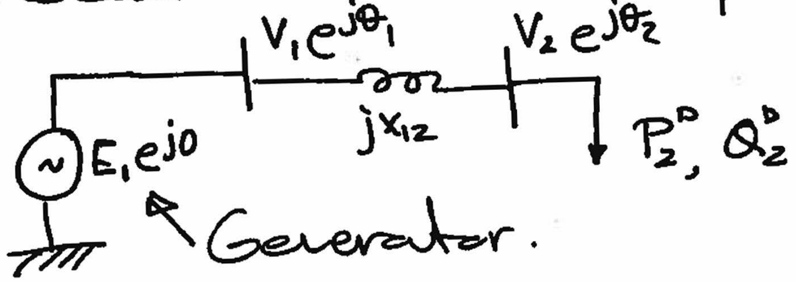
LECTURE 14

10/17/17

CASE I: Single generator

Recall two bus example:



Given: $V_1 = E_1, \theta_1 = 0, P_2 = -P_2^D, Q_2 = -Q_2^D$

To compute: P_1, Q_1, V_2, θ_2

$$P_1 = \frac{E_1 \cdot V_2}{x_{12}} \sin(-\theta_2)$$

$$Q_1 = \frac{E_1^2}{x_{12}} - \frac{E_1 \cdot V_2}{x_{12}} \cos(-\theta_2)$$

$$-P_2^D = \overbrace{\frac{E_1 \cdot V_2}{x_{12}} \sin \theta_2}^{P_2(\theta_2, V_2)}$$

$$-Q_2^D = \overbrace{\frac{V_2^2}{x_{12}} - \frac{E_1 \cdot V_2}{x_{12}} \cos \theta_2}^{Q_2(\theta_2,}$$

This forms a set of closed-equations:
Two unknowns: V_2, θ_2
Two equations.

In the N-R iteration, we use the P & Q balance equation for bus 2 (its P_2 and Q_2 are prespecified, hence this type of bus is referred to as PQ bus.

$$\underbrace{\begin{bmatrix} \theta_2^{\nu+1} \\ V_2^{\nu+1} \end{bmatrix}}_{x^{\nu+1}} = \underbrace{\begin{bmatrix} \theta_2^{\nu} \\ V_2^{\nu} \end{bmatrix}}_{x^{\nu}} - \underbrace{\begin{bmatrix} \left.\frac{\partial P_2}{\partial \theta_2}\right|_{\theta_2^\nu, V_2^\nu} & \left.\frac{\partial P_2}{\partial V_2}\right|_{\theta_2^\nu, V_2^\nu} \\ \left.\frac{\partial Q_2}{\partial \theta_2}\right|_{\theta_2^\nu, V_2^\nu} & \left.\frac{\partial Q_2}{\partial V_2}\right|_{\theta_2^\nu, V_2^\nu} \end{bmatrix}^{-1}}_{J(x^\nu)} \underbrace{\begin{bmatrix} P_2(\theta_2^\nu, V_2^\nu) + P_2^D \\ Q_2(\theta_2^\nu, V_2^\nu) + Q_2^D \end{bmatrix}}_{f(x^\nu)}$$

How to extent this case to n buses:

1 - Bus 1 is a voltage source [Slack bus]; i.e., $V_1 = E_1, \theta_1 = 0$ known; P_1, Q_1 not known (to compute) ... $E_1 e^{j0}$

2 - Buses 2 - n are PQ "load" buses; i.e.,

$$\left.\begin{aligned} P_i &= -P_i^D, \quad i = 2, \ldots, n \\ Q_i &= -Q_i^D, \quad i = 2, \ldots, n \end{aligned}\right\} \text{known}$$

$$V_i, \theta_i, \quad i = 2, \ldots, n \; \Big\} \underset{\text{(to compute)}}{\text{unknown}}$$

Bus 1: (2 eqns)

$$P_1 = \overbrace{E_1^2 G_{11} + \sum_{k=2}^{n} V_1 V_k \left[G_{1k} \cos(-\theta_k) + B_{2k} \sin(-\theta_k)\right]}^{P_1(\theta_2, \theta_3, \ldots \theta_n, V_2, V_3, \ldots, V_n)}$$

$$Q_1 = \underbrace{-E_1^2 B_{11} + \sum_{k=2}^{n} V_1 \cdot V_k \left[G_{1k} \sin(-\theta_k) - B_{1k} \cos(-\theta_k)\right]}_{Q_1(\theta_2, \theta_3, \ldots, \theta_n, V_2, V_3, \ldots, V_n)}$$

Bus 2: (2 eqns)

$$-P_2^D = \underbrace{V_2^2 G_{22} + V_2 E_1 \left[G_{12} \cos(+\theta_2) + B_{12} \sin(+\theta_2)\right] + \sum_{k=3}^{n} V_2 V_k \cdot \left[G_{2k} \cos(\theta_2 - \theta_k) + B_{2k} \sin(\theta_2 - \theta_k)\right]}_{P_2(\theta_2, \theta_3, \ldots, \theta_n, V_2, V_3, V_n)}$$

$$-Q_2^D = \underbrace{-V_2^2 B_{22} + V_2 E_1 \left[G_{12} \sin(\theta_2) - B_{12} \cos(\theta_2)\right] + \sum_{k=3}^{n} V_2 V_k \left[G_{2k} \sin(\theta_2 - \theta_k) - B_{2k} \cos(\theta_2 - \theta_k)\right]}_{\to\, Q_2(\theta_2, \theta_3, \ldots, \theta_n, V_2, V_3, \ldots, V_n)}$$

Here P_2^D & Q_2^D are known and the unknows involved (not necessarily all) are at most $\theta_2, V_2, \theta_3, V_3, \ldots, \theta_n, V_n \rightarrow 2n-2$ @ most

Bus $2 < i \le n$: (2 eqns) $\longrightarrow P_i(\theta_2,\theta_3,\dots,\theta_n,V_2,V_3,\dots)$

$$-P_i^D = \sum_{k=1}^{i-1} V_iV_k\left[G_{ik}\cos(\theta_i-\theta_k)+B_{ik}\sin(\theta_i-\theta_k)\right] + V_i^2G_{ii} + \sum_{k=i+1}^{n} V_iV_k\left[G_{ik}\cdot\cos(\theta_i-\theta_k)+B_{ik}\sin(\theta_i-\theta_k)\right]$$

$$-Q_i^D = \sum_{k=1}^{i-1} V_iV_k\left[G_{ik}\sin(\theta_i-\theta_k)-B_{ik}\cos(\theta_i-\theta_k)\right] - V_i^2B_{ii} + \sum_{k=i+1}^{n} V_iV_k\left[G_{ik}\cos(\theta_i-\theta_k)-B_{ik}\sin(\theta_i-\theta_k)\right]$$

$\longrightarrow Q_i(\theta_2,\theta_3,\dots,\theta_n,V_2,V_3,\dots,V_n)$

Here P_i^D & Q_i^D are know and the unknows involved (not necessarily all) are:
$\theta_2, V_2, \theta_3, V_3, \dots, \theta_n, V_n \longrightarrow 2n-2$ @ most

If you consider the "P" and "Q" eqns for buses $i=2,\dots,n.$, you obtain $2n-2$ eqns with each involving a subset of the following unknowns: $\{\theta_2 V_2, \theta_3, V_3, \dots, \theta_n, V_n\}$ $(2n-2)$; Thus this eqns form a closed system.

The N-R iteration in this case looks like

$$X^{\nu+1} = X^{\nu} - \left(J(X^{\nu})\right)^{-1}\left(f(X^{\nu})\right)$$

$$X^{\nu} = [\theta_2^{\nu}, \theta_3^{\nu}, \ldots, \theta_n^{\nu}, V_2^{\nu}, V_3^{\nu}, \ldots, V_n^{\nu}]^T$$

$$J(X^{\nu}) = \left[\begin{array}{cccc|cccc}
\frac{\partial P_2}{\partial \theta_2}\Big|_{X^{\nu}} & \frac{\partial P_2}{\partial \theta_3}\Big|_{X^{\nu}} & \cdots & \frac{\partial P_2}{\partial \theta_n}\Big|_{X^{\nu}} & \frac{\partial P_2}{\partial V_2}\Big|_{X^{\nu}} & \frac{\partial P_2}{\partial V_3}\Big|_{X^{\nu}} & \cdots & \frac{\partial P_n}{\partial V_n}\Big|_{X^{\nu}} \\
\vdots & & & & & & & \\
\frac{\partial P_n}{\partial \theta_2}\Big|_{X^{\nu}} & \frac{\partial P_n}{\partial \theta_3}\Big|_{X^{\nu}} & \cdots & \frac{\partial P_n}{\partial \theta_n}\Big|_{X^{\nu}} & \frac{\partial P_n}{\partial V_2}\Big|_{X^{\nu}} & \frac{\partial P_n}{\partial V_3}\Big|_{X^{\nu}} & \cdots & \frac{\partial P_n}{\partial V_n}\Big|_{X^{\nu}} \\
\hline
\frac{\partial Q_2}{\partial \theta_2}\Big|_{X^{\nu}} & \frac{\partial Q_2}{\partial \theta_3}\Big|_{X^{\nu}} & \cdots & \frac{\partial Q_2}{\partial \theta_n}\Big|_{X^{\nu}} & \frac{\partial Q_2}{\partial V_2}\Big|_{X^{\nu}} & \frac{\partial Q_2}{\partial V_3}\Big|_{X^{\nu}} & \cdots & \frac{\partial Q_2}{\partial V_n}\Big|_{X^{\nu}} \\
\vdots & & & & \vdots & & & \\
\frac{\partial Q_n}{\partial \theta_2}\Big|_{X^{\nu}} & \frac{\partial Q_n}{\partial \theta_3}\Big|_{X^{\nu}} & \cdots & \frac{\partial Q_n}{\partial \theta_n}\Big|_{X^{\nu}} & \frac{\partial Q_n}{\partial V_2}\Big|_{X^{\nu}} & \frac{\partial Q_n}{\partial V_3}\Big|_{X^{\nu}} & \cdots & \frac{\partial Q_n}{\partial V_n}\Big|_{X^{\nu}}
\end{array}\right]$$

$$f(X^{\nu}) = \begin{bmatrix} P_2(X^{\nu}) + P_2^D \\ P_3(X^{\nu}) + P_3^D \\ \vdots \\ P_n(X^{\nu}) + P_n^D \\ \hline Q_2(X^{\nu}) + Q_2^D \\ Q_3(X^{\nu}) + Q_3^D \\ \vdots \\ Q_n(X^{\nu}) + Q_n^D \end{bmatrix}$$

CASE II (m generators)

- Bus 1 : "slack bus" $\begin{cases} E_1 e^{j0} \Rightarrow V_1 = E_1,\ \theta_1 = 0 \text{ Known} \\ P_1, Q_1 \end{cases}$
- Bus 2-m : "PV buses" $\begin{cases} P_i^G, V_i^G,\ i=2,\dots,m, \text{ Known} \\ \theta_i, V_i,\ i=2,\dots,m, \text{ unknown} \end{cases}$
- Bus m+1 - n "PQ buses" $\begin{cases} P_i = -P_i^D,\ Q_i = -Q_i^D,\ i=m+1,\dots,n \text{ Known} \\ \theta_i, V_i,\ i=2,\dots,m, \text{ unknown} \end{cases}$

Given: $\underbrace{V_1, \theta_1}_{\text{Slack bus}}, \underbrace{V_2, P_2^G, \dots, V_m, P_m^G}_{\text{PV buses}}, \underbrace{P_{m+1}^D, Q_{m+1}^D, \dots, P_n^D, Q_n^D}_{\text{PQ buses}}$

To compute: $\underbrace{P_1, Q_1}_{\text{slack bus}}, \underbrace{\theta_2, Q_2, \dots, \theta_m, Q_m}_{\text{PV buses}}, \underbrace{\theta_{m+1}, V_{m+1}, \dots, \theta_n, V_n}_{\text{PQ buses}}$

In writing the N-R iteration, we only need to consider:

- The P-balance equation for the PV buses:
  - total of m-1 eqns.
  - Unknowns involved (at most): $\underbrace{\theta_2, \theta_3, \dots, \theta_m}_{\text{PV buses}}, \underbrace{\theta_{m+1}, \theta_{m+2}, \dots, \theta_n, V_{m+1}, V_{m+2}, \dots, V_n}_{\text{PQ buses}}$

  → total of $(m-1) + 2(n-(m+1)+1)$

$$= m - 1 + 2n - 2m - \cancel{2} + \cancel{2}$$
$$= 2n - m - 1 \text{ unknowns}$$

- Both The P & Q balance equations for PQ buses:
  - total of $2(n-(m+1)+1) = \underline{2n-2m}$ equations
  - unknowns involved: the same as for the P balance equns of the PV buses, i.e., $\theta_2, \theta_3, \ldots \theta_m, \theta_{m+1}, \theta_{m+2}, \ldots \theta_n, V_{m+1}, V_{m+2}, \ldots, V_n$

    $\longrightarrow$ for a total of $2n-m-1$ unknowns:

$2n-2m$

- UNKNOWNS: $2n-m-1$
- EQUATIONS: $\underbrace{m-1}_{\text{"P" equns for PV buses}} + \underbrace{(n-m)}_{\text{"P" equns for PQ buses}} + \underbrace{(n-m)}_{\text{"Q" equns for PQ buses}} = 2n-m-1$

$\longrightarrow$ This is a closed system of equns!

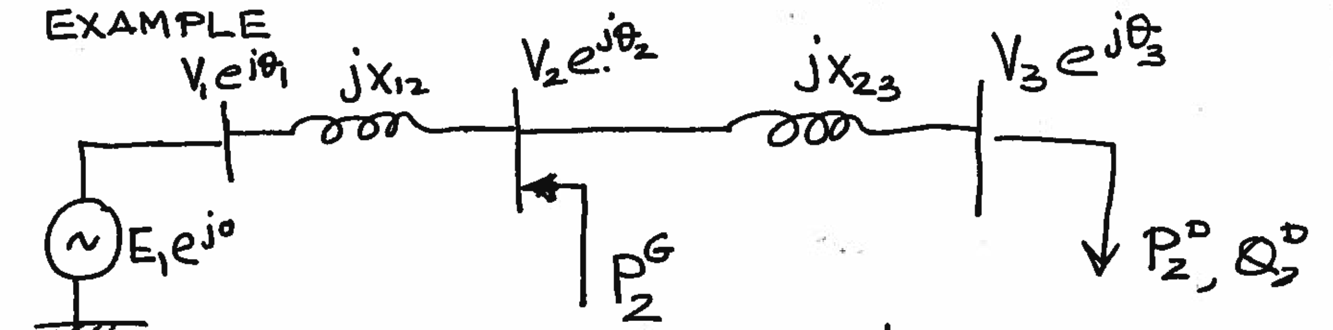
The N-R iteration in this case looks like

$$X^{\nu+1} = X^{\nu} - (J(X^{\nu}))^{-1}(f(X^{\nu}))$$

$$X^{\nu} = [\underbrace{\theta_2^{\nu}, \theta_3^{\nu}, \ldots, \theta_m^{\nu}}_{\text{PV buses}}, \underbrace{\theta_{m+1}^{\nu}, \ldots, \theta_n^{\nu}}_{\text{PQ buses}}, V_{m+1}^{\nu}, V_{m+2}^{\nu}, \ldots, V_n^{\nu}]^T$$

$$f(X^{\nu}) = \begin{bmatrix} P_2(X^{\nu}) - P_2^G \\ \vdots \\ P_m(X^{\nu}) - P_m^G \\ P_{m+1}(X^{\nu}) + P_{m+1}^D \\ \vdots \\ P_n(X^{\nu}) + P_n^D \\ Q_{m+1}(X^{\nu}) + Q_{m+1}^D \\ \vdots \\ Q_n(X^{\nu}) + Q_n^D \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{PV buses} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{PQ buses} \\ \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{PQ buses} \end{matrix}$$

EXAMPLE



$V_2 = E_2$ constant

Slack bus PV bus PQ bus

$$\bar{Y} = jB = j\begin{bmatrix} -\frac{1}{X_{12}} & \frac{1}{X_{23}} & 0 \\ -\frac{1}{X_{12}} & -\frac{1}{X_{12}} - \frac{1}{X_{23}} & \frac{1}{X_{23}} \\ 0 & \frac{1}{X_{23}} & -\frac{1}{X_{23}} \end{bmatrix}$$

- The equs for the slack bus, we already know do not enter the N-R formulation, hence we leave them out

Bus 2:

$$\boxed{P_2^G = \frac{E_1 \cdot E_2}{X_{12}} \sin(\theta_2) + \frac{E_2 V_3}{X_{23}} \sin(\theta_2 - \theta_3)}$$

$$Q_2 = V_2^2 \cdot \left(\frac{1}{X_{12}} + \frac{1}{X_{23}}\right) - \frac{E_1 \cdot V_2}{X_{12}} \cos(\theta_2) - \frac{V_2 V_3}{X_{23}} \cos(\theta_2 - \theta_3$$

Bus 3

$$\boxed{\begin{aligned} -P_3^D &= \frac{E_2 V_3}{X_{23}} \sin(\theta_3 - \theta_2) \\ -Q_3^D &= \frac{V_3^2}{X_{23}} - \frac{E_2 V_3}{X_{23}} \cos(\theta_3 - \theta_3) \end{aligned}}$$

$$
\left.
\begin{aligned}
P_2^G &= \frac{E_1 E_2}{X_{12}} \sin(\theta_2) + \frac{E_2 V_3}{X_{23}} \sin(\theta_2 - \theta_3) \\
-P_3^D &= \frac{E_2 V_3}{X_{23}} \sin(\theta_3 - \theta_2) \\
-Q_3^D &= \frac{V_3^2}{X_{23}} - \frac{E_2 V_3}{X_{23}} \cos(\theta_2 - \theta_3)
\end{aligned}
\right\} \text{Three equations}
$$

Unknowns : $\theta_2, \theta_3, V_3 \longrightarrow X = [\theta_2, \theta_3, V_3]^T$

Given : P_2^G, E_2, P_3^D, Q_3^D

N-R iteration :

$$
\begin{bmatrix} \theta_2^{\nu+1} \\ \theta_3^{\nu+1} \\ V_3^{\nu+1} \end{bmatrix}
=
\begin{bmatrix} \theta_2^{\nu} \\ \theta_3^{\nu} \\ V_3^{\nu} \end{bmatrix}
-
\begin{bmatrix}
\frac{E_1 E_2}{X_{12}} \cos\theta_2 + \frac{E_2 V_3^\nu}{X_{23}} \cos(\theta_2^\nu - \theta_3^\nu) & -\frac{E_2 V_3^\nu}{X_{23}} \cos(\theta_2^\nu - \theta_3^\nu) & \frac{E_2}{X_{23}} \sin(\theta_2^\nu - \theta_3^\nu) \\
-\frac{E_2 V_3^\nu}{X_{23}} \cos(\theta_3^\nu - \theta_2^\nu) & \frac{E_2 V_3^\nu}{X_{23}} \cos(\theta_3^\nu - \theta_2^\nu) & \frac{E_2}{X_{23}} \sin(\theta_3^\nu - \theta_2^\nu) \\
+\frac{E_2 V_3^\nu}{X_{23}} \sin(\theta_2^\nu - \theta_3^\nu) & -\frac{E_2 V_3^\nu}{X_{23}} \sin(\theta_2^\nu - \theta_3^\nu) & \frac{2V_3^\nu}{X_{23}} - \frac{E_2}{X_{23}} \cos(\theta_2^\nu - \theta_3^\nu)
\end{bmatrix}
\times
\begin{bmatrix}
\frac{E_1 E_2}{X_{12}} \sin(\theta_2^\nu) + \frac{E_2 V_3^\nu}{X_{23}} \sin(\theta_2^\nu - \theta_3^\nu) - P_2^G \\
\frac{E_2 V_3^\nu}{X_{23}} \sin(\theta_3^\nu - \theta_2^\nu) + P_3^D \\
\frac{(V_3^\nu)^2}{X_{23}} - \frac{E_2 V_3^\nu}{X_{23}} \cos(\theta_2^\nu - \theta_3^\nu) + Q_3^D
\end{bmatrix}
$$