# ECE 476 - Power System Analysis Fall 2017 <br> Homework 9 

Reading: Sections 11.1 - 11.3 in Chapter 11 of textbook.
In-class quiz: Thursday, November 30, 2017.
Problem 1. Given a turbine-generator unit rated 100 MVA with a per unit inertia constant, $H=5 \mathrm{sec}$.
(a) Calculate the kinetic energy stored at synchronous speed (3600 rpm).
(b) Compare the answer from (a) with the kinetic energy of a 10 -ton truck going at 60 mph .
(c) Suppose that the generator is delivering 100 MW and then at $t=0$, the line circuit breakers open. Calculate the shaft acceleration in $\mathrm{rad} / \mathrm{sec}^{2}$.
(d) At this rate, how long does it take for $\delta$ to increase from $\delta^{0}$ to $\delta^{0}+2 \pi$ radians.

Problem 2. Consider the spring-mass system shown in Fig. 1.

$$
m \frac{d^{2} x}{d t^{2}}+a \frac{d x}{d t}+\sin x=f(t)
$$



Figure 1: Spring-mass system for Problem 2.
(a) For $t<0, f(t)=0.5$. Find the steady-state $x^{0}$.
(b) For $0 \leq t, f(t)=0.5+\Delta f(t)$, where $\Delta f(t)$ is small. Find a linear differential equation that describes $\Delta x=x-x^{0}$ in terms of $\Delta f(t)$.
(c) Find the natural frequencies of the linearized system found in part (b) with $m=1.0$ and $a=0.01$. Suppose that $\Delta f(t)=0.1 u(t)$, where $u(t)$ is the unit step function. Roughly sketch $x(t)$ for $t \geq 0$, showing the initial value, final value, frequency, and decay of the lightly damped oscillatory component.

Problem 3. Consider the single-machine infinite-bus system shown in Fig. 2. The generator is described by the classical model, with $X_{d}^{\prime}=1.0$ p.u., and delivers steady-state power $P_{G}=0.5 \mathrm{p} . \mathrm{u}$. to an infinite bus through a transmission line with reactance $X_{L}=0.4$ p.u. Assume that $|E|=1.8$ p.u., $V_{\infty}=1 \angle 0^{\circ}$ p.u., $H=5 \mathrm{sec}$,


Figure 2: Single-machine infinite-bus system for Problem 3.

1. Find the two possible steady-state (equilibrium) values of power angle $\delta$ that lie in the interval $[0,2 \pi]$.
2. Which of these two equilibrium values are we likely to observe in practice? Hint: Linearize around the equilibrium value and consider the natural frequencies found from linear differential equation in $\Delta \delta$.

Problem 4. Suppose that in Problem 3 at $t=0$, the line circuit breakers open. $P_{M}^{0}=0.5$ p.u. remains constant. Calculate $\delta$ and $\dot{\delta}$ at the end of 1 sec under two assumptions and compare:
(a) $D=0$
(b) $D=0.001$

Problem 5. Refer to Fig. 3 and assume that $P_{M}=1.0$ p.u., $|E|=1.5$ p.u., and $X_{d}^{\prime}=0.9$ p.u.


Figure 3: System for Problem 5.
(a) At $t=0$ the circuit breakers open and remain open. Determine if the transient is stable.
(b) Repeat (a) for $X_{d}^{\prime}=0.6$ p.u.

