11/02/17

LECTURE 18

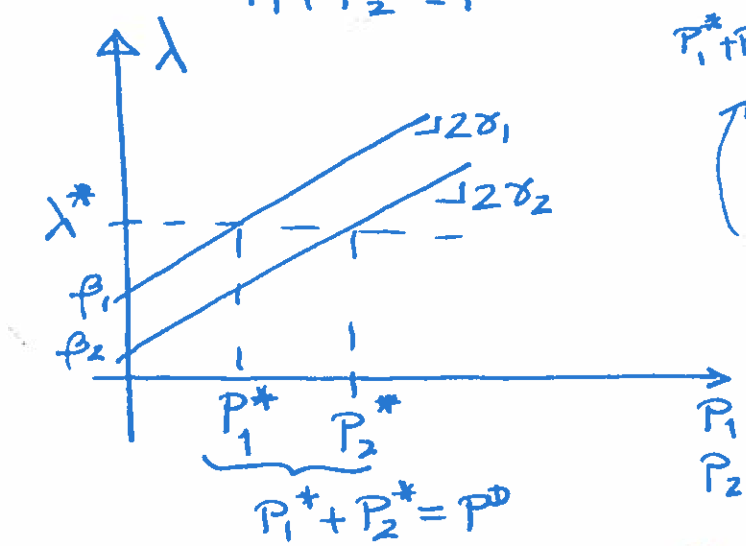
Recall the economic dispatch problem for a system with two buses and no generator capacity limit constraints:

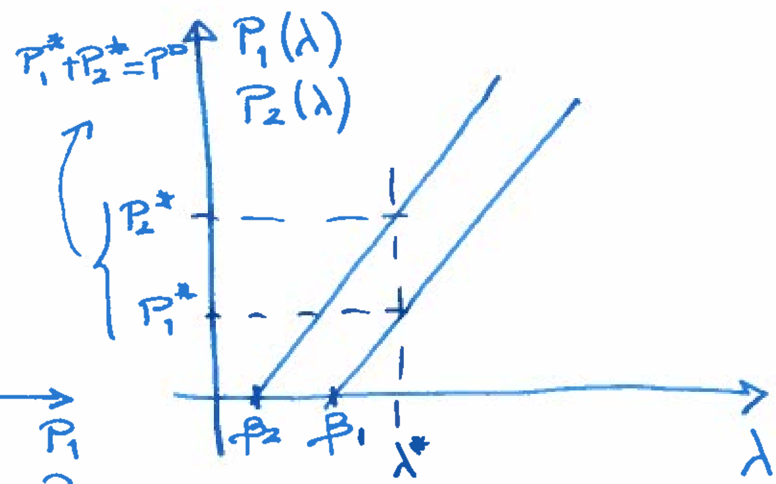
$$\underset{P_1, P_2}{\text{minimize}} \quad (\alpha_1 + \beta_1 P_1 + \gamma_1 \cdot P_1^2) + (\alpha_2 + \beta_2 P_2 + \gamma_2 P_2^2)$$

$$\text{subject to} \quad P_1 + P_2 = P^D$$

We saw that the optimal solution can be obtained by solving

$$\beta_1 + 2\gamma_1 P_1^* = \lambda^*$$
$$\beta_2 + 2\gamma_2 P_2^* = \lambda^*$$
$$P_1^* + P_2^* = P^D$$





$$\left. \begin{aligned} P_1(\lambda) &= \frac{\beta_1 - \lambda}{2\gamma_1} \\ P_2(\lambda) &= \frac{\beta_2 - \lambda}{2\gamma_2} \end{aligned} \right\}$$

$$\boxed{P_i = \frac{\beta_i - \lambda}{2\gamma_i} \quad (A)}$$

λ-ITERATION FOR UNCONSTRAINED CASE

- Next, we discuss an iterative procedure to obtain P_1^*, P_2^*, λ^*. While this procedure is not necessary for the case with no generator constraints, it is the basis for a numerical algorithm to solve the more general case with constraints.

- The algorithm maintains two estimates of λ^*, denoted by λ_L^ν and λ_H^ν, where $\nu = 0, 1, 2, \dots$, and iteratively updates them as follows:

[Initialization]

Choose λ_L^0 so that $\underbrace{\sum_{i=1}^{m} P_i(\lambda_L^0)}_{\sum_{i=1}^{m} \frac{\beta_i - \lambda_L^0}{2\gamma_i}} - P^D < 0$, $i=1,2,\dots m$

Choose λ_H^0 so that $\underbrace{\sum_{i=1}^{m} P_i(\lambda_H^0)}_{\sum_{i=1}^{m} \frac{\beta_i - \lambda_H^0}{2\gamma_i}} - P^D \geq 0$, $i=1,2,\dots,m$

Choose some tolerance level, $\varepsilon > 0$,

While $|\lambda_H^\nu - \lambda_L^\nu| > \varepsilon$

[S1] $\lambda_M^\nu = \dfrac{\lambda_L^\nu + \lambda_H^\nu}{2}$

[S2] Evaluate $P_i(\lambda_M^\nu)$ for all $i = 1, 2, \ldots, m$ → use expression (A)

and define:

$$P_i^\nu := P_i(\lambda_M^\nu) = \frac{\beta_i - \lambda_M^\nu}{2\gamma_i}, \quad \forall i = 1, 2, \ldots, m$$

[S3] If $\sum_{i=1}^{m} P_i^\nu - P^D < 0$

then $\lambda_H^{\nu+1} = \lambda_H^\nu$

$\lambda_L^{\nu+1} = \lambda_M^\nu$

else $\lambda_H^{\nu+1} = \lambda_M^\nu$

$\lambda_L^{\nu+1} = \lambda_L^\nu$

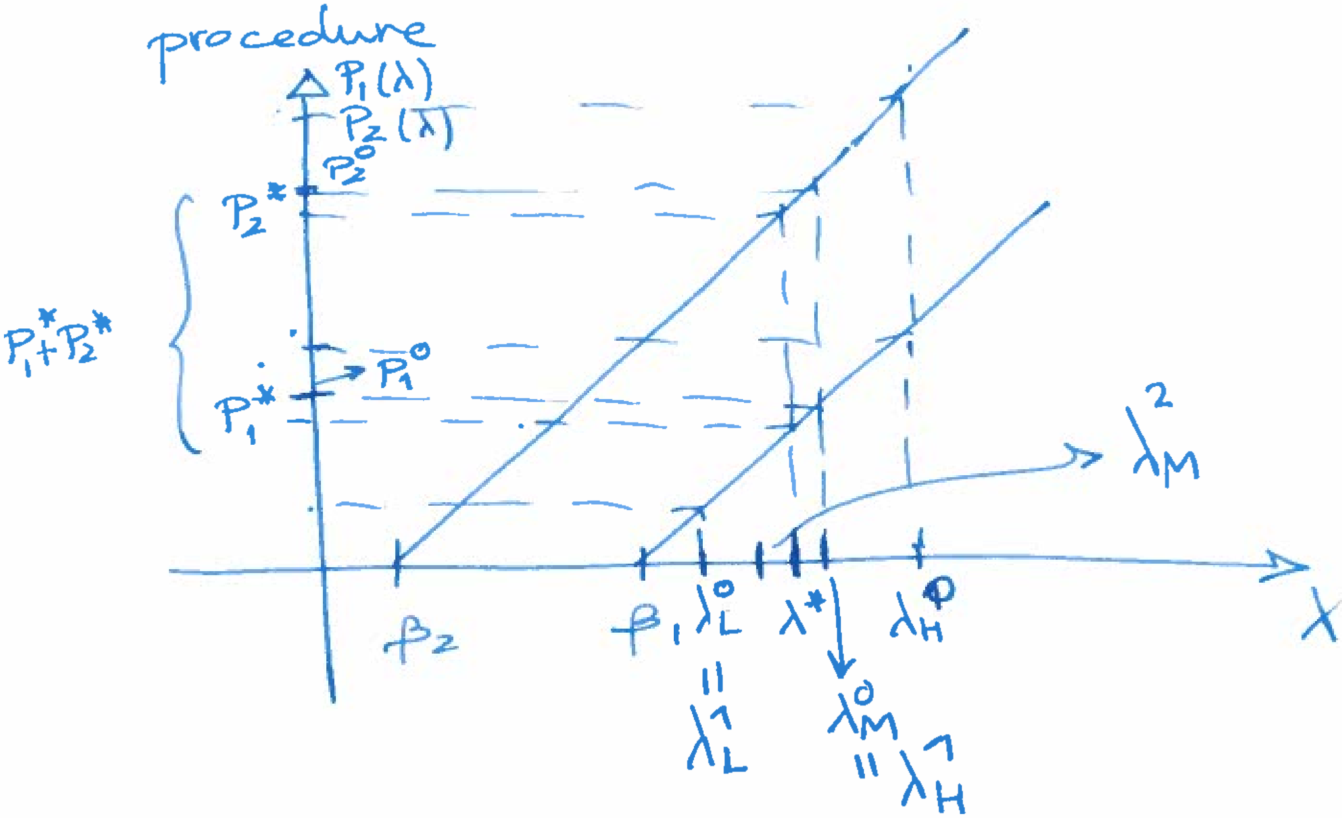
This is how the λ_H^ν and λ_L^ν values get updated

endif

end while

This is essentially a bisection algorithm because at every step the interval $(\lambda_L^\nu, \lambda_H^\nu)$ gets cut in half.

Graphical interpretation of the λ-iteration procedure



λ-ITERATION FOR CONSTRAINED CASE

Next, we discuss the more general case when generator constraints are included

$$\underset{P_1, P_2, \dots, P_m}{\text{minimize}} \sum_{i=1}^{m} \left(\alpha_i + \beta_i P_i + \gamma_i P_i^2\right)$$

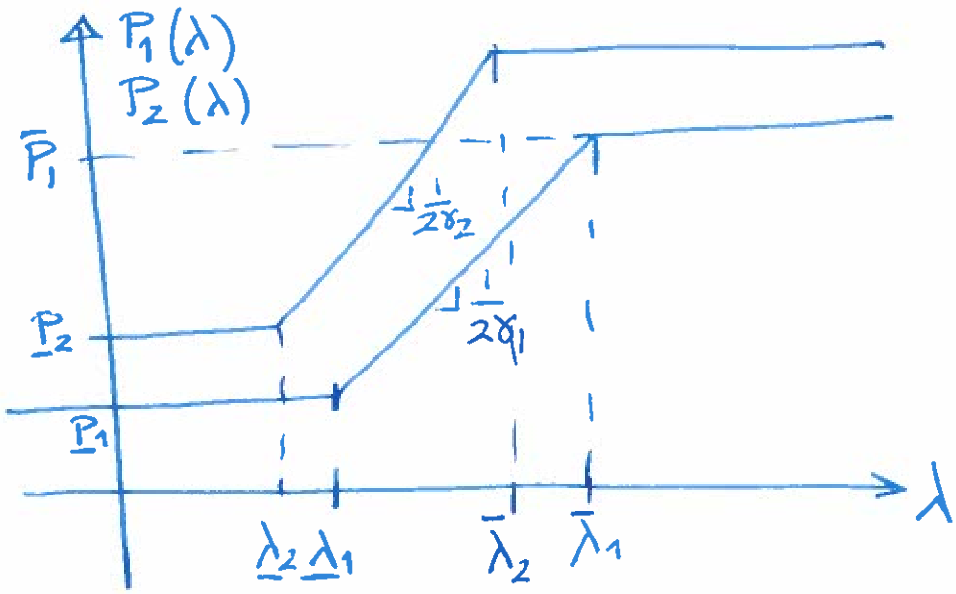
$$\text{subject to} \quad P_1 + P_2 + \dots + P_m = P^D$$

$$0 \leq \underline{P_i} \leq P_i \leq \overline{P_i} \quad , \forall i = 1, 2, \dots, m.$$

In this case, the function $P_i(\lambda)$'s are as follows:

$$P_i(\lambda) = \begin{cases} \dfrac{\beta_i - \lambda}{2\gamma_i}, & \underline{\lambda}_i < \lambda \leq \overline{\lambda}_i \\ \underline{P}_i, & \underline{\lambda}_i > \lambda \\ \overline{P}_i, & \overline{\lambda}_i < \lambda, \end{cases} \quad (B)$$

where $\underline{\lambda}_i = \beta_i + 2\gamma_i \underline{P}_i$, and $\overline{\lambda}_i = \beta_i + 2\gamma_i \overline{P}_i$



The λ- iteration algorithm is very similar to that for the unconstrained case, except for the initialization step, and the evaluation of $P_i(\lambda_M^\nu)$, which uses expression (B) above, instead of expression (A).

[Initialization]

$$\lambda_L^0 = \min\{\underline{\lambda}_1, \underline{\lambda}_2, \dots, \underline{\lambda}_m\}$$
$$\lambda_H^0 = \max\{\overline{\lambda}_1, \overline{\lambda}_2, \dots, \overline{\lambda}_m\}$$

This initialization results in :

$$\sum_{i=1}^m P_i(\lambda_L^0) = \sum_{i=1}^m \underline{P}_i$$
$$\sum_{i=1}^m P_i(\lambda_H^0) = \sum_{i=1}^m \overline{P}_i$$

Thus if the problem is feasible i.e., $\sum_{i=1}^m \underline{P}_i \leq P^D \leq \sum_{i=1}^m \overline{P}_i$, it guarantees $\sum_{i=1}^m P_i(\lambda_L^0) - P^D < 0$, and $\sum_{i=1}^m P_i(\lambda_L^H) - P^D > 0$

Choose some tolerance level, $\varepsilon > 0$,

while $|\lambda_H^\nu - \lambda_L^\nu| > \varepsilon$

[S1] = $\lambda_M^H = \dfrac{\lambda_L^\nu + \lambda_H^\nu}{2}$

[S2] $$P_i^\nu := P_i(\lambda_M^\nu) = \begin{cases} \dfrac{\beta_i - \lambda_M^\nu}{2\gamma_i}, & \text{if } \underline{\lambda}_i < \lambda_M^\nu \leq \overline{\lambda}_i \\ \underline{P}_i, & \text{if } \lambda_M^\nu < \underline{\lambda}_i \\ \overline{P}_i, & \text{if } \lambda_M^\nu > \overline{\lambda}_i \end{cases}$$

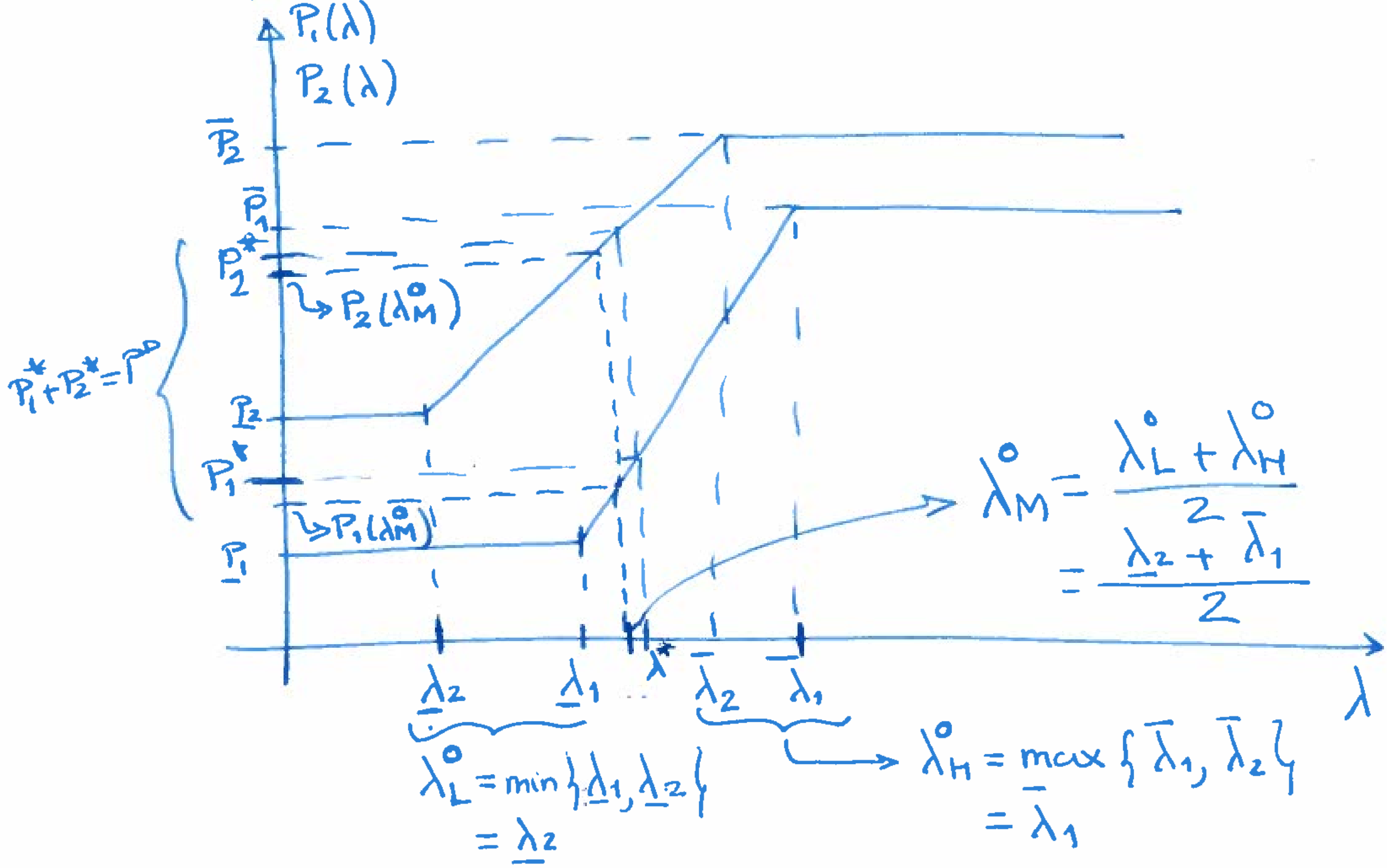
[S3] If $\sum_{i=1}^m P_i^\nu - P^D < 0$

then $\lambda_H^{\nu+1} = \lambda_H^\nu$
$\lambda_L^{\nu+1} = \lambda_M^\nu$

else $\lambda_H^{\nu+1} = \lambda_M^\nu$
$\lambda_L^{\nu+1} = \lambda_L^\nu$

endif

Graphical interpretation of the λ-iteration



Clearly $\overline{P}_1 + \overline{P}_2 > P_1^* + P_2^* = P^D$ (The problem is
$\underline{P}_1 + \underline{P}_2 < P_1^* + P_2^* = P^D$ feasible)