10/24/17

LECTURE 16

STOPPING CRITERIA

$$P_i = \sum_{K=1}^{n} V_i V_K \left(G_{iK} \cos(\theta_i - \theta_K) + B_{iK} \sin(\theta_i - \theta_K) \right)$$

$$Q_i = \sum_{K=1}^{n} V_i V_K \left(G_{iK} \sin(\theta_i - \theta_K) - B_{iK} \cos(\theta_i - \theta_K) \right)$$

Given: $V_1, \theta_1, P_2^G, V_2, \ldots, P_m^G, V_m, P_{m+1}^D, Q_{m+1}^D, \ldots, P_n^D, Q_n^D$

To compute: $P_1, Q_1, Q_2, \theta_2, \ldots, Q_m, \theta_m, V_{m+1}, \theta_{m+1}, \ldots V_n, \theta_n$

- In a nutshell: find x^* that satisfies $f(x^*) = 0$

So far, we have discussed numerical algorithms of the form

$$x^{\nu+1} = x^{\nu} - \left(J^{-1}(x^{\nu})\right)^{-1} f(x^{\nu})$$

- And for ν large enough, we hope that x^{ν} is closed to x^*.
- Is there a way for us to determine that our closeness to x^* is good enough?
- Choose $\varepsilon > 0$. At each iteration ν, compute

$$e^{\nu} := x^{\nu+1} - x^{\nu}$$

- Compute the 2-norm or ∞-norm of e^{ν}:

$$\|e^{\nu}\|_2 = \sqrt{(e_1^{\nu})^2 + (e_2^{\nu})^2 + \ldots (e^{\nu})^2} < \varepsilon, \text{ stop.}$$

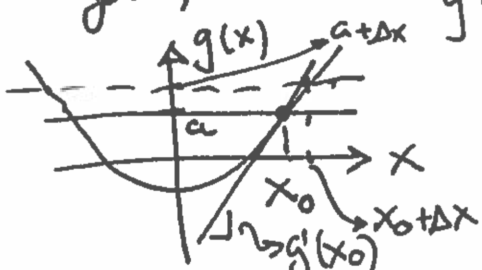
$$\|e^{\nu}\|_{\infty} = \max_i |e_i^{\nu}| < \varepsilon, \text{ stop.}$$

DC POWER FLOW

- So far, the numerical algorithms we have discussed can potentially getting us to the true solution if we keep iterating.
- We are going now to develop a method that would give us the solution approximately.
- The method relies on linearizing the power flow equations around the the flat voltage solution, i.e., $V_1 = 1, V_2 = 1, \ldots, V_n = 1$, $\theta_1 = \theta_2 = \ldots = \theta_n = 0$.

BASICS OF LINEARIZATION

$$g(x) = a$$



$g(x_0) = a$

what happens if we perturb a by Δa, the solution changes from x_0 to Δx:

$$g(x_0 + \Delta x) = a + \Delta a$$

What's Δx? We can solve again $g(x) = a + \Delta a$ or look for an approximation.

Taylo's series expansion of $g(\cdot)$ around x_0:

$$\cancel{g(x_0)} + g'(x_0)\Delta x + \underbrace{h.o.t.}_{\text{neglect.}} = \cancel{a} + \Delta a$$

$$g'(x_0)\Delta x = \Delta a \rightarrow \Delta x \simeq (g'(x_0))^{-1} \Delta a$$

The new solution

$$x_1 = x_0 + \Delta x \simeq x_0 + (g'(x_0))^{-1} \Delta a$$

- Another way to derive the DC power flow is by making some simplifications to the power flow equations consistent with some simplifying assumption about the system topology and operating point of the system.

TOPOLOGY ASSUMPTIONS

T1. No shunt elements

T2. $r_{ij} < x_{ij}$ for series elements $\Rightarrow G_{ik} \simeq 0$

OPERATIONAL ASSUMPTIONS

O1. $\theta_i - \theta_k$ is small for all pairs $\{i,k\}$ that are electrically connected

O2. $V_i \simeq 1$ p.u. $\forall i$.

With those assumptions, the active power balance equations reduce to:

$$P_i = \sum_{k=1}^{n} B_{ik} (\theta_i - \theta_k)$$

$$= B_{i1} \cdot (\theta_i - \theta_1) + B_{i2}(\theta_i - \theta_2) + \dots + B_{in}(\theta_i - \theta_n)$$

$$= \underbrace{\Big(\sum_{\substack{k=1 \\ k\neq i}}^{n} B_{ik}\Big)}\theta_i - \sum_{\substack{k=1 \\ k\neq i}}^{n} B_{ik}\theta_k$$

$$= - B_{ii}\,\theta_i - \sum_{k=1}^{n} B_{ik}\,\theta_k$$

Since we know $P_2 = P_2^G, \ldots, P_m = P_m^G,$
$P_{m+1} = -P_{m+1}^D, \ldots, P_n = -P_n^D,$

and we also know $\theta_1 = 0$, we can write

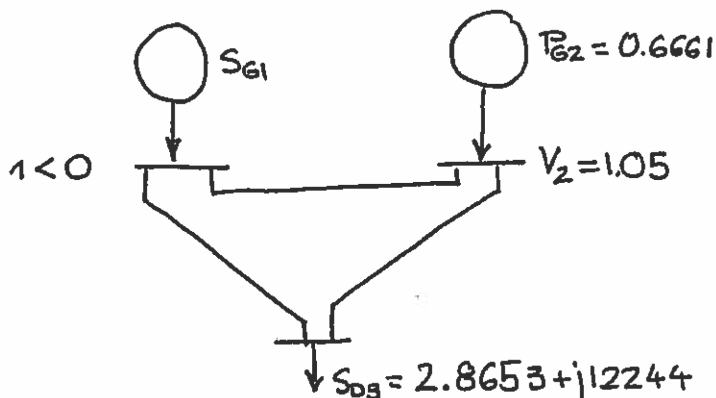
$$\underbrace{\begin{bmatrix} P_2 \\ P_3 \\ \vdots \\ P_n \end{bmatrix}}_{P} = - \underbrace{\begin{bmatrix} B_{22} & B_{23} & \cdots & B_{2n} \\ B_{32} & B_{33} & \cdots & B_{3n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n2} & B_{n3} & \cdots & B_{nn} \end{bmatrix}}_{\tilde{B}} \underbrace{\begin{bmatrix} \theta_2 \\ \theta_3 \\ \vdots \\ \theta_n \end{bmatrix}}_{\theta}$$

$$P = -\tilde{B}\theta$$

$\tilde{B} \in \mathbb{R}^{(n-1)\times(n-1)}$ is invertible; thus, we can solve for θ:

$$\boxed{\theta \simeq -\tilde{B}^{-1} \cdot P}$$

EXAMPLE



$Z_L = j0.1$

$Y_c = j0.01$

$$\bar{Y} = \begin{bmatrix} -j19.98 & j10 & j10 \\ j10 & -j19.98 & j10 \\ j10 & j10 & -j19.98 \end{bmatrix}$$

Bus ②

$P \left\{ V_2 \cdot V_1 \cdot B_{21} \cdot \sin(\theta_2 - \theta_1) + V_2 V_3 B_{23} \sin(\theta_2 - \theta_3) = P_{G2} \right.$

Bus ③

$Q \left\{ -V_3 V_1 B_{31} \cos(\theta_3 - \theta_1) - V_3 V_1 \cdot B_{32} \cos(\theta_3 - \theta_2) - V_3^2 \cdot B_{23} = -Q_{D3} \right.$

$P \left\{ V_3 \cdot V_1 \cdot B_{31} \cdot \sin(\theta_3 - \theta_1) + V_3 \cdot V_2 \cdot B_{32} \sin(\theta_3 - \theta_2) = -P_{D3} \right.$

$$x^{\nu+1} = x^{\nu} - J^{-1}(x^{\nu}) \cdot f(x)$$

$$x = [\theta_2 \ \theta_3 \ V_3]'$$

Bus ②

$$10.5 \cdot \sin\theta_2 + 10.5 \cdot V_3 \sin(\theta_2 - \theta_3) = 0.6661$$

Bus ③

$$10 \cdot V_3 \sin\theta_3 + 10.5 \cdot V_3 \cdot \sin(\theta_3 - \theta_2) = -\cancel{1.2244} \ -2.8653$$

$$-10 \cdot V_3 \cdot \cos\theta_3 - 10.5 V_3 \cos(\theta_3 - \theta_2) - 19.98 V_3^2 = -1.2244$$

We can rewrite the above equations as:

$$\overbrace{10.5 \cdot \sin\theta_2 + 10.5 V_3 \cdot \sin(\theta_3 - \theta_3)}^{P_2(x)} - 0.6661 = 0$$

$$\overbrace{10 \cdot V_3 \sin\theta_3 + 10.5 V_3 \sin(\theta_3 - \theta_2)}^{P_3(x)} \pm 2.8653 = 0$$

$$\overbrace{-10 V_3 \cos\theta_3 - 10.5 V_3 \cos(\theta_3 - \theta_2) - 19.98 V_3^2}^{Q_3(x)} + 1.2244 = 0$$

→ This is now in the form $f(x) = 0$

Jacobian elements:

(1) $$\frac{\partial P_2}{\partial \theta_2} = 10.5\cos\theta_2 + 10.5\cdot V_3\cos(\theta_2-\theta_3)$$

(2) $$\frac{\partial P_2}{\partial \theta_3} = -10.5\,V_3\cos(\theta_2-\theta_3)$$

(3) $$\frac{\partial P_2}{\partial V_3} = 10.5\sin(\theta_2-\theta_3)$$

(4) $$\frac{\partial P_3}{\partial \theta_2} = -10.5\,V_3\cos(\theta_3-\theta_2)$$

(5) $$\frac{\partial P_3}{\partial \theta_3} = 10\cdot V_3\cos\theta_3 + 10.5\,V_3\cos(\theta_3-\theta_2)$$

(6) $$\frac{\partial P_3}{\partial V_3} = 10\cdot\sin\theta_3 + 10.5\sin(\theta_3-\theta_2)$$

(7) $$\frac{\partial Q_3}{\partial \theta_2} = -10.5\,V_3\sin(\theta_3-\theta_2)$$

(8) $$\frac{\partial Q_3}{\partial \theta_3} = 10\cdot V_3\sin\theta_3 + 10.5\,V_3\sin(\theta_3-\theta_2)$$

(9) $$\frac{\partial Q_3}{\partial V_3} = -10\cos\theta_3 - 10.5\cos(\theta_3-\theta_2) + 39.96\,V_3$$

$$J = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_3}{\partial \theta_2} & \frac{\partial Q_3}{\partial \theta_3} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix}$$

We can now start the iterative process

$$x^{\nu+1} - x^{\nu} = -J^{-1}(x^{\nu})\,f(x^{\nu})$$

$$\underbrace{\underbrace{\begin{bmatrix}\theta_2^{\nu+1}\\ \theta_3^{\nu+1}\\ V_3^{\nu+1}\end{bmatrix} - \begin{bmatrix}\theta_2^{\nu}\\ \theta_3^{\nu}\\ V_3^{\nu}\end{bmatrix}}_{\begin{bmatrix}\Delta\theta_2^{\nu}\\ \Delta\theta_3^{\nu}\\ \Delta V_3^{\nu}\end{bmatrix}}}_{\Delta X^{\nu}} = -J^{-1}(X^{\nu})\cdot\begin{bmatrix}P_2(X^{\nu}) - P_{G2}\\ P_3(X^{\nu}) + P_{D3}\\ Q_3(X^{\nu}) + Q_{D3}\end{bmatrix}$$

$\nu = 0$:

We need an initial guess. An easy one is $\theta_2 = \theta_3 = 0$, $V_3 = 1$ (this is referred to as a flat start)

$$\begin{bmatrix} P_2(\cancel{X}^0) - P_{G2} \\ P_3(X^0) + P_{D3} \\ Q_3(X^0) + Q_{D3} \end{bmatrix} = \begin{bmatrix} 0 & -0.661 \\ 0 & +2.8653 \\ -0.52 & +1.2244 \end{bmatrix} = \begin{bmatrix} -0.6661 \\ 2.8653 \\ 0.7044 \end{bmatrix}$$

$$J^0 = \begin{bmatrix} 21 & -10.5 & 0 \\ -10.5 & 20.5 & 0 \\ 0 & 0 & 19.46 \end{bmatrix} \longrightarrow (J^0)^{-1} = \begin{bmatrix} 0.0640 & 0.0328 & 0 \\ 0.0328 & 0.0656 & 0 \\ 0 & 0 & 0.0514 \end{bmatrix}$$

Now:

$$\begin{bmatrix} \Delta\theta_2^0 \\ \Delta\theta_3^0 \\ \Delta V_3^0 \end{bmatrix} = -(J^0)^{-1} \cdot \begin{bmatrix} P_2(X^0) - P_{G2} \\ P_3(X^0) + P_{D3} \\ Q_3(X^0) + Q_{D3} \end{bmatrix} = - \begin{bmatrix} 0.0640 & 0.0328 & 0 \\ 0.0328 & 0.0666 & 0 \\ 0 & 0 & 0.0514 \end{bmatrix} \cdot \begin{bmatrix} -0.6661 \\ 2.8653 \\ 0.7044 \end{bmatrix}$$

$$= \begin{bmatrix} -2.9393^\circ \\ -9.5111^\circ \\ -0.0362 \end{bmatrix} \longrightarrow \begin{bmatrix} \theta_2^1 \\ \theta_3^1 \\ V_3^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -2.9395 \\ -9.8111 \\ -0.0362 \end{bmatrix} = \begin{bmatrix} -2.9395 \\ -9.5111 \\ 0.9638 \end{bmatrix}$$

$\nu = 1$

$$\begin{bmatrix} P_2(X^1) - P_{G2} \\ P_3(X^1) + P_{D3} \\ Q_3(X^1) + Q_{D3} \end{bmatrix} = \begin{bmatrix} -0.0463 \\ 0.1145 \\ 0.2251 \end{bmatrix} \qquad J^1 = \begin{bmatrix} 20.5396 & -10.0534 & 1.2017 \\ -10.0534 & 19.5589 & -2.8541 \\ 1.1582 & -2.7508 & 18.2199 \end{bmatrix}$$

$$(J^1)^{-1} = \begin{bmatrix} 0.0651 & 0.0336 & 0.0010 \\ 0.0336 & 0.0696 & 0.0087 \\ 0.0009 & 0.0084 & 0.0561 \end{bmatrix} \longrightarrow \begin{bmatrix} \Delta\theta_2^1 \\ \Delta\theta_3^1 \\ \Delta V_3^1 \end{bmatrix} \longrightarrow$$

$$\longrightarrow \begin{bmatrix} \theta_2^2 \\ \theta_3^2 \\ V_3^2 \end{bmatrix} = \begin{bmatrix} -3.0023 \\ -9.9924 \\ 0.9502 \end{bmatrix}$$

- The exact answer is $\begin{bmatrix}\theta_2\\ \theta_3\\ V_3\end{bmatrix} = \begin{bmatrix}-3\\ -10\\ 0.95\end{bmatrix}$, so in a couple of iterations we already got very close! The largest error is ~ 0.8%.

- Obviously in the actual problem, we do not know the answer, so we continue iterating until

$$|P_2(x^{\nu+1}) - P_{G2}| < \varepsilon$$
$$|P_3(x^{\nu+1}) + P_{D3}| < \varepsilon$$
$$|Q_3(x^{\nu+1}) + Q_{D3}| < \varepsilon$$

Once we have a good solution (close enough), we need to compute $S_{G1} = P_{G1} + jQ_{G1}$, and Q_{G2}

$$P_{G1} = V_1 \cdot V_2 \cdot B_{12} \cdot \sin(\theta_1 - \theta_2) + V_1 \cdot V_3 \cdot B_{13} \sin(\theta_1 - \theta_3)$$
$$= 10.5 \cdot \sin 3.0023 + 9.502 \cdot \sin 9.9924 = 2.1987$$

$$Q_{G1} = 0.1365$$

$$Q_{G2} = -1.6395$$