Administrivia

- HW6 is due next Friday (11/12) at 8pm
- HW7 will be due Friday (11/19) at 8pm (posted next week)

- Participation grade update:
  - Guest lecture attendance
  - Self-evaluation + staff evaluation
  - (bonus) A fun contribution to the course (ex: a robot fun fact; silly meme)

- Extra Credit Opportunities
  - Lab 6 will contribute up to 10% towards Exam 1
    - More information will be given in lab!
  - Example video will contribute up to 5% towards your overall grade
Administrivia – Extra Credit Video

• Submit an informational/tutorial video for some course topic
• The video should be ~5 minutes in length. You will be graded on the following:
  – How challenging is the topic?
  – Did you provide an interesting motivating example?
  – How well did you explain the topic?
  – Did you provide an informative worked example?
  – How effective are the visualizations used?
• Check out examples on the website, and/or 3Blue1Brown or the Kahn Academy on YouTube
Administrivia - Project

• Project Update 2++ is due 11/13 at midnight
• Project Presentation recorded as videos submitted by 11/29
  • Final video length TBD, but roughly 3 minutes
  • Describe/motivate your problem, show a quick demo, and a blooper (if time)
  • Peer grading in class
• Final Project Report: Upload written sections to Gradescope
  • Includes: Introduction, Task Description, Experimental Setup, Data & Results, and Summary & Challenges
  • You will be graded on quality of writing (e.g., clarity, grammar, typos, etc.), and thoroughness of analysis. You do not need to show 100% success, but a deep understanding of your system.

+ Team assessment
Meet Shakey the Robot:
An Experiment in Robot Planning and Learning

1. An operator types the command "push the block off the platform" at a computer console.
2. Shakey looks around, identifies a platform with a block on it, and locates a ramp in order to reach the platform.
3. Shakey then pushes the ramp over to the platform, rolls up the ramp onto the platform, and pushes the block off the platform.
Control Paradigm

desired behavior (robot position) → controller → actuators and transmissions → dynamics/kinematics of robot and env.

sensors

forces/torques

$f(\theta)$

$T_{sb}(\theta)$

motions and forces
Trajectories and Paths

• The specification of a robot state as a function of time is called a **trajectory**
  \[ \Theta: [0, T] \rightarrow \mathbb{R}^n \sim \Theta(t) \text{ gives joint angles at time } t \]

• Using forward kinematic maps, we can obtain the position of each link given as joint angles
  • The trajectory of the end-effector is then \( T_{sb}(\theta(t)) \)

• A **path** is a set of points

\[ \Rightarrow \text{path} + \text{specification of time} \]

\[ \Rightarrow \text{yields a trajectory} \]
Normalized Trajectories

• **Path** $\theta(s)$ maps a scalar path parameter $s \in [0,1]$ to a point in the robot's configuration space:
  \[
  \theta: [0, 1] \rightarrow \Theta \\
  \theta(0) \text{ is start of path} / \theta(1) \text{ is end of path}
  \]

• A **time-scaling** $s(t)$ is a monotonically increasing function:
  \[
  s: [0, T] \rightarrow [0, 1] \\
  \text{path + time-scaling define } t_{\text{ray}} \\
  \theta(+) \sim \theta(s(+))
  \]
Straight-Line Paths

• Given $\theta_0$ and $\theta_1$, find straight-line path:

$$\theta(s) = \theta_0 + s(\theta_1 - \theta_0), \quad s \in [0, 1]$$

• Is this in the task or configuration space?
  • Straight lines in joint space do not lead to straight lines in end-effector/task space

• Straight line in task space:

$$X(s) = X_0 + s(X_1 - X_0)$$

$SE(3)$
Straight-line Paths

$$\Theta_{i,\text{min}} \leq \Theta_i \leq \Theta_{i,\text{max}}$$
Straight-line Paths

"straight" in joint space
Straight-line Paths

\[
\begin{align*}
\theta_2 \text{(deg)} & \quad \theta_1 \text{(deg)} \\
-90 & \quad -90 \\
90 & \quad 90 \\
180 & \quad 180 \\
\theta_{\text{start}} & \quad \theta_{\text{end}}
\end{align*}
\]
Straight lines in SE(3)

In $\mathbb{R}^2$, straight lines are characterized by a constant velocity

$$\gamma(t) = \gamma_0 + \nu(t) \quad \Rightarrow \quad \dot{\gamma} = \nu$$

Recall: $\dot{T} = T \ [S]$ if $S$ is constant, then: $T(t) = T_0 e^{[S]t}$

Given $X_0$ and $X_1$, straight line in SE(3) is:

$$X(s) = X_0 e^{\log (X_0^{-1}X_1) \cdot s}$$

Given:

$X_{\text{start, end}} = X_{\text{start}, SE(3)}^{-1} X_{\text{end}, SE(3)}$

$X_{\text{start, end}} = X_{\text{start}} X_{\text{end}, SE(3)}^{-1}$
Straight lines in SE(3)

• We can decouple rotation and translation:

\[ X = (R, p) \quad \rightarrow \quad p(s) = p_0 + s(p_1 - p_0) \]
\[ R(s) = R_0 e^{s \log(R_1 R_0^T)} \]

• Now pass to **IK solver** to translate into joint space!
Time-scaling of straight-line paths

- Time scaling ensures that the motion is smooth and constraints are met

\[
\dot{\Theta}(s) = \Theta_0 + s(\Theta_1 - \Theta_0)
\]

\[
\frac{d\Theta(t)}{dt} = \Theta(s) = \frac{ds}{dt}(\Theta_1 - \Theta_0)
\]

\[
\ddot{\Theta}(s) = \frac{d^2s}{dt^2}(\Theta_1 - \Theta_0)
\]

choose time-scaling \( s(t) \) to limit \( \dot{\Theta} \) and \( \ddot{\Theta} \)

\[\text{often use parametric form of } s(t) \text{ like polynomials}\]
Polynomial Time-Scaling (1)

Let \( s(t) = a_0 + a_1 t + \frac{a_2 t^2}{2} + \frac{a_3 t^3}{3} \) \( t \geq 0 \) point-to-point motion in time \( T \) imposes constraints:

\[
\begin{align*}
    s(0) &= \dot{s}(0) = 0, & \text{and} & & s(T) = 1 + \dot{s}(T) = 0.
\end{align*}
\]

\( \dot{s}(t) = a_1 + 2a_2 t + 3a_3 t^2 \)

Evaluate \( 1 + 2 \) at \( t = 0 \) and \( t = T \) and solve:

\[
\begin{align*}
    a_0 &= 0, & a_1 &= 0, & a_2 &= \frac{3}{T^2}, & a_3 &= -\frac{2}{T^3}.
\end{align*}
\]

\[
\Rightarrow s(t) = \frac{3t^2}{T^2} - \frac{2t^3}{T^3}
\]
Polynomial Time-Scaling (2)
Summary

• Defined **paths, time-scaling, and trajectories**
• Looked at how to find **straight-line paths** in various spaces
• We choose a **parametrization** $s(t)$, and computed the resulting velocity and acceleration profiles of the trajectory
  • Using a third-order polynomial, we tuned their maximal values to meet requirements with one parameter $T$
• We can follow the same procedure with different parametrizations for $s(t)$ (e.g., polynomials of order 5, trapezoidal functions, splines, etc.)
  • Having more parameters allows us to meet more constraints. For example, using a fifth order polynomial, we can ensure that $\ddot{\theta}(0) = \ddot{\theta}(T) = 0$, meaning no jerk at beginning and end of the motion
• **Next topics** are on different concepts of / approaches to planning when the path may not be given