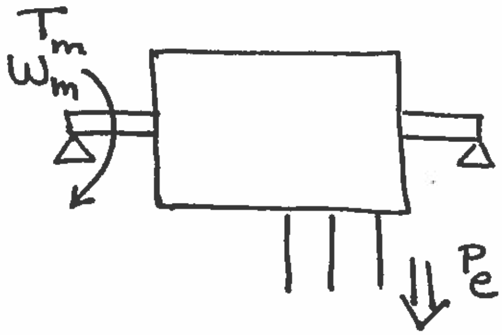
LECTURE 21

11/16/17

DERIVATION OF THE SWING EQUATION.



T_m: mechanical torque in the shaft applied by prime mover

ω_m: rotor mechanical velocity

P_e: electrical power

Newton's second law

$\omega_m = \frac{d\theta_m}{dt}$,

θ_m: rotor angular position w.r.t. a stationary reference

$$J \cdot \frac{d\omega_m}{dt} = T_m - T_e - K(\omega_m - \omega_s);$$

- $T_e := \frac{P_e}{\omega_m}$; this is the electrical torque
- $K(\omega_m - \omega_s)$: friction losses, where ω_s is the synchronous speed of the generator.
- J : Generator inertia

UNITS :

- T_m, T_e [N·m]
- ω_m [rad/s]
- J [Kg·m²]
- K [N·m·s]

- Refer the rotor position to a reference frame rotating at synchronous speed:

$$\theta_m = \omega_s t + \delta_m$$

$$\frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2};$$

- Thus, we obtain:

$$J \cdot \frac{d^2\delta_m}{dt^2} = T_m - T_e - K\frac{d\delta_m}{dt}$$

- Multiply by ω_m to get:

$$\omega_m \cdot J \cdot \frac{d^2\delta_m}{dt^2} = \underbrace{\omega_m T_m}_{P_m:\ \text{mechanical power [W]}} - \overbrace{\omega_m T_e}^{\text{electrical power [W]}} - K\omega_m \cdot \frac{d\delta_m}{dt}$$

- Divide by the generator base power, S_B:

$$\frac{J\omega_m}{S_B}\frac{d^2\delta_m}{dt^2} = \underbrace{\frac{\omega_m T_m}{S_B}}_{\text{mechanical power [P.U.]}} - \underbrace{\frac{\omega_m T_e}{S_B}}_{\text{Electrical power [P.U.]}} - \frac{K\omega_m}{S_B}\frac{d\delta_m}{dt}$$

We make the approximation that

$$\omega_m = \omega_s + \frac{d\delta}{dt} \simeq \omega_s; \text{ Thus}$$

$$\frac{J\omega_s}{S_B} \cdot \frac{d^2\delta_m}{dt^2} = P_m - P_e - \frac{K\omega_s}{S_B} \cdot \frac{d\delta_m}{dt}$$

$$\frac{J\cdot\omega_s}{S_B}\cdot\frac{2\omega_s}{2\omega_s} = \underbrace{\frac{\frac{1}{2}J\omega_s^2}{S_B}}_{=:H}\cdot\frac{2}{\omega_s}$$

$=: H$ [Stored kinetic energy @ synchronous speed / rated power]

H: falls in a very narrow range; between 1 and 10 p.u.·s; whereas J varies drastically.

$$H\cdot\frac{2}{\omega_s}\cdot\frac{d^2\delta_m}{dt^2} = P_m - P_e - \frac{K\omega_s}{S_B}\frac{d\delta_m}{dt}$$

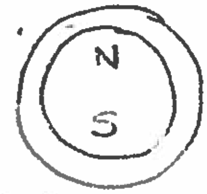
$$\omega_s = \frac{2}{P}\omega_0$$

"synchronous"

nominal electrical frequency

Synchronous speed,

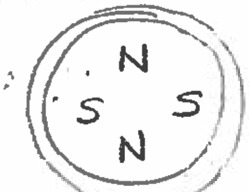
P: # of poles of the machine.



$P = 2$

Clearly:

$$\omega_m = \frac{2}{P}\omega \rightarrow \frac{d\theta_m}{dt} = \frac{2}{P}\frac{d\theta}{dt}$$



$P = 4$

electrical frequency

mechanical velocity

$$\theta_m = \omega_s t + \delta_m$$

$$\delta_m = \frac{2}{P}\delta$$

power angle

$$\underbrace{\frac{P}{2}\theta_m}_{\theta} = \underbrace{\frac{P}{2}\omega_s}_{\omega_0} t + \underbrace{\frac{P}{2}\delta}_{\delta}$$

$$H \cdot \frac{2}{\omega_s} \cdot \frac{d^2 \delta_m}{dt^2} = P_m - P_e - \frac{K\omega_s}{S_B} \cdot \frac{d\delta_m}{dt}$$

$$H \cdot \frac{2}{\frac{\cancel{2}}{\cancel{p}}\omega_0} \cdot \frac{d^2}{dt^2}\left(\frac{\cancel{2}}{\cancel{p}}\delta\right) = P_m - P_e - \frac{K\frac{2}{p}\omega_0}{S_B} \cdot \frac{d}{dt}\left(\frac{2}{p}\delta\right)$$

$$\underbrace{H \cdot \frac{2}{\omega_0}}_{=:M} \frac{d^2\delta}{dt^2} = P_m - P_e - \underbrace{\frac{4K\omega_0}{p^2 S_B}}_{=:D} \frac{d}{dt}\delta$$

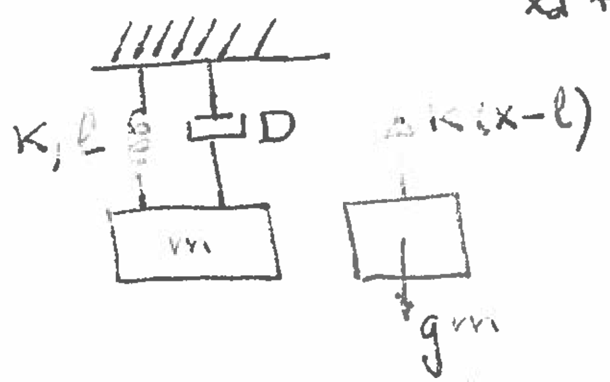
$$M \cdot \frac{d^2\delta}{dt^2} = P_m - P_e - D \cdot \frac{d}{dt}\delta$$

$$\boxed{M \frac{d^2\delta}{dt^2} = -D \cdot \frac{d}{dt}\delta + P_m - P_e}$$ swing equation

In state space form:

$$\boxed{\begin{aligned} \frac{d\delta}{dt} &= \omega - \omega_0 \\ M\frac{d\omega}{dt} &= -D \cdot (\omega - \omega_0) + P_m - P_e \end{aligned}}$$

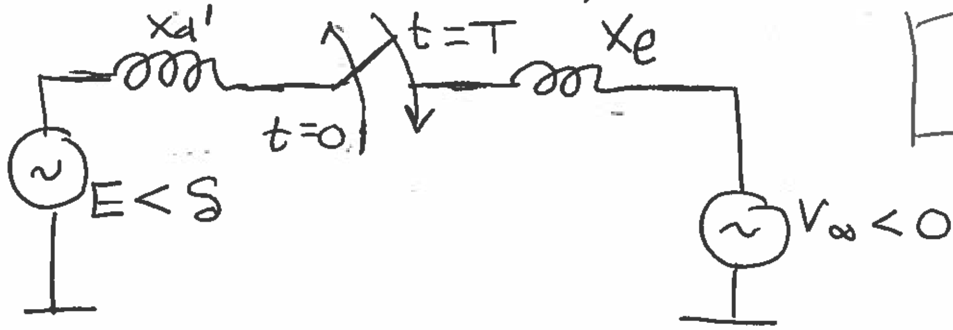
This is analogous to a mass suspended by a spring when $P_e = \frac{E \cdot V_\infty}{x_d' + x_e} \sin\delta$ (refer to Fig. on page



$$\frac{d^2x}{dt^2} = -D\frac{dx}{dt} - K(x-\ell) + m$$

$$M\frac{d^2\delta}{dt^2} = -D\frac{d\delta}{dt} - \frac{EV_\infty}{x_d' + x_e}\sin\delta + P$$

TRANSIENT STABILITY



$$\boxed{X = X_d' + X_e}$$

- We assume that an event causing the transmission line occurs @ $t=0$
- Prior to the line opening, the system is in steady state.
- Once the line open, it remains as such for some time T, at which it recloses, the question is what happens with the system after the line recloses?

δ

STAGE 1 $\quad t<0$

- System is operating in steady state.

$$\left.\begin{cases} \dfrac{d\delta}{dt} = \omega - \omega_s \\ \dfrac{d\omega}{dt} = P_m - \dfrac{E V_\infty}{X}\sin\delta \end{cases}\right\} \quad \frac{d\delta}{dt} = \frac{d\omega}{dt} = 0 \longrightarrow \boxed{\delta_0 = \frac{P_m}{E V_\infty / X}}$$

STAGE 2 $\quad 0 < t \leq T$

The transmission line opens and the system dynamics evolves according to

$$\begin{cases} M \dfrac{d^2\delta}{dt} = P_m \\ \delta(0) = \delta_0 \\ \dot{\delta}(0) = 0 \end{cases}$$

The solution can be found by integrating twice

$$\dot{\delta}(t) = \frac{P_M}{M} t + \cancel{\dot{\delta}(0)}^{\,0} \quad [\mathrm{rad/s}]$$

$$\delta(t) = \frac{1}{2} \frac{P_M}{M} t^2 + \delta(0)$$

$$= \frac{1}{2} \frac{P_M}{M} t^2 + \delta_0$$

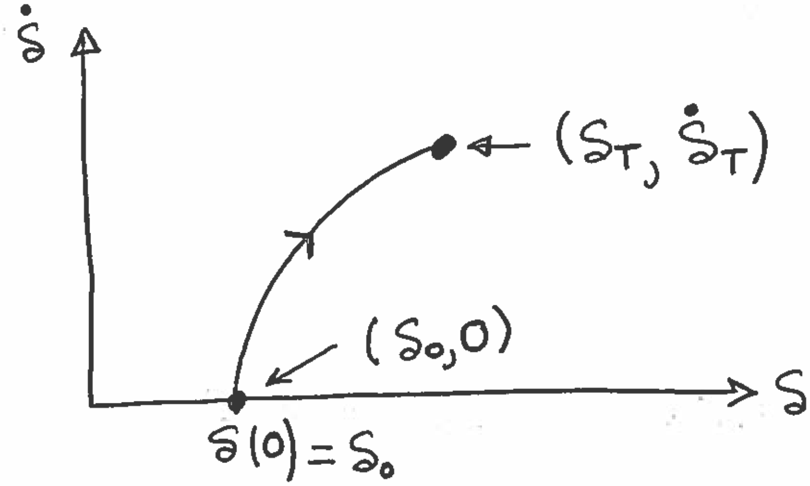
We can eliminate t to describe a relation between $\dot{\delta}(t)$ and $\delta(t)$

$$\left(\frac{M}{P_M} \dot{\delta}(t)\right)^2 = t^2$$

$$\delta(t) = \frac{1}{2} \frac{\cancel{P_M}}{\cancel{M}} \cdot \left(\frac{M}{P_M} \dot{\delta}(t)\right)^2 + \delta_0$$

$$\boxed{\delta(t) - \delta_0 = \frac{1}{2} \frac{M}{P_M} \left(\dot{\delta}(t)\right)^2}$$

$$\delta(t) - \delta_0 = \frac{1}{2}\,\frac{M}{P_M}\left(\dot{\delta}(t)\right)^2$$



STAGE 3 $\quad t > T$

The transmission line is reclosed at time $t = T$, the system will now evolve according

$$\begin{cases} M\dfrac{d^2\delta}{dt^2} = P_m - \dfrac{EV_\infty}{X}\sin\delta \\ \delta(0) = \delta_T = \delta_0 + \dfrac{M}{2P_M}T^2 \\ \dot{\delta}(0) = \dot{\delta}_T = \dfrac{P_M}{M}T \end{cases}$$

- We are interested in the properties of this system.
- We can study its behavior using energy techniques.