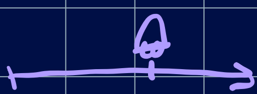
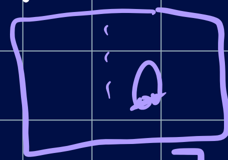
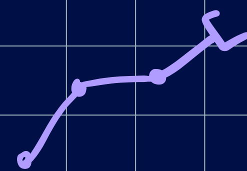
a vector is an array of numbers that represent a point in space



$[a_1]$

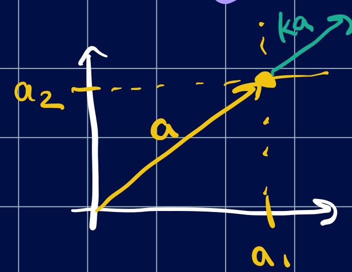


$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

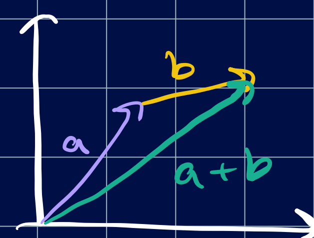


$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = a$

$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$



$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

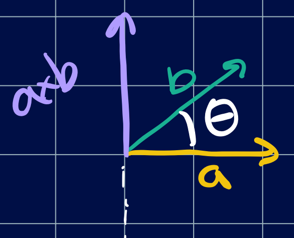


the <u>dot product</u> of two vectors that returns a scalar

$$a \cdot b = b \cdot a = \sum_i a_i b_i$$
$$= \|a\| \cdot \|b\| \cos\theta$$

→ if $a \cdot b = 0$, then vectors ar orthogonal the cross product $a \times b$ return the vec that is perpendicular to both a + b

$$a \times b = \|a\| \cdot \|b\| \sin\theta \cdot \hat{n}$$



$b \times a = -a \times b$

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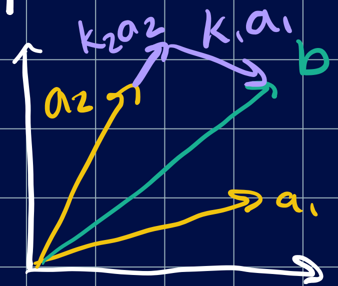
collection of vecs: $\{a_1, \ldots, a_n\}$
vec b is linearly dependent from if:

$$b = \sum k_i a_i$$

if no k_i exists, b is linearly independent



Matrix: $A: \overset{\text{rows}}{n} \times \overset{\text{col}}{m}$

$$A = \underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & \ddots & \vdots \\ a_{n1} & & \cdots & a_{nm} \end{bmatrix}}_{\text{collection of col vec}} \quad \text{collection of row vec} \begin{bmatrix} a_{1*}^T \\ a_{2*}^T \\ \vdots \\ a_{n*}^T \end{bmatrix}$$

$$[\underset{a_{:1}}{a_{*1}} \; a_{*2} \; \cdots \; a_{*m}]$$

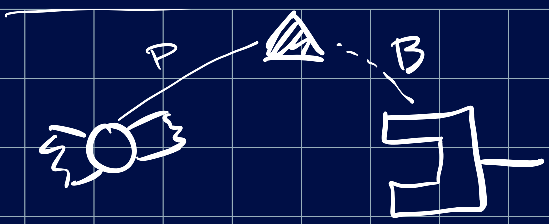
Matrix Operations and Rank

Common Matrix Operations

- Multiplication by a scalar
- Sum (commutative, associative)
- Multiplication by a vector
- Product (not commutative)
- Transposition
- Inversion (if square, full rank)

Matrix Rank

- Rank is determined by the maximum number of linearly independent rows (columns)
- If A is $m \times n$, then
  - $\text{rank}(A) \geq 0$
  - $\text{rank}(A) \leq \min(m, n)$
- $\text{rank}(A)$ can be computed by finding the rows that are linearly dependent, Gaussian elimination, and/or by counting the number of non-zero rows



Matrix B can represent position of sensor relative to robot

Bp give obj pose wrt robot

Matrix - Vector Products

$$Ab = \begin{bmatrix} a_{1*}^T \\ \vdots \\ a_{n*}^T \end{bmatrix} b = \begin{bmatrix} a_{1*}^T \cdot b \\ \vdots \\ a_{n*}^T b \end{bmatrix} = \sum_k a_{*k} b_k$$

i^{th} component of Ab is $a_{i*}^T \cdot b$

vec Ab is lin. dependent from collection of vecs $\{a_{*i}\}$ w/ coeff $\{b_i\}$

Matrix Matrix Products

$$C = AB = \begin{bmatrix} a_{1*}^T \cdot b_{*1} & \cdots & a_{1*}^T \cdot b_{*m} \\ \vdots & \ddots & \vdots \\ a_{n*}^T \cdot b_{*1} & \cdots & a_{n*}^T b_{*m} \end{bmatrix}$$

$$= \begin{bmatrix} Ab_{*1} & Ab_{*2} & \cdots & Ab_{*m} \end{bmatrix}$$

C_{*1} C_{*2}

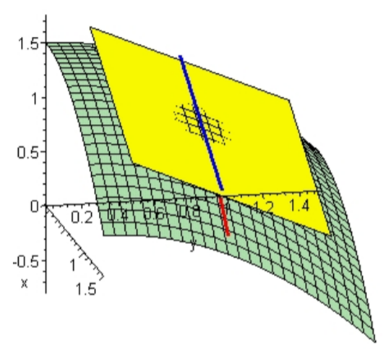
Matrix Inverse:

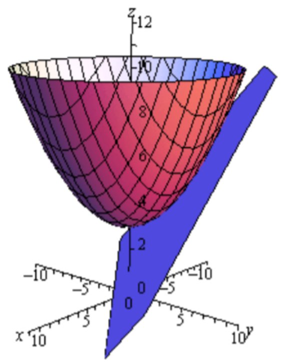
$AB = I \longrightarrow B = A^{-1}$

if A is full rank, then unique A^{-1} exists

Jacobian Matrix

Gives the orientation of the **tangent plane**
to the vector-valued function at a given point





review materials for linear algebra

- The Matrix Cookbook
- Linear Algebra Done Right
- Textbooks on Linear Algebra by Gilbert Strang

Summary

- Introduced course content
- Reviewed Vector and Matrix representations and operations
- **Rotation matrices** are an example of **orthogonal matrices** that have many practical uses in robotics
- The **Jacobian** contains the partial derivatives for a vector valued function