10/31/17

LECTURE 17

ECONOMIC DISPATCH

- MOTIVATION
- PROBLEM FORMULATION
  - CLASSICAL ECONOMIC DISPATCH

MOTIVATION

So far, we spent time figuring out how to solve the power flow problem:

- Given: $V_1, \theta_1, P_2^G, V_2, \ldots, P_m^G, V_m, P_{m+1}^D, Q_{m+1}^D, \ldots, P_n^D, Q_n^D$
- Compute: $P_1^G, Q_1^G, \theta_2, Q_2^G, \ldots, \theta_m, Q_m^G, \theta_{m+1}, V_{m+1}, \ldots, \theta_n, V_n$

→ We fix the power produced by each generator, but what is the <u>criteria</u> for fixing it?

The criteria is the following

(i) Cost of operation (ECONOMIC DISPATCH)

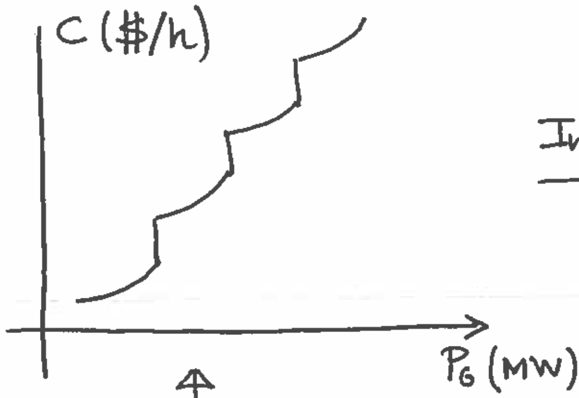
(ii) Reliability

- In ECE 476, we focus on (i)
- The total cost of operation includes:
  (i) Fuel
  (ii) Labor
  (iii) Maintenance

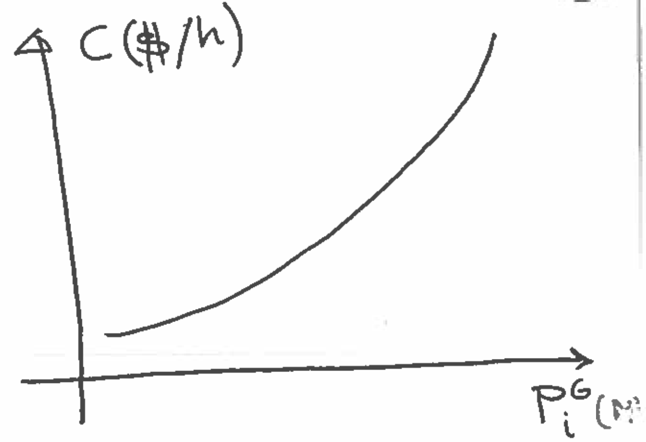
- To simplify the problem, we will only focus on fuel <u>costs</u>.
- We will assume that fuel-cost curves for each generating units are available.
- The cost of the fuel used per-unit is a function of the generator power output P_i^G

$$\boxed{C\,(\$/h) = \mathcal{C}_i\left(P_i^G\right)}$$

- Fuel-cost curves are usually not smooth functions; however, a good approximation of the fuel-cost curve of a generator is a piece-wise quadratic cost function



In 476 →



The discontinuities are due to the firing of equipment such as additional boilers come online as the power output is increased.

$$\mathcal{C}_i\left(P_i^G\right) = \alpha_i + \beta_i P_i^G + \gamma_i \left(P_i^G\right)^2$$

We will assume a quadratic function.

PROBLEM FORMULATION

- We are given a system with m generators committed and all the P_i^D, Q_i^D, $i=m+1,\dots,n$.
- The problem is to pick the P_i^G's and V_i, $i=1,2,\dots,m$ so as to minimize the total cost

$$C_T := \sum_{i=1}^{m} C_i(P_i^G)$$

subject to

(i) Satisfaction of the power flow eqns

$$P_i = \sum_k V_i V_k \left[G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k) \right]$$

$$Q_i = \sum_k V_i V_k \left[G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k) \right]$$

(ii) Constraints on generator power

$$\underline{P}_i^G \leq P_i^G \leq \overline{P}_i^G, \quad i=1,2,\dots,m$$

(iii) line power flows

$$|P_{ij}| \leq \overline{P}_{ij}, \quad \text{all lines}$$

(iv) Voltage magnitudes

$$\underline{V}_i \leq V_i \leq \overline{V}_i, \quad i=1,2,\dots,m,\dots n$$

A few comments are in place:

(i) The power flow eqns. must be satisfied (i.e., they are equality constraints on the optimization)

(ii) The upper limit on P_i^G is set by thermal limits on the turbine/generator unit, while the lower limit is set by boiler and/or thermodynamic considerations

(iii) The constraints on voltage keep the system voltages from varying too far from their rated or nominal voltages

(iv) The reasons for constraints on the transmission-line ~~paramet~~ powers relate to thermal and stability limits.

The minimization of a cost function subject to equality and inequality constraints is a problem in optimization that is treated by a branch of applied mathematics called nonlinear programming

ECE 490: INTRODUCTION TO OPTIMIZATION

CLASSICAL ECONOMIC DISPATCH

(i) We assume no losses:

$$\sum_{i=1}^{m} P_i^G = \sum_{i=m+1}^{n} P_i^D$$

(ii) We ignore all the inequality constraints;

Thus, the problem reduces to

$$\underset{P_1^G, P_2^G, \dots P_G^m}{\text{minimize}} \sum_{i=1}^{m} C_i(P_i^G) = \sum_{i=1}^{m} \left(\alpha_i + \beta_i P_i^G + \gamma_i (P_i^G)^2\right)$$

$$\text{subject to} \quad \sum_{i=1}^{m} P_i^G = \underbrace{\sum_{i=m+1} P_i^D}_{P^D}$$

- This is a constrained optimization problem
- We can reformulate the problem so as to get rid of the equality constraint.
- First, let's look at the case when we do have the equality constraint in place

$$\underset{P_1^G, P_2^G, \dots, P_m^G}{\text{minimize}} \sum_{i=1}^{m} \underbrace{\left(\alpha_i + \beta_i P_i^G + \gamma_i (P_i^G)^2\right)}_{C_i(P_i^G)}$$

$$\nabla C_T(P_1^G, P_2^G, \ldots, P_m^G) = 0$$

$$\left.\begin{array}{ll}
\dfrac{\partial C_1(P_1^G)}{\partial P_1^G} = 0 & \beta_1 + 2\gamma_1 P_1^G = 0 \\[2ex]
\dfrac{\partial C_2(P_2^G)}{\partial P_2^G} = 0 & \beta_2 + 2\gamma_2 P_2^G = 0 \\
\vdots & \vdots \\
\dfrac{\partial C_m(P_m^G)}{\partial P_m^G} = 0 & \beta_m + 2\gamma_m P_m^G = 0
\end{array}\right\} \rightarrow P_i^G = \frac{-\beta_i}{2\gamma_i}$$

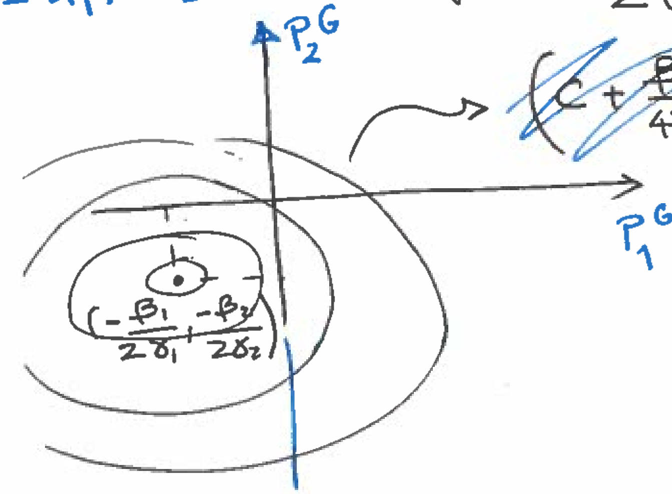
$\uparrow$

In our setup the solution does not make any physical sense

In 2-D

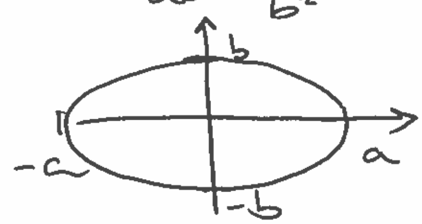
$$\underset{P_1^G, P_2^G}{\text{minimize}} \sum_{i=1}^{2} \left(\alpha_i + \beta_i P_i^G + \gamma_i (P_i^G)^2\right) = C$$

$$-\alpha_1 - \alpha_2 + C = \gamma_1\left(P_1^G + \frac{\beta_1}{2\gamma_1}\right)^2 + \gamma_2\left(P_2^G + \frac{\beta_2}{2\gamma_2}\right)^2 - \frac{\beta_1^2}{4\gamma_1} - \frac{\beta_2^2}{4\gamma_2}$$

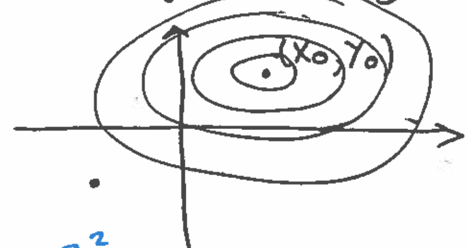


$$\cancel{\left(C + \frac{\beta_1^2}{4\gamma_1} + \frac{\beta_2^2}{4\gamma_2}\right)^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$



$$\gamma_1\left(P_1^G + \frac{\beta_1}{2\gamma_1}\right)^2 + \gamma_2\left(P_2^G + \frac{\beta_2}{2\gamma_2}\right)^2 = C + \frac{\beta_1^2}{4\gamma_1} + \frac{\beta_2^2}{4\gamma_2}$$

$$-\alpha_1 - \alpha_2$$

$$a^2 = \frac{C + \frac{\beta_1^2}{4\gamma_1} + \frac{\beta_2^2}{4\gamma_2}}{\gamma_1} \qquad b^2 = \frac{C + \frac{\beta_1^2}{4\gamma_1} + \frac{\beta_2^2}{4\gamma_2}}{\gamma_2}$$

- Let's put the constraint back

$$\underset{P_1^G, P_2^G, \ldots, P_m^G}{\text{minimize}} \sum_{i=1}^{m} \left(\alpha_i + \beta_i P_i^G + \gamma_i (P_i^G)^2\right)$$

$$\text{subject to} \quad \sum_{i=1}^{m} P_i^G = P^D$$

- The Key idea is to turn the problem above into an unconstrained optimization problem and solve as before (i.e., by making the gradient of the cost fn. equal to zero)

- To this end, we introduce a so-called Lagrange multiplier λ and reformulate the problem as follows.

$$\underset{P_1^G, P_2^G, \ldots, P_m^G, \lambda}{\text{minimize}} \underbrace{\sum_{i=1}^{m} \left(\alpha_i + \beta_i P_i^G + \gamma_i (P_i^G)^2\right) + \lambda \left(P^D - \sum_{i=1}^{m} P_i^G\right)}_{C_T(P_1^G, P_2^G, \ldots, P_m^G, \lambda)}$$

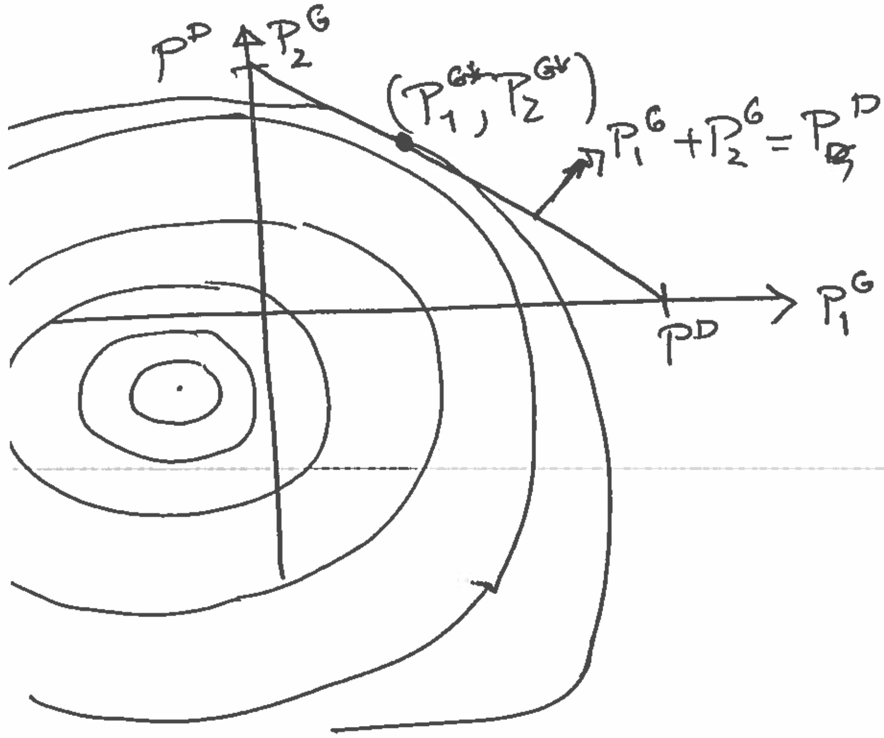
$$\nabla C_T(P_1^G, P_2^G, \ldots, P_m^G, \lambda)$$

$$\frac{\partial C_T}{\partial P_i^G} = \beta_i + 2\gamma_i P_i^G - \lambda, \quad i = 1, 2, \ldots m$$

$$\frac{\partial C_T}{\partial \lambda} = P^D - \sum_{i=1}^{m} P_i$$

$m=2$:

$$\left.\begin{aligned} \beta_1 + 2\gamma_1 P_1^G &= \lambda \\ \beta_2 + 2\gamma_2 P_2^G &= \lambda \\ P_1^G + P_2^G - P^D &= 0 \end{aligned}\right\}$$ 3 eqns and 3 unknowns



- The solution of minimum cost is when C_T is tanget to the constraint at exactly one point.
- For the ellipse to be tangent to the constraint, it implies that the normal vector to the constraints $n = [1,1]^T$ and gradient of the ellipse must be paralel, which is equivalent to say that there exist some λ such that $\nabla C_T = \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\nabla C_T(P_1^G, P_2^G) - \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0 \equiv \nabla \left(C_T(P_1^G, P_2^G) - \lambda (P_1^G + P_2^G - P_D) \right)$$

$$P_1^G + P_2^G = P^D$$

$$\Vert$$

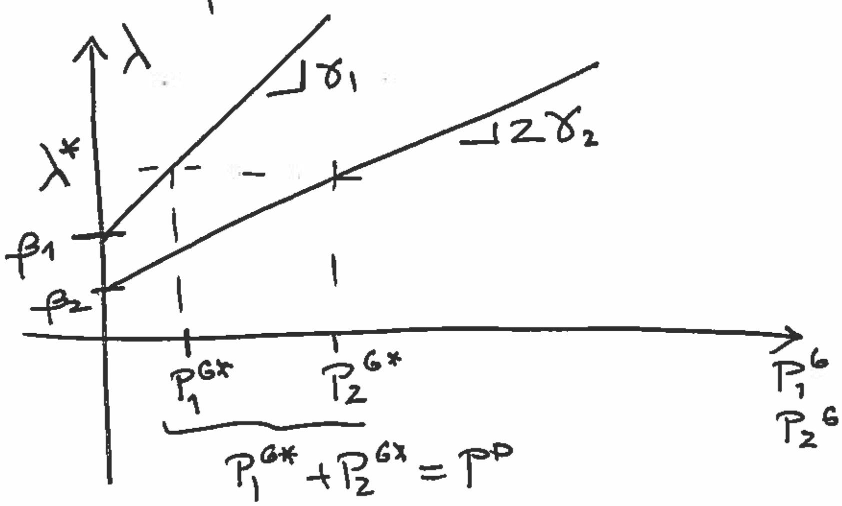
$$\underset{P_1^G, P_2^G, \lambda}{\text{minimize}} \left(C_T(P_1^G, P_2^G) + \lambda (P^D - P_1^G - P_2^G \right.$$

$$\left.\begin{aligned} \beta_1 + 2\gamma_1 P_1^G - \lambda &= 0 \\ \beta_2 + 2\gamma_2 P_2^G - \lambda &= 0 \\ P_1^G + P_2^G - P_D &= 0 \end{aligned}\right\} \rightarrow \begin{bmatrix} 2\gamma_1 & 0 & -1 \\ 0 & 2\gamma_2 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_1^G \\ P_2^G \\ \lambda \end{bmatrix} = \begin{bmatrix} -\beta_1 \\ -\beta_2 \\ P^D \end{bmatrix}$$

GRAPHICAL INTERPRETATION

$$\lambda = \beta_1 + 2\gamma_1 P_1^G$$

$$\lambda = \beta_2 + 2\gamma_2 P_2^G$$



λ^* solves
$$\begin{cases} \dfrac{dC_i(P_i^G)}{dP_i^G} = \lambda, & \forall i = 1, 2, \ldots, m \\ \displaystyle\sum_{i=1}^{m} P_i^G = P_D \end{cases}$$