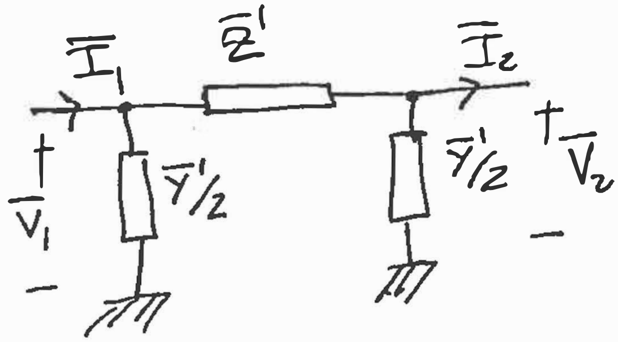
LECTURE 7 09/19/17

LUMPED-PARAMETER CIRCUIT MODEL (π MODEL



$$\bar{V}_1 = \bar{V}_2 + \bar{Z}'\left(\bar{I}_2 + \frac{\bar{Y}'}{2}\bar{V}_2\right)$$

$$= \left(1 + \bar{Z}' \cdot \frac{\bar{Y}'}{2}\right)\bar{V}_2 + \bar{Z}' \cdot \bar{I}_2$$

$$\bar{Z}' = \bar{Z}_c \cdot \sin h\gamma d$$

$$\bar{Z}' = \sqrt{\frac{\bar{Z}}{\bar{Y}}} \cdot \frac{\gamma d}{\gamma d} \sinh \gamma d$$

$$= \sqrt{\frac{\bar{Z}}{\bar{Y}}} \cdot \sqrt{\bar{Z} \cdot \bar{Y}} \cdot d \cdot \frac{\sinh \gamma d}{\gamma d}$$

$$= \underbrace{\bar{Z} \cdot d}_{\bar{Z}_1} \cdot \frac{\sinh \gamma d}{\gamma d}$$

$$\bar{Z}' = \bar{Z}_1 \cdot \frac{\sinh \gamma d}{\gamma d}, \quad \bar{Z}_1 = \bar{Z} \cdot d$$

$$1 + \bar{Z}' \cdot \frac{\bar{Y}'}{2} = \cosh \gamma d$$

$\tanh \frac{\gamma d}{2}$

$$\frac{\bar{Y}'}{2} = \frac{1}{\bar{Z}'}(\cosh \gamma d - 1) = \frac{1}{\bar{Z}_c} \frac{\cosh \gamma d - 1}{\sinh \gamma d}$$

$$= \frac{1}{\bar{Z}_c} \tanh \frac{\gamma d}{2} = \frac{\gamma d/2}{\bar{Z}_c} \frac{\tanh \frac{\gamma d}{2}}{\frac{\gamma d}{2}}$$

$\bar{Y}$

$$= \frac{\sqrt{\bar{Z}\bar{Y}}\, d/2}{\sqrt{\bar{Z}/\bar{Y}}} \frac{\tanh \frac{\gamma d}{2}}{\frac{\gamma d}{2}} = \frac{\bar{Y} d}{2} \frac{\tanh \frac{\gamma d}{2}}{\frac{\gamma d}{2}}$$

$$\boxed{\begin{aligned} \bar{Z}' &= \bar{Z}_1 \cdot \frac{\sinh \gamma d}{\gamma d} \\ \frac{\bar{Y}'}{2} &= \frac{\bar{Y}}{2} \frac{\tanh \gamma d/2}{\gamma d/2} \end{aligned}}$$

$$\bar{Z}_1 = \bar{z} \cdot d$$

$$\frac{\bar{Y}}{2} = \bar{y}/2$$

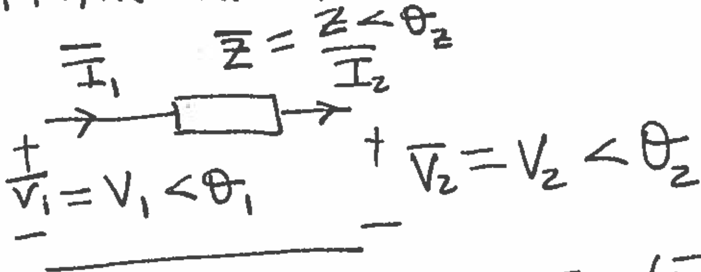
SIMPLIFIED MODELS

length d	$\bar{Z}'$	$\bar{Y}'/2$
$d > 200$ miles	$\bar{Z}_1 \dfrac{\sinh \gamma d}{\gamma d}$	$\dfrac{\bar{Y}}{2} \dfrac{\tanh \gamma d/2}{\gamma d/2}$
50 miles $< d < 200$ miles	$\bar{Z}$	$\bar{Y}/2$
$d < 50$ miles	$\bar{Z}_1$	0

LOSELESS TRANSMISSION LINE

- $r = 0$
- $\bar{Z}_c = \sqrt{\cancel{j\omega} \ell \dfrac{1}{\cancel{j\omega} c}} = \sqrt{\dfrac{\ell}{c}} = \sqrt{\dfrac{L}{C}}$ $[\Omega]$
- The characteristic impedance in this case is referred to as the SURGE IMPEDANCE

POWER TRANSFER THROUGH A SHORT TRANSMISSION LINE



$$\overline{S}_{12} = \overline{V}_1 \cdot \overline{I}_1^{\,*} = \overline{V}_1 \cdot \frac{(\overline{V}_1 - \overline{V}_2)^*}{Z^*}$$

$$\overline{V}_1 = V_1\angle\theta_1 \quad , \quad \overline{V}_2 = V_2\angle\theta_2 \quad , \quad \overline{Z} = Z\angle\theta_z$$

$$\overline{S}_{12} = V_1\angle\theta_1 \; \frac{V_1\angle -\theta_1 - V_2\angle -\theta_2}{Z\angle -\theta_z}$$

$$= \frac{V_1^2}{Z}\angle\theta_z - \frac{V_1V_2}{Z}\angle\theta_1 - \theta_2 + \theta_z$$

If the line is lossless $\theta_z = \frac{\pi}{2}$

$$\overline{S}_{12} = \frac{V_1^2}{Z}\angle\frac{\pi}{2} - \frac{V_1V_2}{Z}\angle\theta_1 - \theta_2 + \frac{\pi}{2}$$

$$P_{12} = \mathrm{Re}\{\overline{S}_{12}\} = -\frac{V_1V_2}{Z}\cos\left(\theta_1 - \theta_2 + \frac{\pi}{2}\right)$$

$$= \frac{V_1V_2}{Z}\sin(\theta_1 - \theta_2)$$

$$Q_{12} = \mathrm{Im}\{\overline{S}_{12}\} = \frac{V_1^2}{Z} - \frac{V_1V_2}{Z}\sin\left(\overbrace{\theta_1 - \theta_2}^{\alpha} + \frac{\pi}{2}\right)$$

$$= \frac{V_1^2}{Z} - \frac{V_1V_2}{Z}\cos(\underbrace{\theta_1 - \theta_2}_{\alpha})$$

TRANSFORMERS

- THE NEED FOR TRANSFORMERS

(i) The generation of electric power is usually done at voltages that range between 11 and 13 KV

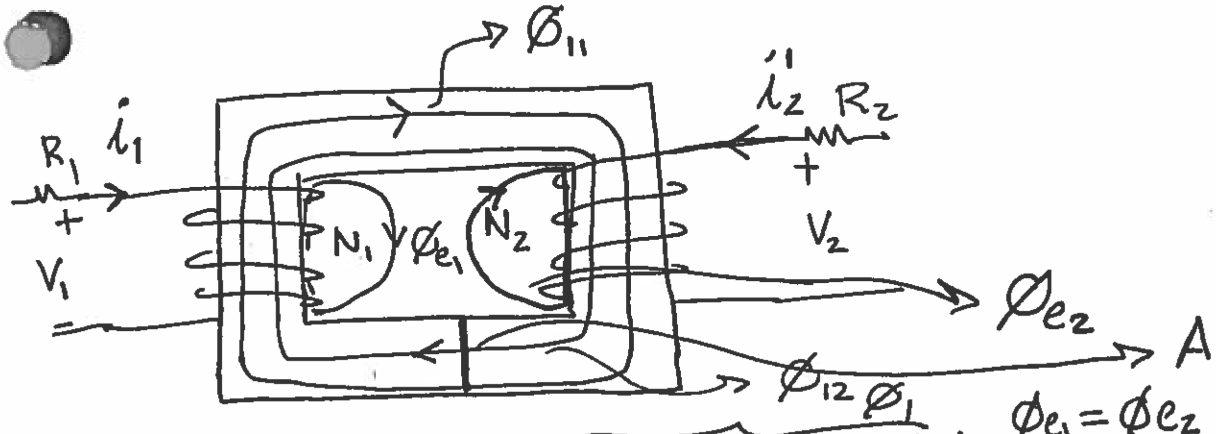
(ii) Efficient power transmission over long distances requires much higher voltages that those at which power is generated. Typically values used in power transmission are 138, 230, 345, 765 KV

(iii) Distribution of power to our neighbor. and industrial plants is done at much lower voltages: 2400 – 4600 V

(iv) In our houses, the voltage is 240/120 V

In view of (i) – (iv), we need a way to step voltages up and down. The device that permits AC voltages to be stepped UP & DOWN is the transformers. [DC voltages can also be stepped up and down using power electronics → ECE 464/469.]

1Ø IDEAL TRANSFORMER MODEL



mutual fl

$Øe_1 = Øe_2 = 0$

$$\left\{\begin{array}{l} \lambda_1 = N_1 \cdot Ø_1 = N_1\,(Ø_{11} + Ø_{12} + Øe_1) \\ \lambda_2 = N_2\,Ø_2 = N_2\,(\underbrace{Ø_{22} + Ø_{21} + Øe_2}_{Ø_2}) \\ V_1 = \dfrac{d\lambda_1}{dt} + R_1 i_1 \\ V_2 = \dfrac{d\lambda_2}{dt} + R_2 i_2' \end{array}\right.$$

$\rightarrow Ø_1 = Ø_2 = Ø_m$

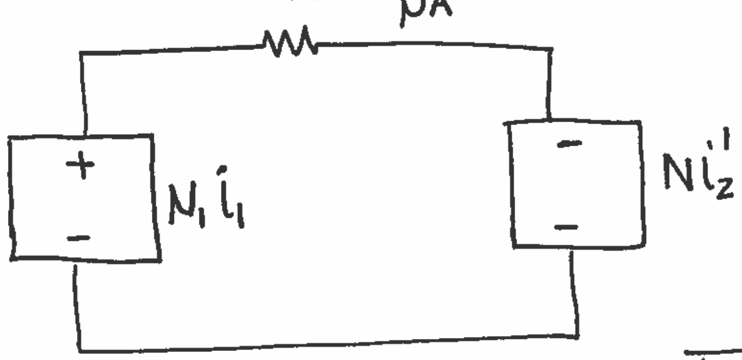
$\rightarrow R_1 = R_2 = 0$

$$V_1 = \frac{d\lambda_1}{dt} = N_1 \frac{dØ_m}{dt}$$
$$V_2 = \frac{d\lambda_2}{dt} = N_2 \frac{dØ_m}{dt}$$
$$\rightarrow \boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}}$$

Assumptions for an ideal transformer:

- No losses: $R_1 = R_2 = 0$ (also no magnetic losses)
- No leakage fluxes: $Øe_1 = Øe_2$
- Magnetic core material has infinite permeabilit

$$\mathcal{R} = \frac{l}{\mu A}$$



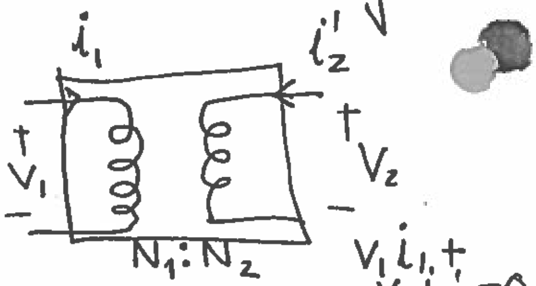
$\mu \rightarrow \infty \rightarrow$

$\mathcal{R} \rightarrow 0$

$$N_1 i_1 + N_2 i_2' = 0$$

$$\boxed{\frac{i_1}{i_2'} = -\frac{N_2}{N_1}}$$

The two fundamental relations for a single-phase transformer are:

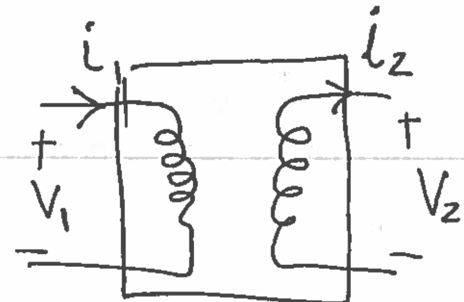


$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}} \qquad \boxed{\frac{i_1}{i_2'} = -\frac{N_2}{N_1}}$$

$V_1 i_1 + V_2 i_2' = 0$

We can reverse the current on the "2" side:

$i_2 := -i_2'$

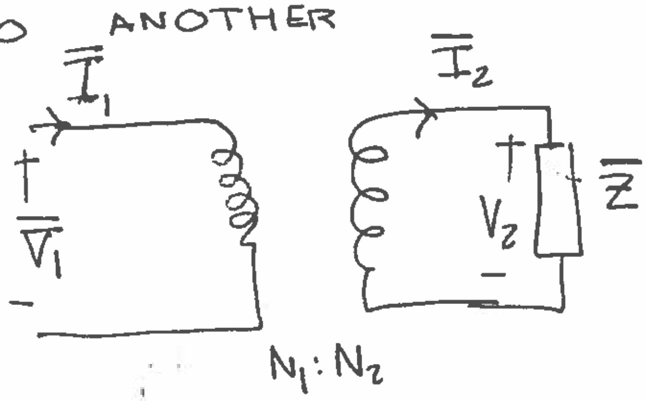


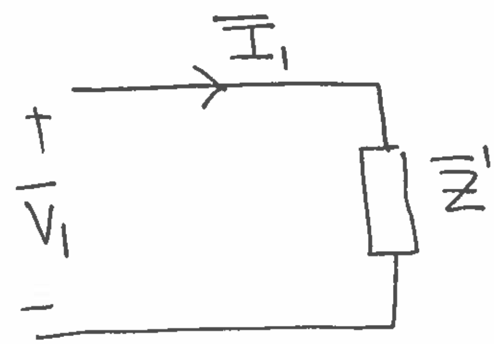
$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}} \qquad \boxed{\frac{i_1}{i_2} = \frac{N_1}{N_2}}$$

$$\underbrace{V_1 i_1}_{\text{Power in}} = \underbrace{V_2 i_2}_{\text{Power out}}$$

<u>Turn ratio</u>: $\boxed{a := \frac{N_1}{N_2}}$

REFERRING AN IMPEDANCE FROM ONE SIDE TO ANOTHER





$\overline{V}_2 = \overline{I}_2 \cdot \overline{Z}$ $\qquad \frac{\overline{V}_1}{\overline{V}_2} = \frac{N_1}{N_2} \longrightarrow \overline{V}_2 = \frac{N_2}{N_1}\overline{V}_1$

T $\qquad \frac{\overline{I}_1}{\overline{I}_2} = \frac{N_2}{N_1} \longrightarrow \overline{I}_2 = \frac{N_1}{N_2}\overline{I}_1$

$$\frac{N_2}{N_1}\overline{V}_1 = \frac{N_1}{N_2}\overline{I}_1 \cdot \overline{Z} \longrightarrow \overline{V}_1 = \overline{I}_1 \underbrace{\left(\frac{N_1}{N_2}\right)^2 \overline{Z}}_{\overline{Z}'}$$

$$\boxed{\overline{Z}' = \left(\frac{N_1}{N_2}\right)^2 \overline{Z}}$$