10/19/17

LECTURE 15

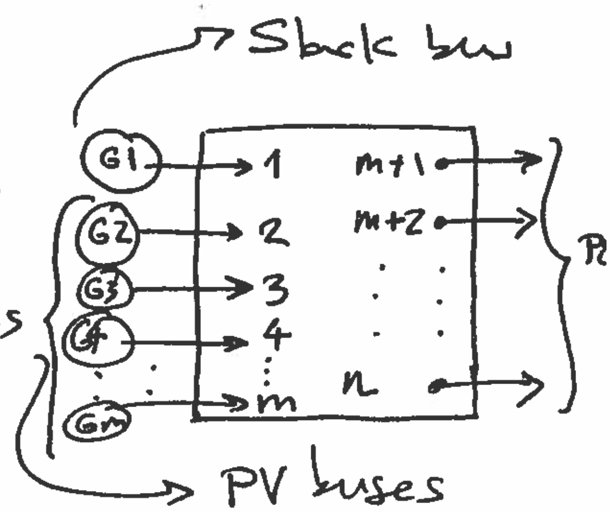
JACOBIAN STRUCTURE

Understanding the general form of the entries of the power flow Jacobian provides intuition on the system behavior, as well as some justification for some variations of the Newton-Raphson algorithm that are computational / faster.

$$x^{\nu+1} = x^{\nu} - (J(x^{\nu}))^{-1} \cdot f(x)$$

$$x = [\underbrace{\theta_2, \theta_3, \ldots, \theta_m}_{\text{PV buses}}, \underbrace{\theta_{m+1}, \theta_{m+2}, \ldots, \theta_n}_{\text{PQ buses}}, \underbrace{V_{m+1}, V_{m+2}, \ldots, V_n}_{\text{PQ buses}}]^T$$

$$\begin{bmatrix} P_2(x) - P_2^G \\ P_3(x) - P_3^G \\ \vdots \\ P_m(x) - P_m^G \\ P_{m+1}(x) + P_{m+1}^D \\ \vdots \\ P_n(x) + P_n^D \\ \hline Q_{m+1}(x) + Q_{m+1}^D \\ Q_{m+2}(x) + Q_{m+2}^D \\ \vdots \\ Q_n(x) + Q_n^D \end{bmatrix} \begin{array}{l} \left.\vphantom{\begin{matrix} P \\ P \\ \vdots \\ P \end{matrix}}\right\} \text{PV buses} \\ \left.\vphantom{\begin{matrix} P \\ \vdots \\ P \end{matrix}}\right\} \text{PQ buses} \\ \left.\vphantom{\begin{matrix} Q \\ Q \\ \vdots \\ Q \end{matrix}}\right\} \text{PQ buses} \end{array}$$



$$J(X) = \begin{bmatrix} \left.\dfrac{\partial P}{\partial \theta}\right|_x & \left.\dfrac{\partial P}{\partial V}\right|_x \\ \left.\dfrac{\partial Q}{\partial \theta}\right|_x & \left.\dfrac{\partial Q}{\partial V}\right|_x \end{bmatrix} \longrightarrow (2n-m-1) \times (2n-m-1)$$

dimensional matrix

$$\left.\frac{\partial P}{\partial \theta}\right|_x \in \mathbb{R}^{(n-1)\times(n-1)} \quad \left((n-1) \times (n-1) \text{ matrix} \right)$$

$$\left.\frac{\partial P}{\partial V}\right|_x \in \mathbb{R}^{(n-1)\times(n-m)}$$

$$\left.\frac{\partial Q}{\partial \theta}\right|_x \in \mathbb{R}^{(n-m)\times(n-1)}$$

$$\left.\frac{\partial Q}{\partial V}\right|_x \in \mathbb{R}^{(n-m)\times(n-m)}$$

$$\left.\frac{\partial P}{\partial \theta}\right|_x = \begin{bmatrix} \left.\dfrac{\partial P_2}{\partial \theta_2}\right|_x & \cdots & \left.\dfrac{\partial P_2}{\partial \theta_n}\right|_x \\ \vdots & & \\ \left.\dfrac{\partial P_n}{\partial \theta_2}\right|_x & \cdots & \left.\dfrac{\partial P_n}{\partial \theta_n}\right|_x \end{bmatrix}; \quad \left.\frac{\partial P}{\partial V}\right|_x = \begin{bmatrix} \left.\dfrac{\partial P_2}{\partial V_{m+1}}\right|_x & \cdots & \left.\dfrac{\partial P_2}{\partial V_n}\right|_x \\ \vdots & & \\ \left.\dfrac{\partial P_n}{\partial V_{m+1}}\right|_x & \cdots & \left.\dfrac{\partial P_n}{\partial V_n}\right|_x \end{bmatrix}$$

$$\left.\frac{\partial Q}{\partial \theta}\right|_x = \begin{bmatrix} \left.\dfrac{\partial Q_{m+1}}{\partial \theta_2}\right|_x & \cdots & \left.\dfrac{\partial Q_{m+1}}{\partial \theta_n}\right|_x \\ \vdots & & \\ \left.\dfrac{\partial Q_n}{\partial \theta_2}\right|_x & \cdots & \left.\dfrac{\partial Q_n}{\partial \theta_n}\right|_x \end{bmatrix}; \quad \left.\frac{\partial Q}{\partial V}\right|_x = \begin{bmatrix} \left.\dfrac{\partial Q_{m+1}}{\partial V_{m+1}}\right|_x & \cdots & \left.\dfrac{\partial Q_{m+1}}{\partial V_n}\right|_x \\ \vdots & & \\ \left.\dfrac{\partial Q_n}{\partial V_{m+1}}\right|_x & \cdots & \left.\dfrac{\partial Q_n}{\partial V_n}\right|_x \end{bmatrix}$$

(i) Entries of $\frac{\partial P}{\partial \theta}$

$$\begin{cases} \left.\frac{\partial P_i}{\partial \theta_i}\right|_X = -Q_i(X) - B_{ii}V_i^2 \\ \left.\frac{\partial P_i}{\partial \theta_j}\right|_X = V_i V_j \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)\right] \end{cases}$$

(ii) Entries of $\frac{\partial P}{\partial V}$

$$\begin{cases} \left.\frac{\partial P_i}{\partial V_i}\right|_X = \left.\frac{P_i(X)}{V_i}\right|_X + G_{ii}V_i \\ \left.\frac{\partial P_i}{\partial V_j}\right|_X = V_i \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)\right] \end{cases}$$

(iii) Entries of $\frac{\partial Q}{\partial \theta}$

$$\begin{cases} \left.\frac{\partial Q_i}{\partial \theta_i}\right|_X = P_i(X) - G_{ii}V_i^2 \\ \left.\frac{\partial Q_i}{\partial \theta_j}\right|_X = -V_i V_j \left[G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \right. \end{cases}$$

(iv) Entries of $\frac{\partial Q}{\partial V}$

$$\begin{cases} \left.\frac{\partial Q_i}{\partial V_i}\right|_X = \left.\frac{Q_i}{V_i}\right|_X - B_{ii}V_i \\ \left.\frac{\partial Q_i}{\partial V_j}\right|_X = V_i \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)\right] \end{cases}$$

$$x[t] \equiv x^t \Rightarrow x[\nu] \equiv x^\nu$$

Recall that in the scalar case, we had

$$x[t+1] = x[t] - \left(\left.\frac{df}{dx}\right|_{x[t]}\right)^{-1} f(x[t])$$

or more generally

$$x[t+1] = x[t] - \cancel{\left.\frac{df}{dx}\right|} m(t)^{-1} f(x[t])$$

In the vector case, can we substitute $J(x[t])$ by some appropriately chosen matrix $M(x[t]) \neq J(x[t])$, i.e.,

$$x[t+1] = x[t] - (M(x[t]))^{-1} f(x[t]) \; ?$$

- Two approximations:

$$M(x[t]) = \left.\begin{bmatrix} \frac{\partial P}{\partial \theta} & 0 \\ 0 & \frac{\partial Q}{\partial V} \end{bmatrix}\right|_{x[t]}$$

$$\boxed{\begin{array}{l} \theta_i - \theta_j \simeq 0 \\ G_{ij} \simeq 0 \end{array}}$$

i.e., make the off-diagonal block matrices zero $\longrightarrow$ This is called <u>DECOUPLED POWER FLOW</u>

FAST DECOUPLED POWER FLOW

$$\theta_i - \theta_j = 0$$
$$G_{ij} = 0$$
$$V_i = V_j \approx 1, \ \forall i, j$$

$$\frac{\partial P_i}{\partial \theta_i} = -\overbrace{V_i V_j \cdot \underbrace{\sum B_{ij}}_{0}}^{0\,!} - B_{ii} V_i^2 = -B_{ii}$$

$$\frac{\partial P_i}{\partial \theta_j} = -V_i V_j B_{ij} = -B_{ij}$$

$$\frac{\partial P}{\partial \theta} = -[V]\tilde{B}[V]$$

$$[V] = \begin{bmatrix} V_{m+1} & & \\ & V_{m+2} & \\ & & \ddots \ V_n \end{bmatrix}$$

$$\frac{\partial Q_i}{\partial V_i} = \frac{\cancelto{0}{Q_i}}{\cancel{V_i}} - B_{ii} V_i$$

$$\frac{\partial Q_i}{\partial V_j} = -V_i B_{ij}$$

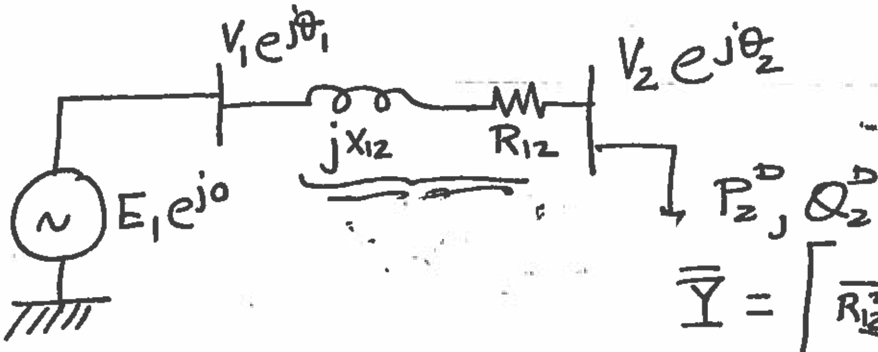
$$\frac{\partial Q}{\partial V} = [V]\tilde{\tilde{B}}$$

$$\frac{\partial Q}{\partial V} = -[V]\tilde{\tilde{B}}$$

$$\tilde{B} = \begin{bmatrix} B_{22} & \cdots & B_{2n} \\ \vdots & & \\ B_{nn} & \cdots & B_{nn} \end{bmatrix}$$

$$\tilde{\tilde{B}} = \begin{bmatrix} B_{mm} & \cdots & B_{mn} \\ \vdots & & \\ B_{nm} & \cdots & B_{mn} \end{bmatrix}$$

$$\begin{bmatrix} \theta[k+1] \\ V[k+1] \end{bmatrix} = \begin{bmatrix} \theta[k] \\ V[k] \end{bmatrix} \bar{\zeta} \begin{bmatrix} -\tilde{B} & 0 \\ 0 & -\tilde{\tilde{B}} \end{bmatrix} \begin{bmatrix} P(\theta[k], V[k]) - P \\ Q(\theta[k], V[k] - Q \end{bmatrix}$$



$$\bar{Y}_{12} = \frac{R_{12} - jX_{12}}{R_{12}^2 + X_{12}^2}$$

$$\bar{Y} = \underbrace{\begin{bmatrix} \frac{R_{12}}{R_{12}^2+X_{12}^2} & \frac{-R_{12}}{R_{12}^2+X_{12}^2} \\ \frac{-R_{12}}{R_{12}^2+X_{12}^2} & \frac{R_{12}}{R_{12}^2+X_{12}^2} \end{bmatrix}}_{G} + j \underbrace{\begin{bmatrix} \frac{-X_{12}}{R_{12}^2+X_{12}^2} & \frac{X_{12}}{R_{12}^2+X_{12}^2} \\ \frac{X_{12}}{R_{12}^2+X_{12}^2} & \frac{-X_{12}}{R_{12}^2+X_{12}^2} \end{bmatrix}}_{B}$$

$$-P_2^D = G_{22} \cdot V_2^2 + G_{12} V_1 \cdot V_2 \cdot \cos(\theta_2) + B_{12} V_1 V_2 \sin(\theta_2)$$

$$-Q_2^D = -B_{22} V_2^2 + G_{21} V_2 \cancel{V_1} \sin(\theta_2) - B_{21} V_2 \cancel{V_1} \cdot \cos(\theta_2)$$

$$\begin{bmatrix} \theta_2^{\nu+1} \\ V_2^{\nu+1} \end{bmatrix} = \begin{bmatrix} \theta_2^{\nu} \\ V_2^{\nu} \end{bmatrix} - \begin{bmatrix} -G_{12} E_1 \cdot V_2^\nu \sin(\theta_2^\nu) + B_{12} E_1 V_2^\nu \cos\theta_2^\nu \\ G_{21} V_2^\nu E_1 \cdot \cos(\theta_2^\nu) + B_{21} V_2^\nu E_1 \sin(\theta_2^\nu) \\ -2B_{22} V_2^\nu + G_{21} E_1 \sin(\theta_2^\nu) \\ 2G_{22} V_2^\nu + G_{12} E_1 \cos\theta_2^\nu \end{bmatrix} \cdot f(x)$$

$\rightarrow -B_{21} \cdot E_1 \cos$

What happens if we assume $\theta_2^\nu - \theta_1^\nu \simeq 0$ and $G_{12} \simeq 0$

$$2G_{22} V_2^\nu + G_{12} E_1 \cos\theta_2^\nu \simeq 0$$

$$G_{21} V_2 E_1 \cos(\theta_2^\nu) + B_{21} E_1 V_2^\nu \sin(\theta_2) \simeq 0$$

→ We can eliminate the off diagonals and decouple the iterations

↳ Decoupled N-R

$$\begin{bmatrix}\theta_2^{\nu+1}\\ V_2^{\nu+1}\end{bmatrix} = \begin{bmatrix}\theta_2^{\nu}\\ V_2^{\nu}\end{bmatrix} - \begin{bmatrix} G_{12}E_1V_2^{\nu}\sin(\theta_2^{\nu}) + B_{12}E_1V_2^{\nu}\cos\theta_2^{\nu} & 0 \\ 0 & -2B_{22}V_2^{\nu} + G_{21}E_1\sin\theta_2^{\nu} - B_{21}E_1\cos(\theta_2) \end{bmatrix} \times f(x^{\nu})$$

What happens if we actually impose

$G_{12} = 0 \quad \theta_2^{\nu} - \theta_1^{\nu} = 0$ and $V_2^{\nu} = 1 \quad E_1 = 1$

$B_{12} = B_{21} = -B_{22}$

$$\begin{bmatrix}\theta_2^{\nu+1}\\ V_2^{\nu+1}\end{bmatrix} = \begin{bmatrix}\theta_2^{\nu}\\ V_2^{\nu}\end{bmatrix} - \begin{bmatrix} \overset{-B_{22}}{B_{12}} & 0 \\ 0 & \underbrace{-2B_{22} - B_{21}}_{-B_{22}} \end{bmatrix} f(x)$$

$B_{21} + B_{22} = 0$

$-B_{21} = B_{22}$

$$\begin{bmatrix}\theta_2^{\nu+1}\\ V_2^{\nu+1}\end{bmatrix} = \begin{bmatrix}\theta_2^{\nu}\\ V_2^{\nu}\end{bmatrix} - \begin{bmatrix} B_{12} & 0 \\ 0 & -B_{22} \end{bmatrix} f(x)$$

$$= \begin{bmatrix}\theta_2^{\nu}\\ V_2^{\nu}\end{bmatrix} - \begin{bmatrix} -B_{22} & 0 \\ 0 & -B_{22} \end{bmatrix} f(x)$$

$-B_{22} \rightarrow -\tilde{B}$

$-B_{22} \rightarrow -\tilde{\tilde{B}}$

Fast Decoupled Newton-Raphson

In this case: $\tilde{B} = B_{22}$,

$\tilde{\tilde{B}} = -B_{22}$