Lecture 02: Differential Equations Review

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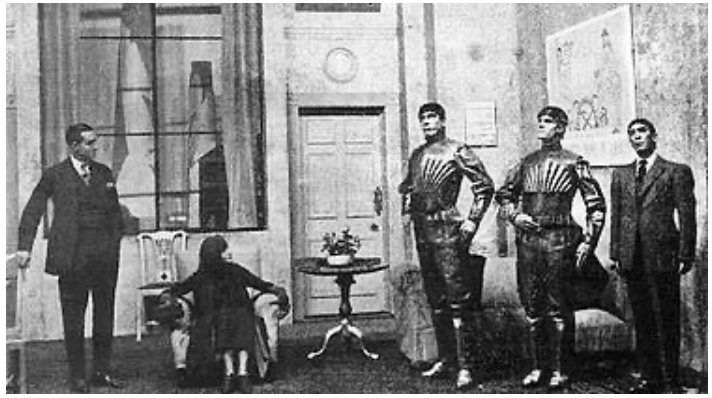
January 19, 2023

Where does the word *robot* come from?

- The term *robot* was first used in a play published by the Karel Čapek in 1921
  - Plays off the Czech word *robota*, meaning servitude
- R.U.R. (Rossum's Universal Robots) was a satire, robots were manufactured beings that performed all unpleasant manual labor
- The term robot has replaced the popular use of the word automation and/or android; however, the robots in the play are not mechanical devices, but artificial biological organisms that may be mistaken for humans



Announcements

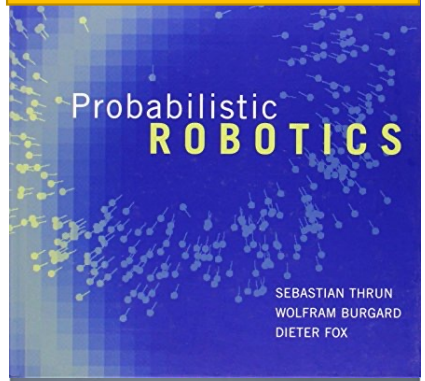
- Homework 1 deadline extended to next Friday
- Reach out to me if you did not receive the email with the important links

Course Components

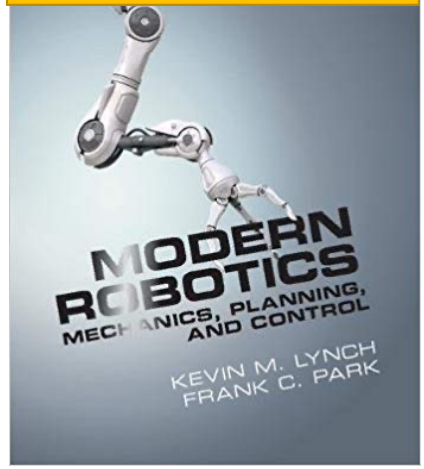
– 10% Reflections
– 20% Homework
– 40% Exams (13.333% each)
– 20% Laboratory
– 10% Final Lab

+ Extra Credit

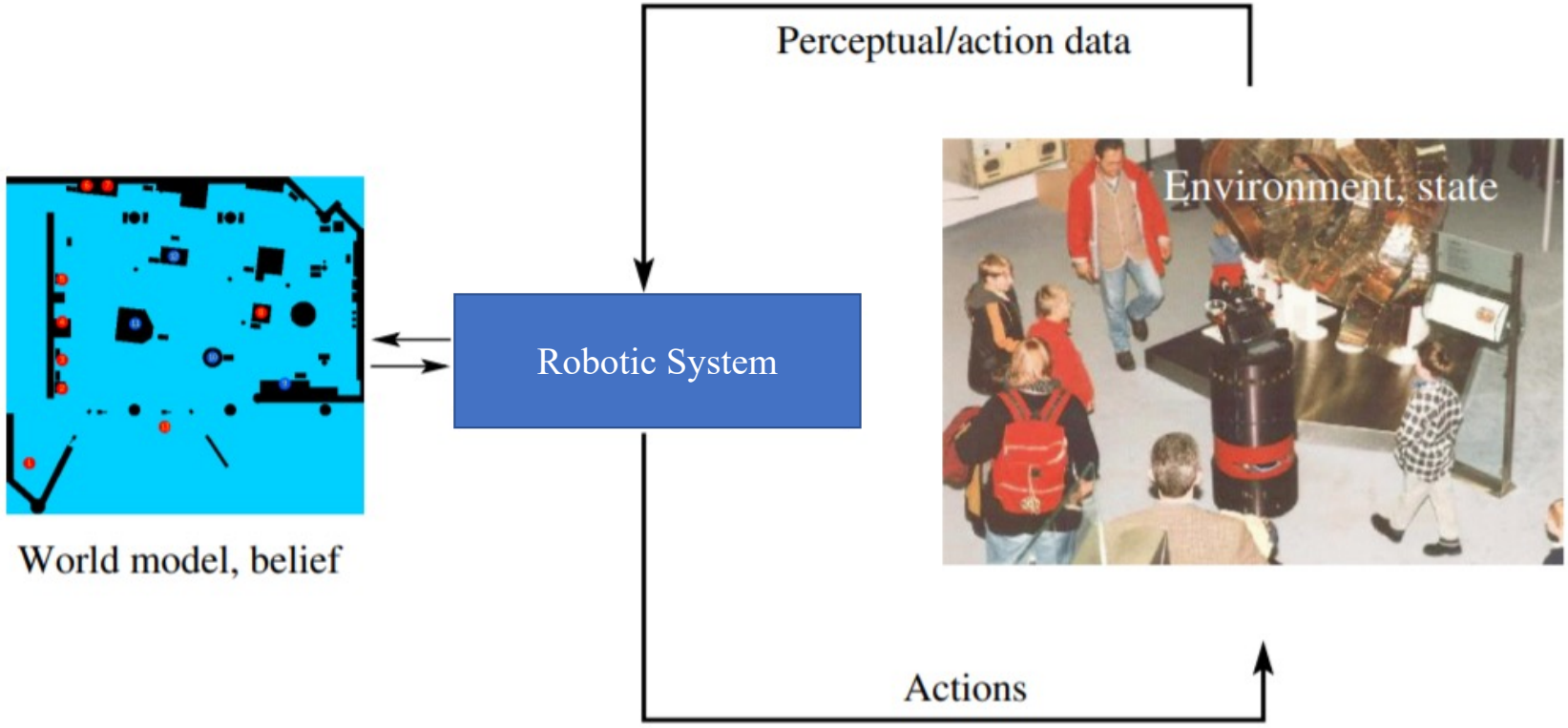
Suggested:



Required:



Robot States and the Environment



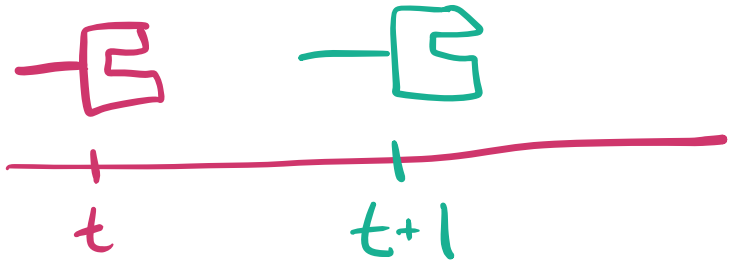
Robot States and the Environment

- **State** represents the environment as well as the robot, for example:
  - location of walls or objects
  - *pose of the robot*
- **Environment interaction** comes in the form of
  - Sensor measurements
  - Control actions
- **Internal representation (or belief)** of the state of the world
  - In general, the state (or the world) cannot be measured directly
  - Perception is the process by which the robot uses its sensors to obtain information about the state of the environment

Now back to review notes!

Inspiration from Lukas Luft and Wolfram Burgard

Diff. Eq. Notes from Roy Dong



$$x(t+\Delta t) = x(t) + v \cdot \Delta t$$

In general, we describe motion w/ diff eq.

$$\frac{dx}{dt} = \dot{x}(t) = f(x(t)) = \underline{\dot{x} = f(x)}$$

What does it mean when we say a fn $x(\cdot)$ satisfies the diff eq?

① take fn $x(\cdot)$ ② differentiate it: $\dot{x}(t)$

③ plug in to $f(x(t))$ ④ does it match? $\forall t$

ex

$\Rightarrow x(t) = \exp(at) = e^{at}$

Does this satisfy $\dot{x} = \underline{ax}$?

$\dot{x}(t) = a e^{at}$

$f(x(t)) = a x(t) = a e^{at}$ ✓

x_0

Given an initial condition $x(0)$ + a diff eq: $\dot{x} = f(x)$, there is exactly one function $x(t)$ that satisfies both

ex $\dot{x} = ax$, $x(0) = c$

$\rightarrow x(t) = c e^{at}$

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we can put the diff eq in integral form:

$$x(t) = x(0) + \int_0^t f(x(s))\,ds$$

Recall:

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

note that e^t is the function whose derivative is itself:

$$\frac{d}{dt} e^t = e^t$$

If we consider the soln to:

$x(0) = 1, \quad \dot{x}(t) = x(t)$

$\underline{e}^1$ is defined as $x(1)$

Let's look at the derivative w/ small steps:

$$\dot{x} = f(x) \rightsquigarrow x(t + 1/n) \approx x(t) + 1/n\, f(x(t))$$

$$\dot{x} = x \rightarrow x(t + 1/n) \approx (1 + 1/n)\, x(t)$$

$$x(0) = 1, \quad x(1/n) \approx 1 + 1/n$$

$$x(2/n) \approx (1 + 1/n)(1 + 1/n) = (1 + 1/n)^2$$

$$\vdots$$

$$x(1) \approx (1 + 1/n)^n$$

if n is large (step size is small),

$$x(1) = \lim_{n \to \infty} (1 + 1/n)^n$$

let's look for a soln to: $\dot{x} = x$

suppose it's an infinite polynomial w/ $x(0)=0$

$$x(t) = \overset{a_0}{1} + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 \cdots$$

$$\dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2 + \cdots$$

can $x(t) = \dot{x}(t)$ $\forall t$?

$a_1 = 1$

$2a_2 = a_1$

$3a_3 = a_2$

$a_1 = 1$

$a_2 = 1/2$

$a_3 = 1/6$

$\vdots$

$a_n = 1/n!$

$$e^t = \sum_{n=0}^{\infty} \frac{1}{n!} t^n$$