Lecture 02: Differential Equations Review

Prof. Katie Driggs-Campbell
January 19, 2023
Where does the word *robot* come from?

- The term *robot* was first used in a play published by the Karel Čapek in 1921
  - Plays off the Czech word *robota*, meaning servitude
- R.U.R. (Rossum's Universal Robots) was a satire, robots were manufactured beings that performed all unpleasant manual labor
- The term robot has replaced the popular use of the word automation and/or android; however, the robots in the play are not mechanical devices, but artificial biological organisms that may be mistaken for humans
Announcements

• Homework 1 deadline extended to next Friday
• Reach out to me if you did not receive the email with the important links
Course Components

– 10% Reflections
– 20% Homework
– 40% Exams (13.333% each)
– 20% Laboratory
– 10% Final Lab

+ Extra Credit
Robot States and the Environment

Robotic System

Perceptual/action data

Environment, state

World model, belief

Actions
Robot States and the Environment

• **State** represents the environment as well as the robot, for example:
  • location of walls or objects
  • *pose of the robot*

• **Environment interaction** comes in the form of
  • Sensor measurements
  • Control actions

• **Internal representation (or belief)** of the state of the world
  • In general, the state (or the world) cannot be measured directly
  • Perception is the process by which the robot uses its sensors to obtain information about the state of the environment
Now back to review notes!

Inspiration from Lukas Luft and Wolfram Burgard
Diff. Eq. Notes from Roy Dong
\[ x(t + \Delta t) = x(t) + v \cdot \Delta t \]

In general, we describe motion w/ diff eq.

\[ \frac{dx}{dt} = \dot{x}(t) = f(x(t)) \]

what does it mean when we say a fn \( x(t) \) satisfies the diff eq?

1. Take fn \( x(t) \)
2. Differentiate it: \( \dot{x}(t) \)
3. Plug in to \( f(x(t)) \)
4. Does it match? \( \Delta t \)
\[8\]

\[
\begin{align*}
\Rightarrow x(t) &= \exp(at) = e^t \\
\text{Does this satisfy } &\dot{x} = a\underline{x} \text{?} \\
\dot{x}(t) &= a \, e^t \\
f(x(t)) &= a \, x(t) = a \, e^t \quad \checkmark
\end{align*}
\]
Given an initial condition $x(0)$ and a diff eq: $\dot{x} = f(x)$, there is exactly one function $x(t)$ that satisfies both.

$$\begin{align*}
\text{ex} & \quad \dot{x} = ax, \quad x(0) = c \\
\rightarrow & \quad x(t) = ce^{at}
\end{align*}$$

we can put the diff eq in integral form:

$$x(t) = x(0) + \int_0^t f(x(s)) \, ds$$
Recall:
\[ e = \lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^n \]

Note that \( e^t \) is the function whose derivative is itself:
\[ \frac{d}{dt} e^t = e^t \]

If we consider the solution to:
\[ x(0) = 1, \quad x(t) = x(t) \]

\( e^t \) is defined as \( x(1) \)
Let's look at the derivative w/ small steps:
\[ \dot{x} = f(x) \Rightarrow x(t + \frac{1}{n}) \approx x(t) + \frac{1}{n} f(x(t)) \]
\[ \dot{x} = x \Rightarrow x(t + \frac{1}{n}) \approx (1 + \frac{1}{n}) x(t) \]
\[ x(0) = 1, \quad x(\frac{1}{n}) \approx 1 + \frac{1}{n} \]
\[ x(\frac{2}{n}) \approx (1 + \frac{1}{n})(1 + \frac{1}{n}) = (1 + \frac{1}{n})^2 \]
\[ \vdots \]
\[ x(1) \approx (1 + \frac{1}{n})^n \]

If \( n \) is large (step size is small),
\[ x(1) = \lim_{n \to \infty} (1 + \frac{1}{n})^n \]
Let's look for a solution to: \( \dot{x} = x \)

Suppose it's an infinite polynomial with \( x(0) = 0 \):

\[
x(t) = \sum_{n=0}^{\infty} a_n t^n
\]

\[
\dot{x}(t) = \sum_{n=1}^{\infty} n a_n t^{n-1}
\]

Can \( x(t) = \dot{x}(t) \) for all \( t \)?

1. \( a_0 = 1 \)
2. \( 2a_2 = a_1 \)
3. \( 3a_3 = a_2 \)

\[
a_1 = 1 \\
a_2 = \frac{1}{2} \\
a_3 = \frac{1}{6} \\
\vdots \\
a_n = \frac{1}{n!}
\]