LECTURE 20

TRANSIENT STABILITY

- In order to operate as an interconnected system all of the generators (and other synchronous machines) must remain in synchronism with one another.

→ Keep in mind that the frequency of the system is ~~tight~~ intimately related to the speed at which machines turn.

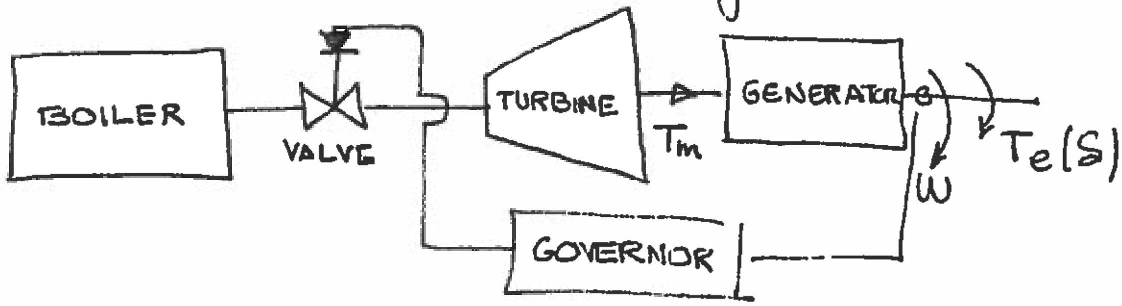
→ Synchronism requires that (for two pole machines) the rotorn turn at exactly the same speed.

→ Loss of sync results in a condition in which no net power can be transferred between the machine.

→ A system is said to be transiently unstable if following a disturbance one or more of the generators lose sync.

GENERATOR DYNAMIC MODEL

Generator mechanical block diagram



THE CANONICAL PROBLEM

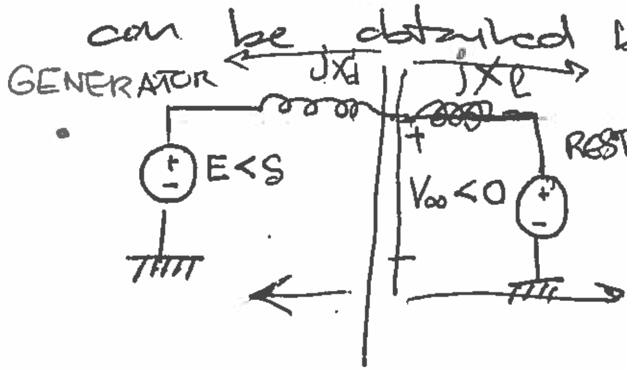
The canonical problem we are trying to solve is to determine whether or not the system remains stable after a disturbance. We have two types of disturbances:

- small disturbances
- Large disturbances

Although power systems are extremely large nonlinear dynamical systems, most stability issues can be appreciated with a very simplistic model: the single machine infinite bus, which assumes a synchronous machine (the stability of which we want to study) is connected to a voltage source that models the rest of the system

SINGLE-MACHINE INFINITE BUS MODEL

- Dynamical models for generators are complicated and beyond the scope of this course (ECE 43 and ECE 576)
- We will use a reduced-order model that can be obtained by making certain assumptions



- We derive the equation that governs the evaluation of δ (the torque angle)

- In per-unit:

$$\boxed{M\ddot{\delta} + D\dot{\delta} = P_m - \overbrace{\frac{E\cdot V_\infty}{X_d' + X_e}\sin\delta}^{\text{Electrical power output}}}$$

P_m : mechanical power
M : per-unit inertia constant
D : friction coefficient
X_d' : machine internal reactance.

- We can rewrite this equations in state-space (430 material)

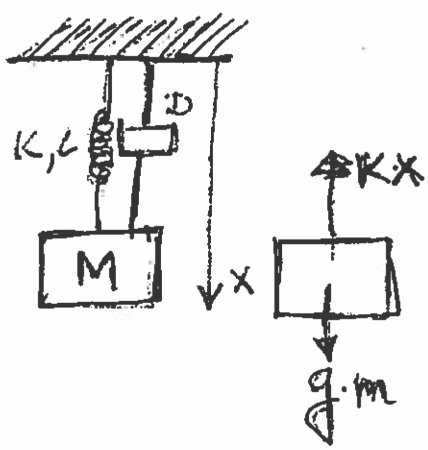
$$\dot{\delta} = \omega_m - \omega_s$$
$$\ddot{\delta} = \dot{\omega}_m$$

$$\boxed{\begin{aligned} \frac{d\delta}{dt} &= \omega_m - \omega_s \\ \frac{d\omega_m}{dt} &= -\frac{D}{M}(\omega_m - \omega_s) - \frac{E\cdot V_\infty}{M(X_d' + X_e)}\sin\delta + \frac{1}{M}P_m \end{aligned}}$$

which is of the form $\dot{x} = f(x)$, where $x = [\delta, \omega_m]'$

$$M\ddot{\delta} + D\dot{\delta} = P_m - \underbrace{P(\delta)}_{\frac{E V_\infty}{X_d' + X_e}\cdot\sin\delta}$$

This eq is analogous to a mass suspended by a spring
nonlinear spring



$$M\ddot{\delta} = -D\dot{\delta} + P_m - P(\delta)$$

$$M\ddot{x} = -D\cdot\dot{x} - K(x - l) + Mg$$

This is why the $M\ddot{\delta} = P_M - P_e(\delta) - D\dot{\delta}$ is referred to as ~~the so-called~~ swing equation.

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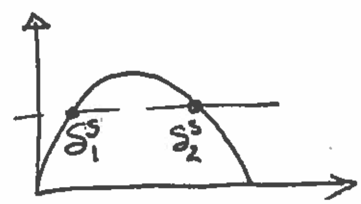
SMALL-SIGNAL STABILITY

- We can think of small disturbances as for example small variations in V_∞ or P_m (the nechanical power slightly fluctuates or the voltage at the infinite bus).

- In this case, we can study the stability of the linearized system around the equilibrium point.

- Eq. point $\dot{\delta} = \dot{\omega}_m = 0 \longrightarrow \omega_m - \omega_s = 0$

$$\delta^s = \arcsin\left(\frac{P_m}{\frac{E \cdot V_\infty}{X_d' + X_L}}\right)$$



Linearization

Renember from 430: $\dot{x} = f(x) \longrightarrow$

$$\longrightarrow \quad x \simeq x^s + \Delta x \quad \longrightarrow \quad \Delta\dot{x} = f(x^s) + \left.\frac{df(x)}{dx}\right|_{x_s} \Delta x$$

If we have inputs, we have an additional term for the input: $\dot{x} = f(x, u) \longleftrightarrow f(x_s, u_n) = 0$

$$\boxed{\Delta\dot{x} = \left.\frac{\partial f(x\,u)}{\partial x}\right|_{x_s, u_n} \Delta x + \left.\frac{\partial f(x,u)}{\partial u}\right|_{x_s, u_n} \Delta u}$$

Let's study the case when the system is in equilibrium for some value V_∞^s and P_m^s and let's perturb it slightly. The linearized model is:

$$\delta = \delta_s + \Delta\delta \longrightarrow \omega = \omega_s + \Delta\omega$$

Let's denote $\quad \omega_r = \omega_m - \omega_s$

$$\frac{d\delta}{dt} = \omega_r$$

$$\frac{d\omega_r}{dt} = -\frac{D\omega_r}{M} - \frac{E \cdot V_\infty}{M(X_d' + X_l)} \sin\delta + \frac{1}{M} P_m$$

$$\frac{d}{dt}\begin{bmatrix} \Delta\delta \\ \Delta\omega_r \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{E V_\infty \cos\delta^s}{M(X_d'+X_l)} & -\frac{D}{M} \end{bmatrix}}_{A} \begin{bmatrix} \Delta\delta \\ \Delta\omega_r \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -\frac{E \cdot V_\infty}{M(X_d'+X_l)} \sin\delta^s \end{bmatrix}}_{B} \Delta V_\infty$$

Eigenvalues of A

$$\lambda^2 + \frac{D}{M}\lambda + \frac{E \cdot V_\infty \cdot \cos\delta^s}{M(X_m' + X_l)} = 0$$

$$\lambda = \frac{-\frac{D}{M} \pm \sqrt{\left(\frac{D}{M}\right)^2 - 4 \cdot \frac{E \cdot V_\infty \cos\delta^s}{M(X_m' + X_l)}}}{2}$$

if $\cos\delta^s > 0$ both $\lambda < 0 \rightarrow$ the system is small-signal stable.

if $\cos\delta^s < 0$ one eigenvalue is $> 0 \rightarrow$ system is unstable