Mechanism design

links & joints

"hard" robot *

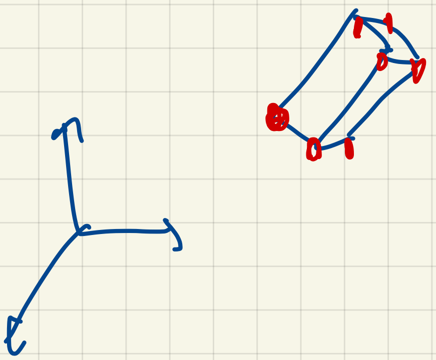
"soft" robot.

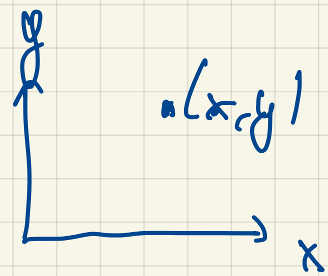
dofs (degree of freedom)

REVOLUTE ← revolute, spherical, prismatic.

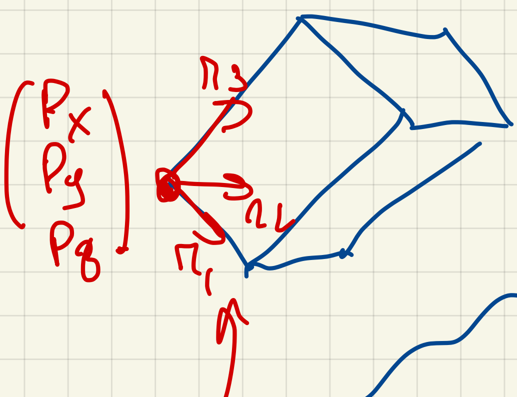
2. Rigid body motions.

position or state of a rigid object in 3D.





EXPONENTIAL COORDINATES

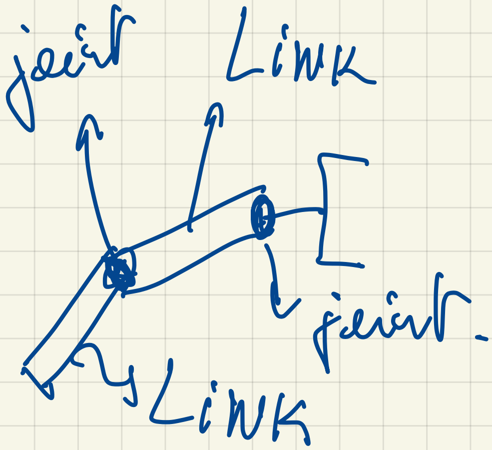


$$\underbrace{\begin{pmatrix} r_1 & r_2 & r_3 & P_x \\ | & | & | & P_y \\ | & | & | & P_z \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{rigid body coordinate}}$$

Affine transformations.

(encodes the desired motion) $\begin{pmatrix} r_1 & r_2 & r_3 & p \\ & & & \vdots \\ 0 & 0 & 0 & 1 \end{pmatrix}$ = (new position)

3. Forward Kinematics map.



· actuate the joint.

⇝ where is the hand?

→ FORWARD KINEMATICS MAP.

$$\underbrace{\theta_1 , \theta_2 , \theta_3 \ldots}_{\text{joint angles}} \longmapsto \underbrace{\begin{pmatrix} | & | & | & p_x \\ r_1 & r_2 & r_3 & p_y \\ | & | & | & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{position of} \\ \text{end effector} \\ \text{hand of robot.}}}$$

Exponential coordinates allow for a very systematic writing of this map.

4. Jacobian or Velocity Kinematics

$$f : \mathbb{R}^2 \to \mathbb{R}^2 : \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x^2 y \\ y^2 \end{pmatrix} \begin{matrix} \to f_1 \\ \to f_2 \end{matrix}$$

Jacobian is $\dfrac{\partial f}{\partial x} := \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix}$

$$f\left(\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}\right) = f(x,y) + \underbrace{\frac{\partial f}{\partial x}\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}}_{\text{order } 1} + \underset{\text{hessian}}{\underset{\uparrow}{h.o.t}}$$

↑

TAYLOR series

in this class: compute the $\boxed{\text{Jacobian}}$ of the forward kinematics map.

uses of Jacobian

· Dynamics

· $\rightsquigarrow$ Inverse kinematics *

· statics * Wrench

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5. Inverse Kinematics.

$\theta_1 \dots \theta_n \longmapsto$ position of robot

Position $\longmapsto \theta_1, \dots \theta_n$

· analytic, & numerical method
(for specific designs)

· Motion planning ; Control ; Dynamics

Vector $x \in \mathbb{R}^n$: $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ $x_i \in \mathbb{R}$

added, multiplied by a constant

inner product : $x^T y$

$$[x_1 \dots x_n] \begin{bmatrix} y_1 \\ \vdots \\ y \end{bmatrix} = \sum x_i y_i$$
$$= x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

· Vectors x, y, z are linearly independent if:

$$\overset{\in \mathbb{R}}{a_1} x + \overset{\in \mathbb{R}}{a_2} y + \overset{\in \mathbb{R}}{a_3} z = 0$$
$$\Leftrightarrow a_1 = a_2 = a_3 = 0$$

Rank of a matrix.

$$A \in \mathbb{R}^{n \times m} = \begin{bmatrix} a_{11} & \cdots & a_{1m} \\ & \ddots & \\ a_{n1} & \cdots & a_{nm} \end{bmatrix}$$

row. column

row

A is full rank if

colum rows of A are linearly indep.

colums

$$Ax = \begin{bmatrix} a_{11} & \cdots & a_{1m} \\ a_{n1} & \cdots & a_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m \\ \\ a_{n1}x_1 \cdots + a_{nm}x_m \end{bmatrix}$$

$A \in \mathbb{R}^{n \times m}$ $\quad$ $B \in \mathbb{R}^{m \times p}$

$$AB \in \mathbb{R}^{n \times p}$$

$$C = AB.$$

$$= \begin{bmatrix} c_{11} & c_{1p} \\ c_{n1} & c_{np} \end{bmatrix}$$

$$c_{11} = \sum_j a_{1j} b_{j1}$$

$$c_{ik} = \sum a_{ij} b_{jk}$$