LECTURE 19

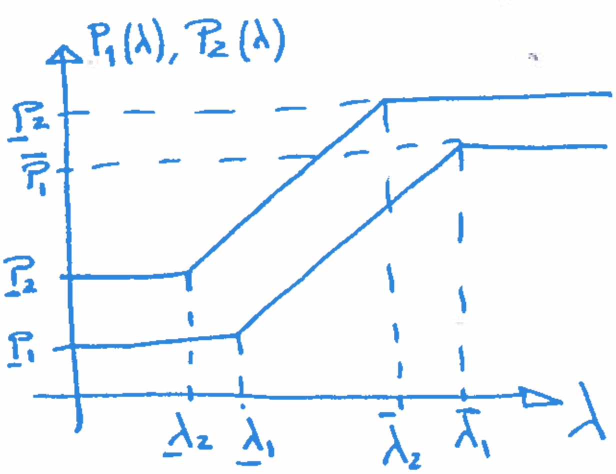
Recall the E.D. problem with generator constraints for a system with two buses:

$$\underset{P_1, P_2}{\text{minimize}} \; (\alpha_1 + \beta_1 P_1 + \gamma_1 P_1^2) + (\alpha_2 + \beta_2 P_2 + \gamma_2 P_2^2)$$

$$\text{subject to} \quad P_1 + P_2 = P^D$$

$$\underline{P}_1 \le P_1 \le \overline{P}_1 \; ; \quad \underline{P}_2 \le P_2 \le \overline{P}_2$$

- We discuss a procedure for how to solve the problem numerically [The λ-iteration]
- Here, we will provide yet another way to solve the problem which gives a bit more insight into the structure.



$$P_1(\lambda) = \begin{cases} \dfrac{\beta_1 - \lambda}{2\gamma_1}, & \underline{\lambda}_1 \le \lambda \le \overline{\lambda}_1 \\ \underline{P}_1, & \underline{\lambda}_1 > \lambda \\ \overline{P}_1, & \overline{\lambda}_1 < \lambda, \end{cases}$$

$$\underline{\lambda}_1 = \beta_1 + 2\gamma_1 \underline{P}_1, \quad \overline{\lambda}_1 = \beta_1 + 2\gamma_1 \overline{P}_1$$

$$P_2(\lambda) = \begin{cases} \dfrac{\beta_2 - \lambda}{2\gamma_2}, & \underline{\lambda}_2 < \lambda \le \overline{\lambda}_2 \\ \underline{P}_2, & \underline{\lambda}_2 > \lambda \\ \overline{P}_2, & \overline{\lambda}_2 < \lambda, \end{cases}$$

$$\underline{\lambda}_2 = \beta_2 + 2\gamma_2 \underline{P}_2, \quad \overline{\lambda}_2 = \beta_2 + 2\gamma_2 \overline{P}_2$$

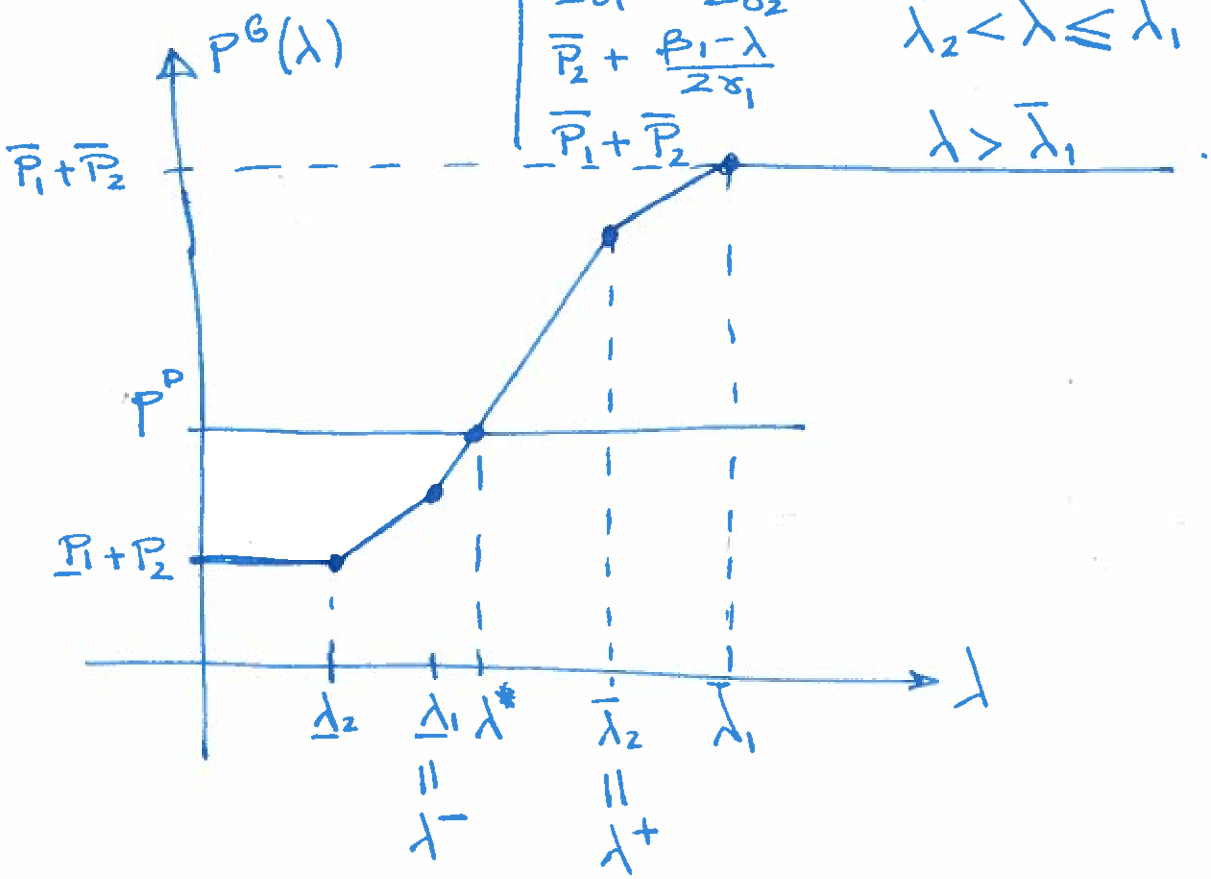
Note that in the λ-iteration, we iterative refine some estimates of λ so as to compute λ^* that satisfies

$$\underbrace{P_1(\lambda^*)+P_2(\lambda^*)}_{P^G(\lambda^*)}=P^D$$

$P^G(\lambda)$ is the total power generated by both units as a function of the variable λ, which is the same for both at optimality; thus, finding the solution λ^* can be done by determining the intersection point of $P^G(\lambda)$ and P^D.

$$P_1(\lambda)+P_2(\lambda)=\begin{cases} \underline{P}_1+\underline{P}_2 & \lambda<\underline{\lambda}_2 \\ \underline{P}_1+\dfrac{\beta_2-\lambda}{2\gamma_2} & \underline{\lambda}_2\leq\lambda<\underline{\lambda}_1 \\ \dfrac{\beta_1-\lambda}{2\gamma_1}+\dfrac{\beta_2-\lambda}{2\gamma_2} & \underline{\lambda}_1\leq\lambda\leq\overline{\lambda}_2 \\ \overline{P}_2+\dfrac{\beta_1-\lambda}{2\gamma_1} & \overline{\lambda}_2<\lambda\leq\overline{\lambda}_1 \\ \overline{P}_1+\overline{P}_2 & \lambda>\overline{\lambda}_1 \end{cases}$$

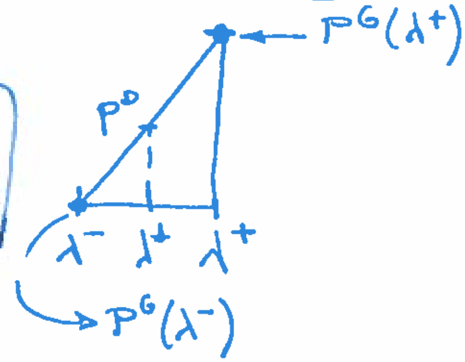
$[\underline{\lambda}_2<\underline{\lambda}_1<\overline{\lambda}_2<\overline{\lambda}_1]$



- The curve $P^G(\lambda)$ changes slope in at most 4 points: $\underline{\lambda}_2$, $\underline{\lambda}_1$, $\bar{\lambda}_2$, $\bar{\lambda}_1$.

- Order $\underline{\lambda}_2, \underline{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_1$ in ascending order; in this case: $\underline{\lambda}_2 < \underline{\lambda}_1 < \bar{\lambda}_2, \bar{\lambda}_1$
- Evaluate $P^G(\lambda)$ for each in ascending order until finding λ^-, which would give the closest $P^G(\lambda^-)$ value to P^D from below, and λ^+, which would give the closest $P^G(\lambda^+)$ from above. In this case $\lambda^- = \underline{\lambda}_1$ and $\lambda^+ = \bar{\lambda}_2$.
- Then, we can interpolate to obtain λ^* as follows:

$$\boxed{\lambda^* = \lambda^- + \frac{P^D - P^G(\lambda^-)}{P^G(\lambda^+) - P^G(\lambda^-)} \cdot (\lambda^+ - \lambda^-)}$$



$$P_1^* = P_1(\lambda^*)$$
$$P_2^* = P_2(\lambda^*)$$

$$\frac{\lambda^+ - \lambda^-}{P^G(\lambda^+) - P^G(\lambda^-)} = \frac{\lambda^* - \lambda^-}{P^D - P^G(\lambda^-)}$$

- This interpolation method provides the exact solution and is a finite-time algorithm: you need to at most check 4 values.
- If the cost function is not quadratic, the interpolation step would be nonlinear

INCORPORATING LOSSES

So far, we have assumed the system is lossless and also ignored whether the resulting P_i's obtained from the optimization will yield an acceptable voltage profile (this can be checked by solving the power flow equations). In reality, the more general problem to be solved is of the form:

$$\underset{P_1, P_2, \ldots, P_m}{\text{minimize}} \sum_{i=1}^{m} (\alpha_i + \beta_i P_i + \gamma_i P_i^2)$$

subject to

(a) $P_i = \sum_k V_i V_k [G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)], \forall i = 1, \ldots n$

(b) $Q_i = \sum_k V_i V_k [G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)], \forall i = 1, \ldots n$

(c) $P_i = -P_i^D, \forall i = m+1, \ldots n$

(d) $Q_i = -Q_i^D, \forall i = m+1, \ldots n$

(e) $\underline{P_i} \le P_i \le \overline{P_i}, \forall i = 1, 2, \ldots, m$

(f) $\underline{Q_i} \le Q_i \le \overline{Q_i}, \forall i = ~~1, 2, \ldots,~~ m$

(g) $\underline{V_i} \le V_i \le \overline{V_i}, \forall i = 1, 2, \ldots, m.$ this is from $m+1, \ldots$ to n

On top of the lossless assumption, previously, we ignored (b), (f), and (g).

Here, we will still ignore (b), (f), and (g), but would like to consider the losses.

- The losses in a power system, denoted by P^L, are clearly equal to the sum of active power injections; thus

$$\sum_{i=1}^{m} P_i + P^L = \overbrace{\sum_{i=1}^{m} P_i^D}^{P^D}$$

If we take a step back to the power flow ~~equations~~ problem; in that case, for given P_i, $i = 2, \dots, n$, and also given Q_i^D's, we were able to solve for the angles at all buses ~~except~~ except for the slack bus (the angle of which is $\theta_1 = 0$) and compute P_1.

Thus, conceptually, the losses in a system are a function of $P_2, P_3, \dots, P_m$:

$$P^L = P^L(P_2, P_3, \dots, P_m)$$

Thus, we can write:

$$\underset{P_1, P_2, \dots, P_m}{\text{minimize}} \sum_{i=1}^{m} (\alpha_i + \beta_i P_i + \gamma_i P_i^2)$$

$$\text{subject to} \quad \sum_{i=1}^{m} P_i = P^D + P^L(P_2, P_3, \dots, P_m)$$

$$\underline{P_i} \le P_i \le \overline{P_i}$$

- Note, that $P^L(\cdot)$ is not known here; we can obtain its value by solving the power flow equations, but in general we do not have an expression for it.
- We will explore how the earlier solution with no losses would change.
- We can solve the problem with no constraints first:

$$\underset{P_1, P_2, \dots, P_m}{\text{minimize}} \underbrace{\sum_{i=1}^{m} (\alpha_i + \beta_i P_i + \gamma_i P_i^2) + \lambda \left(P^D + P^L(P_2, \dots, P_m) - \sum_{i=1}^{m} P_i\right)}_{C(P_1, P_2, \dots, P_m, \lambda)}$$

$$\nabla C(P_1, P_2, \dots, P_m, \lambda) = 0$$

$$\beta_1 + 2\gamma_1 P_1 = \lambda$$

$$\beta_i + 2\gamma_i P_i + \lambda\left(\frac{\partial P^L}{\partial P_i}\Big|_{P_2,P_3,\dots P_m} - 1 \right) = 0, \quad i = 2, \dots, m$$

$$\sum_{i=1}^{m} P_i = P^D + P^L(P_2, P_3, \dots, P_m)$$

$$\beta_i + 2\gamma_i P_i = \lambda \left(1 - \frac{\partial P_L}{\partial P_i}\Big|_{P_1,P_2,\dots,P_m}\right)$$

$$\frac{1}{1 - \frac{\partial P^L}{\partial P_i}\Big|_{P_2,P_2,\dots,P_m}} \underbrace{(\beta_i + 2\gamma_i P_i)}_{\hookrightarrow \frac{dC_i(P_i)}{dP_i}} = \lambda$$

$$\underbrace{L_i(P_2, \dots, P_m) = \left(1 - \frac{\partial P^L}{\partial P_i}\Big|_{P_2,\dots P_m}\right)^{-1}}_{\text{Penalty factor}}, \quad i = 2, \dots, m$$

We are penalizing the generators that have more impact on the losses.

$L_1 := 1 \longrightarrow$ thus $L_i(P_2, \dots, P_m) = \begin{cases} 1, & i = 1 \\ \left(1 - \frac{\partial P^L}{\partial P_i}\Big|_{P_2,\dots P_m}\right)^{-1}, & i = 2, \dots m \end{cases}$

~~Loss~~ Penalty factors vary with the solution (the operating point); but they are relatively constant, so if we assume them to be constant, we have that the losses are linear with $P_2, \dots, P_m$ and we can write

$$\frac{\partial P^L}{\partial P_i} = 1 - \frac{1}{L_i}$$

~~Minimize~~

$$L_i(\beta_i + 2\gamma_i P_i) = \lambda, \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^{m} P_i = P^D + \sum_{i=1}^{m} \left(1 - \frac{1}{L_i}\right) P_i$$

Rearranging, we obtain

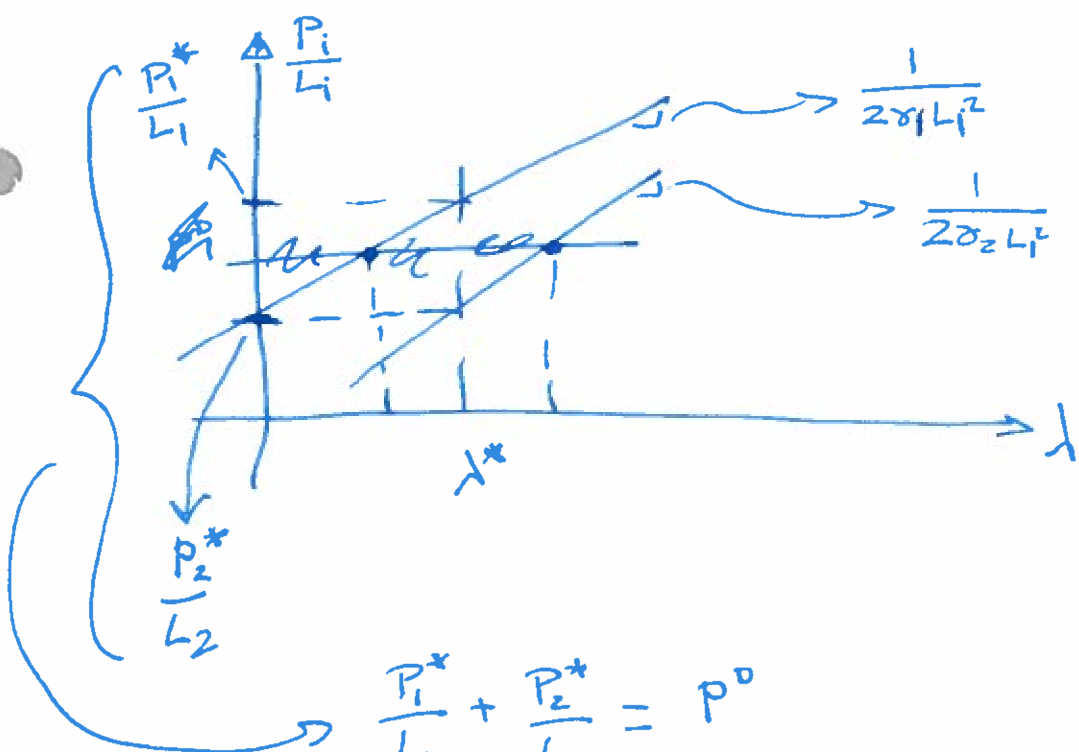
$$L_i(\beta_i + 2\gamma_i P_i) = \lambda, \quad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} \frac{1}{L_i} P_i = P^D,$$

and we could solve for P_i and λ to obtain P_i^* and λ^*.

We can also find a relation between $\frac{P_i}{L_i}$ and λ as follows

$$\frac{P_i}{L_i} = \frac{\lambda - \beta_i}{2\gamma_i L_i^2}$$



$$\frac{P_1^*}{L_1} + \frac{P_2^*}{L_2} = P^D$$

The effect is clear if $L_i > 0$; then the slope of the lines is smaller; thus the P_i^* is smaller.