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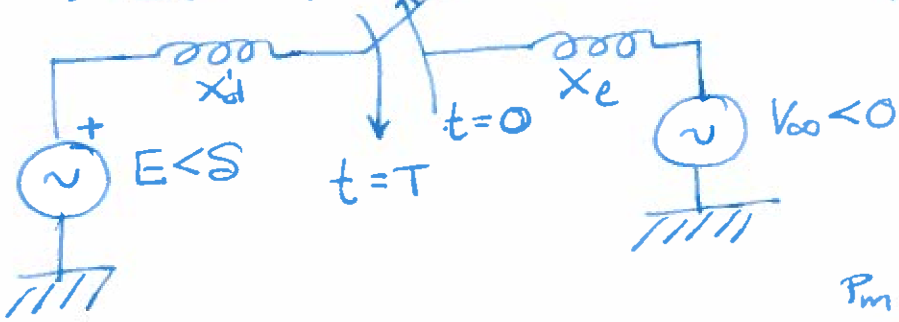
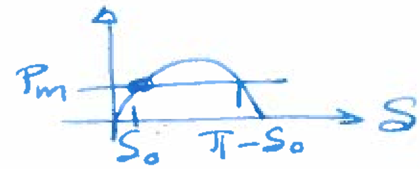
LECTURE 22

$$M\frac{d^2\delta}{dt^2}+D\frac{d\delta}{dt}=P_m-P_e$$



Assume $D=0$

$X := X_d' + X_e$



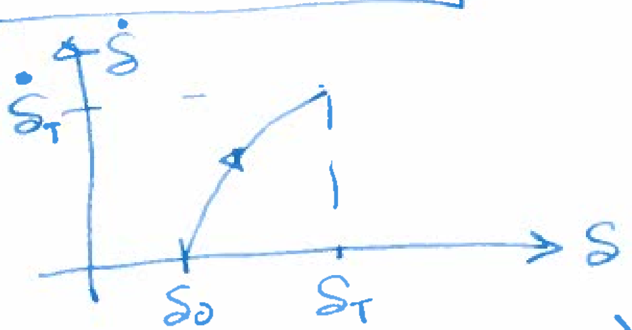
STAGE 1 $(t<0)$

$$\frac{d\delta}{dt}=\frac{d^2\delta}{dt^2}=0 \longrightarrow \quad \delta_0=\arcsin\left(\frac{P_m}{\frac{EV_\infty}{X}}\right)<\frac{\pi}{2}$$

STAGE 2 $(0\le t<T)$

$$M\cdot\frac{d^2\delta}{dt^2}=P_m \longrightarrow \frac{d\delta}{dt}=\frac{P_m}{M}t, \quad \delta=\frac{P_m}{2M}t^2+\delta_0$$

$$\boxed{\delta=\frac{M}{2P_m}\dot{\delta}+\delta_0}$$



$$\boxed{\begin{aligned}\dot{\delta}_T&=\frac{P_m}{M}T\\ \delta_T&=\frac{P_m}{2M}T^2+\delta_0\end{aligned}}$$

STAGE 3 $(t\ge T)$

$$M\frac{d^2\delta(t)}{dt^2}=P_m-\frac{EV_\infty}{X}\sin\delta(t)$$

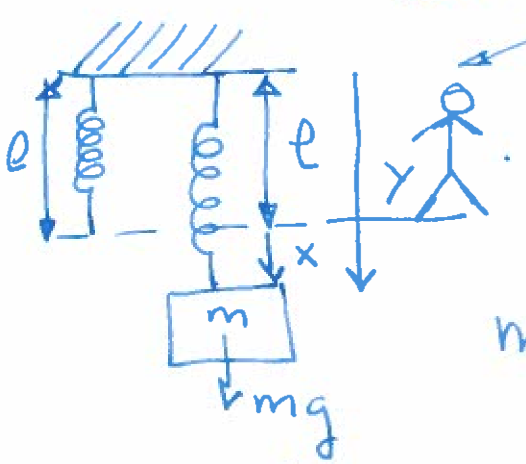
$$\delta(0)=\delta_T=\frac{P_m}{2M}T^2+\delta_0$$

$$\dot{\delta}(0)=\dot{\delta}_T=\frac{P_m}{M}T$$

What happens with $(\delta(t),\dot{\delta}(t))$?

To answer the question, we resort to an energy argument

DETOUR [MASS-SPRING SYSTEM]



$$m\frac{d^2y(t)}{dt} = mg - K(y(t)-\ell)$$

$$x(t) = y(t) - \ell$$

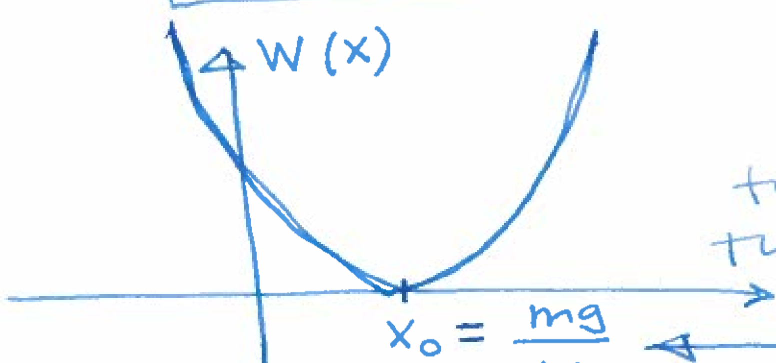
$$m\frac{d^2x(t)}{dt^2} = mg - Kx(t)$$

Potential energy function:

$$\underbrace{W(x_0)}_{0\ [\text{arbitrary}]} - W(x) = \int_{x_0}^{x} \bar{F}\,d\bar{x} = \int_{x_0}^{x} (mg - K\cdot z)\,dz$$

$$W(x) = -mg(x - x_0) + \frac{1}{2}K(x^2 - x_0^2)$$

$$\boxed{W(x) = -mg(x - x_0) + \frac{1}{2}K(x^2 - x_0^2)}$$



equilibrium point [we take the potential energy of this point to be zero.]

$x_0 = \frac{mg}{K}$ ← equilibrium

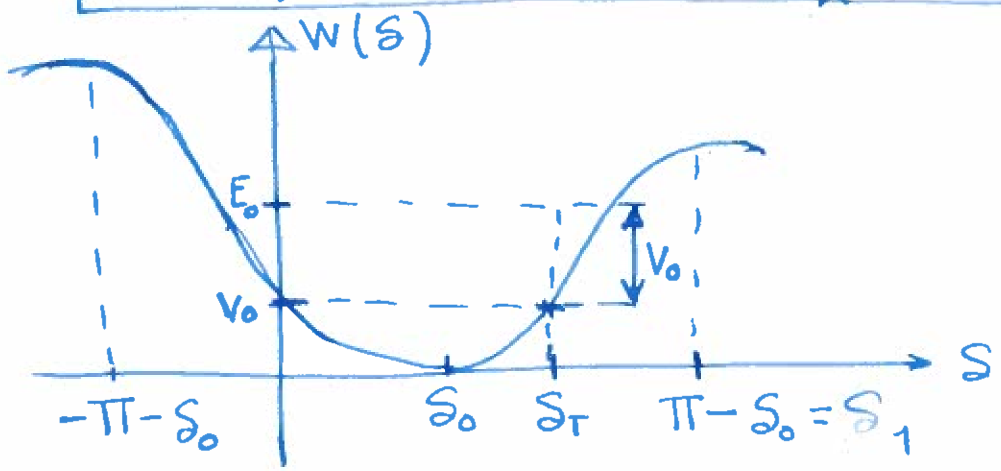
STAGE 3 ANALYSIS

For the swing equation, we have

$$M\cdot\frac{d^2\delta(t)}{dt} = P_m - \frac{EV_\infty}{X}\sin\delta = P(\delta)$$

$$W(\delta) = -\int_{\delta_0}^{\delta} P(u)\,du = -\int_{\delta_0}^{\delta}\left(P_m - \frac{EV_\infty}{X}\sin u\right)du$$

$$\boxed{W(\delta) = -P_m(\delta - \delta_0) - \frac{EV_\infty}{X}(\cos\delta - \cos\delta_0)}$$



At the beginning of STAGE 3, we have $\delta(0) = \delta_T$ and $\dot{\delta}(0) = \dot{\delta}_T$; thus

the potential energy is:

$$W_0 = -P_m(\delta_T - \delta_0) - \frac{EV_\infty}{X}(\cos\delta_T - \cos\delta_0)$$

The system also has kinetic energy because $\dot{\delta}(0) = \dot{\delta}_T \neq 0$:

$$V_0 = \frac{1}{2} M \cdot \dot{\delta}_T^2$$

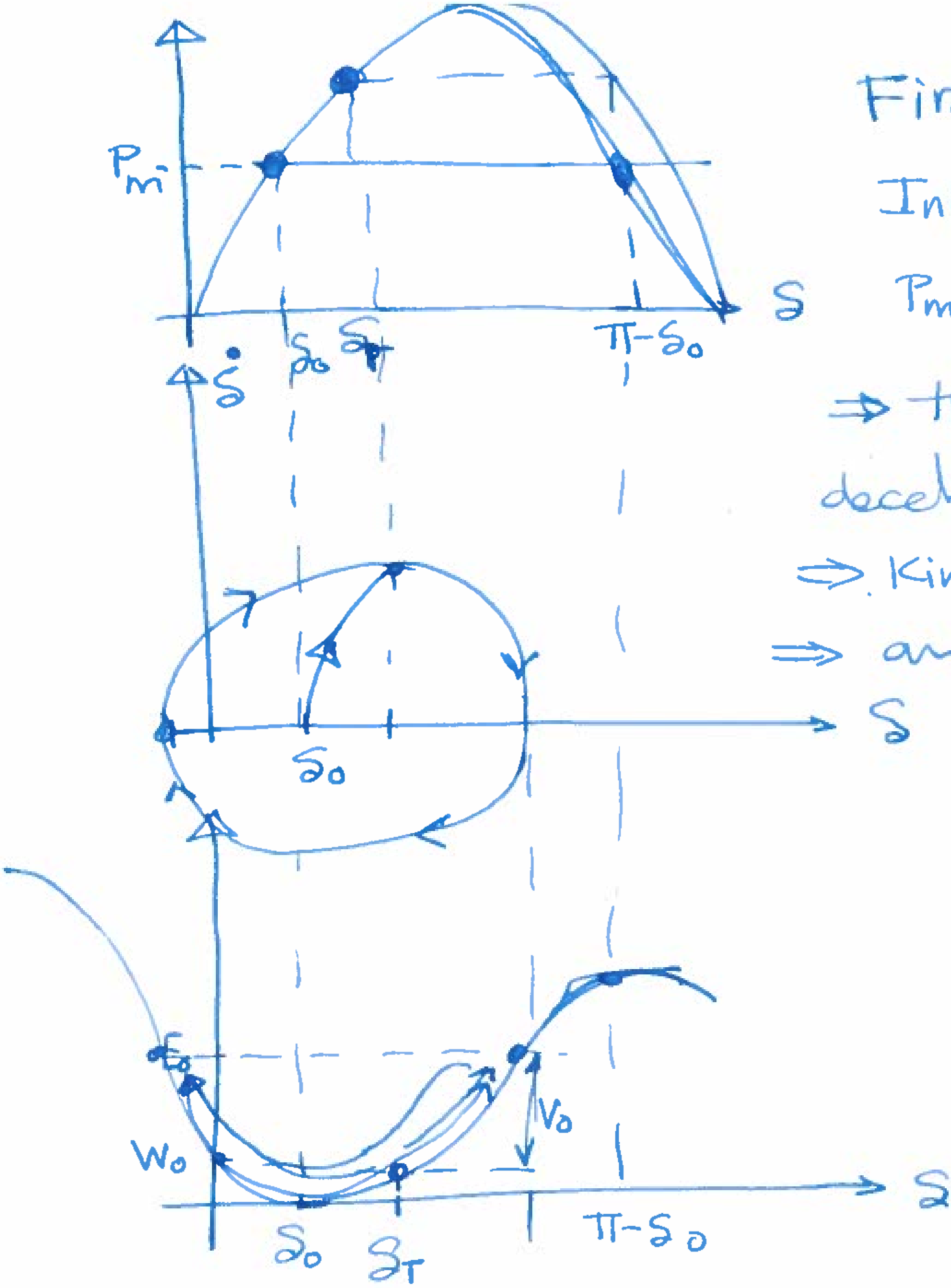
The total energy @ the beginning of stage 3 is:

$$E_0 = \underbrace{\frac{1}{2} M \dot{\delta}_T^2}_{\text{Kinetic}} + \underbrace{\left(-P_m(\delta_T - \delta_0) - \frac{EV_\infty}{X}(\cos\delta_T - \cos\delta_0)\right)}_{\text{Potential}}$$

Because $D \neq 0$; the initial energy will remain constant; i.e.,

$$\boxed{E(t) = E_0, \quad t \geq T}$$

$$\frac{d}{dt} E(\delta(t), \dot{\delta}(t)) = \frac{1}{2} 2M\dot{\delta} \cdot \frac{d}{dt}\dot{\delta} - \left(P_m - \frac{EV_\infty}{X}\sin\delta\right)\dot{\delta}$$
$$= \left(M\ddot{\delta} - P_m + \frac{EV_\infty}{X}\sin\delta\right)\dot{\delta} \doteq 0 \,!$$



First swing:

Initially ($t = T$)

$P_m - \frac{EV_\infty}{X}\cdot\sin\delta_T < 0$

⇒ the system will decelerate ⇒

⇒ Kinetic energy decreases

⇒ angle increases

What's the effect of the friction coefficient?

$$M\ddot\delta = -D\dot\delta + \underbrace{P_m - \frac{EV_\infty}{X}\sin\delta}_{P(\delta)}$$

$$\cancel{\frac{d}{dt}E(\delta(t)) = \frac{\partial E}{\partial \delta}\cdot\frac{d}{}}$$

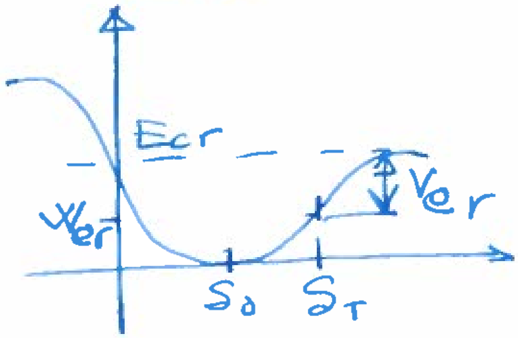
$$\frac{d}{dt}E(\delta(t),\dot\delta(t)) = \frac{\partial E}{\partial\delta}\cdot\frac{d\delta}{dt} + \frac{\partial E}{\partial\dot\delta}\frac{d\dot\delta}{dt}$$

$$= M\cdot\dot\delta\cdot\ddot\delta \cancel{+} - P(\delta)\dot\delta$$

$$= [M\ddot\delta \cancel{+} - P(\delta)]\,\dot\delta = -D\dot\delta^2 < 0$$

EQUAL AREA CRITERION

- Clearly there is an initial energy level that if the system has, it won't be able to recover after the fault has been cleared.



$$E_{cr} = W_{cr} + V_{cr}$$

$$\begin{aligned} E_{cr} &= W(\pi - \delta_0) \\ &= -P_m(\pi - 2\delta_0) \\ &\quad - \frac{EV_\infty}{X}\cos(\pi - \delta_0) + \frac{EV_\infty}{X}\cos\delta_0 \\ &= -P_m(\pi - 2\delta_0) + \frac{2EV_\infty}{X}\cos\delta_0 \end{aligned}$$

- Corresponding to this E_{cr}, there is a critical T.

Mathematically:

$$\frac{1}{2}M\dot{\delta}_T^2 + \int_{\delta_0}^{\delta_T} -P(u)\,du < \int_{\delta_0}^{\pi-\delta_0} -P(u)\cdot du$$

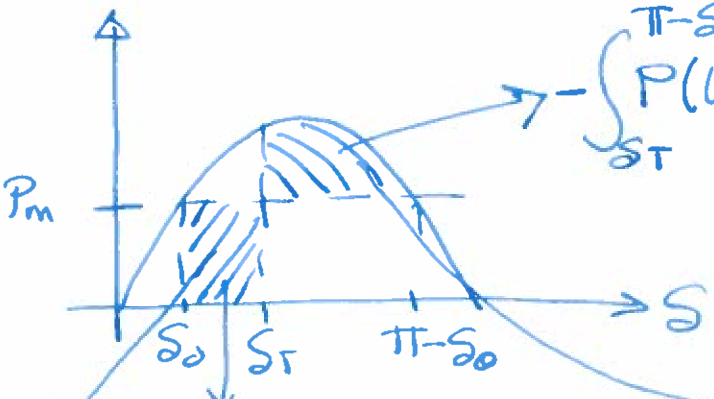
$$\frac{1}{2}M\dot{\delta}_T^2 < \int_{\delta_T}^{\pi-\delta_0} -P(u)\,du$$

We also saw that @ the end of STAGE 2, we had:

$$\delta_T = \delta_0 + \frac{M}{2P_m}\dot{\delta}_T^2 \rightarrow \boxed{P_m(\delta_T - \delta_0) = \frac{1}{2}M\dot{\delta}_T^2}$$

- Thus, the criterion is

$$P_m(\delta_T - \delta_0) < -\int_{\delta_T}^{\pi-\delta_0} P(u)\,du$$



$$-\int_{\delta_T}^{\pi-\delta_0} P(u)\,du = -\int_{\delta_T}^{\pi-\delta_0}\left(P_m - \frac{EV_\infty}{X}\sin u\right)du$$

$$= \int_{\delta_T}^{\pi-\delta_0}\left(\frac{EV_\infty}{X}\sin u - P_m\right)du$$

"decelerating area"

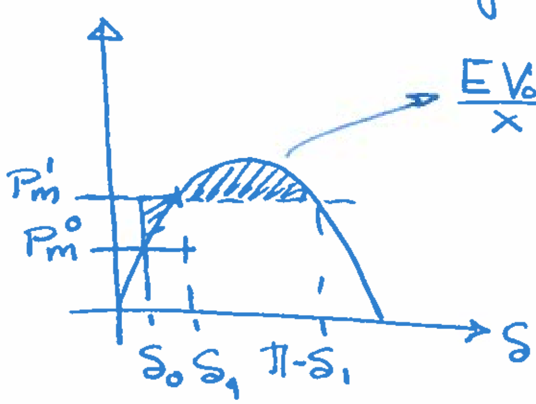
$P_m(\delta_T - \delta_0)$

"Accelerating area"

Critical T: $\quad P_m(\delta_T - \delta_0) = \int_{\delta_0}^{\pi-\delta_0}\left(\frac{EV_\infty}{X}\sin u - P_m\right)du$

OTHER APPLICATIONS OF THE EQUAL-AREA CRITERION

1. Sudden change in P_m



The idea is always the same: need to compare the initial energy after the change with the critical potential energy

In this case, after P_m changes, the potential energy curve changes as well!

$$W(\delta) = -\int_{\delta_1}^{\delta}\left(P_m' - \frac{EV_\infty}{X}\sin u\right)du$$

this is the eq. point we need to use

- Initially, the potential energy the system has is

$$W(\delta_0) = -\int_{\delta_1}^{\delta_0} \left(P_m' - \frac{EV_\infty}{X} \sin u\right) du$$

- No kinetic energy.
- Thus the criterion is

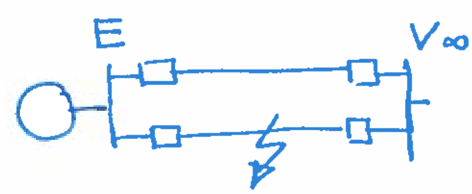
$$W(\delta_0) \le W(\pi - \delta_1)$$

$$\underbrace{-\int_{\delta_1}^{\delta_0} \left(P_m' - \frac{EV_\infty}{X} \sin u\right) du}_{} \le -\int_{\delta_1}^{\pi-\delta_1} \left(P_m' - \frac{EV_\infty}{X} \sin u\right) du$$

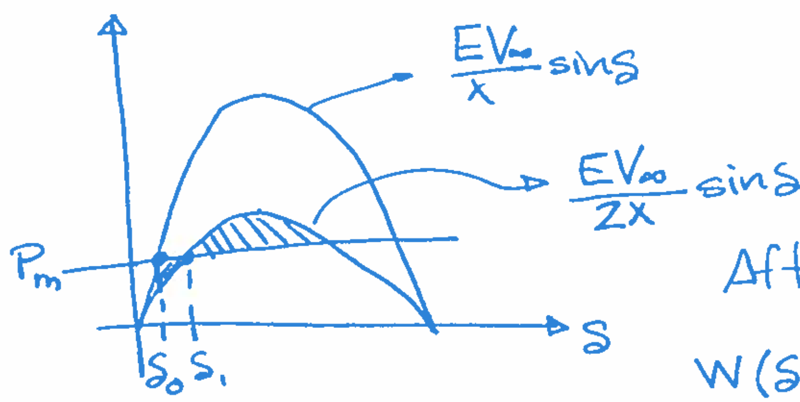
$$\boxed{\int_{\delta_0}^{\delta_1} \left(P_m' - \frac{EV_\infty}{X} \sin u\right) du \le \int_{\delta_1}^{\pi-\delta_1} \left(\frac{EV_\infty}{X} \sin u - P_m'\right) du}$$

consistent with the shaded areas in the figure.

2. Sudden change in X



After the fault occurs X goes to $2 \cdot X$



After the fault (initial potential energy)

$$W(\delta_0) = -\int_{\delta_1}^{\delta_0} \left(P_m - \frac{EV_\infty}{2X} \sin u\right) du$$

- Critical potential energy:

$$W(\pi-\delta_1) = -\int_{\delta_1}^{\pi-\delta_1} \left(P_m - \frac{EV_\infty}{2X}\sin u\right) du$$

- Initially, no kinetic energy; thus, the criterion is

$$W(\delta_0) \leq W(\pi-\delta_1)$$

$$+\int_{\delta_0}^{\delta_1} \left(P_m - \frac{EV_\infty}{2X}\sin u\right) du \leq -\int_{\delta_1}^{\pi-\delta_1} \left(P_m - \frac{EV_\infty}{2X}\sin u\right) du$$

$$\boxed{\int_{\delta_0}^{\delta_1} \left(P_m - \frac{EV_\infty}{2X}\sin u\right) du \leq \int_{\delta_1}^{\pi-\delta_1} \left(\frac{EV_\infty}{2X}\sin u - P_m\right) du}$$

→ consistent with the shaded areas in the figure.