LECTURE 13

10/12/17

NEWTON-RAPSHON FOR SOLVING THE POWER FLOW PROBLEM

- CASE I : SINGLE GENERATOR

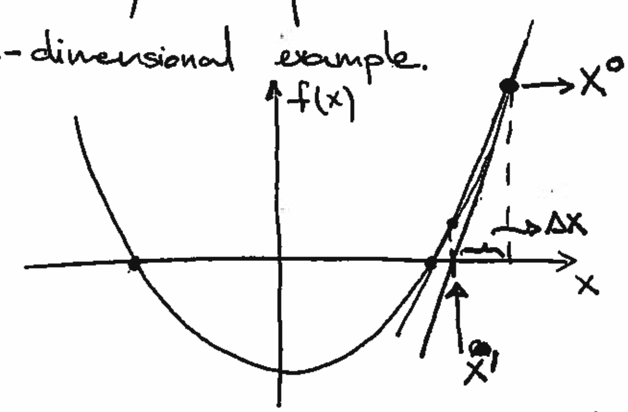
NEWTON - RAPSHON

- Remember, we are trying to solve a set of non-linear equations:

$$P_i = \sum_K V_i \cdot V_K \left(G_{iK} \cos(\theta_i - \theta_K) + B_{iK} \sin(\theta_i - \theta_K) \right)$$

$$Q_i = \sum_K V_i V_K \left(G_{iK} \sin(\theta_i - \theta_K) - B_{iK} \cos(\theta_i - \theta_K) \right)$$

- We will study now another algorithm that iteratively solves the equations above.
- This algorithm is based on the Taylor series expansion of a non-linear function.
- Let's try to gather the basic concept from a one-dimensional example.



Let's try to find the roots of $f(x)$:

$$f(x) = 0$$

We start with some value of x: x^0, and unless we are very lucky: $f(x^0) \neq 0$

The Taylor series expansion of $f(x^0)$, allows us to write:

$$f(x^0+\Delta x) \cong f(x^0) + \left.\frac{df}{dx}\right|_{x^0} \Delta x \quad \leftarrow \text{ This is true for small } \Delta x$$

- In practice, what we are saying is that we can approximate the nonlinear function by a straight line (in the neighborhood of x^0).
- We could actually compute the root of the approximation given by the straight line and obtain Δx

$$f(x^0) + \left.\frac{df}{dx}\right|_{x^0} \Delta x = 0 \rightarrow \Delta x = -f(x^0)\left(\left.\frac{df}{dx}\right|_{x^0}\right)^{-1}$$

but $\Delta x = x^1 - x^0 \rightarrow x^1 = x^0 - f(x^0)\left(\left.\frac{df}{dx}\right|_{x^0}\right)^{-1}$

We can use now x^1 instead of x^0 and do the same:

$$f(x^1) \neq 0, \text{ but } f(x^1+\Delta x) \simeq f(x^1) + \left.\frac{df}{dx}\right|_{x^1} \underbrace{(x^2 - x^1)}_{\Delta x}$$

$$\rightarrow \quad x^2 = x^1 - \left(\left.\frac{df}{dx}\right|_{x^1}\right)^{-1} f(x^1)$$

The iteration scheme is clear:

$$x^{\nu+1} = x^{\nu} - \left(\left.\frac{df}{dx}\right|_{x^{\nu}}\right)^{-1} f(x^{\nu})$$

x^0 the initial guess.

In a multi-dimensional case:

$$
\begin{aligned}
f_1(x_1, x_2, \dots, x_n) &= 0 \qquad & x &\triangleq [x_1, x_2, \dots x_n]' \\
f_2(x_1, x_2, \dots, x_n) &= 0 & \Delta x &\triangleq [\Delta x_1, \Delta x_2, \dots, \Delta x_n]' \\
&\ \vdots \\
f_n(x_1, x_2, \dots, x_n) &= 0 & x^0 &\triangleq [x_1^0, x_2^0, \dots, x_n^0]'
\end{aligned}
$$

Now the Taylor series expansion becomes:

$f_1(x_1^0, x_2^0, \dots, x_n^0)$:

- $f_1(x_1^0 + \Delta x_1,\ x_2^0 + \Delta x_2, \dots, x_n^0 + \Delta x_n) =$

$$
= f_1(x_1^0, x_2^0, \dots, x_n^0) + \left.\frac{\partial f_1}{\partial x_1}\right|_{x^0} \Delta x_1 + \left.\frac{\partial f_1}{\partial x_2}\right|_{x^0} \Delta x_2 + \dots + \left.\frac{\partial f_1}{\partial x_n}\right|_{x^0} \Delta x_n
$$

$$
= f_1(x^0) + \left[\left.\frac{\partial f_1}{\partial x_1}\right|_{x^0} \ \left.\frac{\partial f_1}{\partial x_2}\right|_{x^0} \cdots \left.\frac{\partial f_1}{\partial x_n}\right|_{x^0} \right] \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}
$$

We can do something similar for the others, and we get:

$$
f(x^0 + \Delta x) \simeq f(x^0) + \underbrace{\begin{bmatrix}
\left.\frac{\partial f_1}{\partial x_1}\right|_{x^0} & \left.\frac{\partial f_1}{\partial x_2}\right|_{x^0} & \cdots & \left.\frac{\partial f_1}{\partial x_n}\right|_{x^0} \\
\left.\frac{\partial f_2}{\partial x_1}\right|_{x^0} & \left.\frac{\partial f_2}{\partial x_2}\right|_{x^0} & \cdots & \left.\frac{\partial f_2}{\partial x_n}\right|_{x^0} \\
\vdots & & & \\
\left.\frac{\partial f_n}{\partial x_1}\right|_{x^0} & \left.\frac{\partial f_n}{\partial x_2}\right|_{x^0} & \cdots & \left.\frac{\partial f_n}{\partial x_n}\right|_{x^0}
\end{bmatrix}}_{J(x^0)} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} = 0
$$

$J(x^0) \leftarrow$ Jacobian matrix

The iteration scheme becomes

$$
x^{\nu+1} = x^{\nu} - J^{-1}(x^{\nu}) \cdot f(x^{\nu})
$$

APPLICATION OF NEWTON-RAPHSON TO SOLVE THE POWER FLOW PROBLEM

Remember:

$$P_i = \sum_K V_i \cdot V_K \left[G_{iK} \cdot \cos(\theta_i - \theta_K) + B_{iK} \sin(\theta_i - \theta_K) \right]$$

$$Q_i = \sum_K V_i \cdot V_K \left[G_{iK} \sin(\theta_i - \theta_K) - B_{iK} \cos(\theta_i - \theta_K) \right]$$

In general $P_i = P_{Gi} - P_{Di}$ and $Q_i = Q_{Gi} - Q_{Di}$, we can rewrite the equations as:

$$\sum_K V_i V_K \left[G_{iK} \cos(\theta_i - \theta_K) + B_{iK} \sin(\theta_i - \theta_K) \right] - P_{Gi} + P_{Di} = 0$$

$$\sum_K V_i \cdot V_K \left[G_{iK} \cos(\theta_i - \theta_K) - B_{iK} \sin(\theta_i - \theta_K) \right] - P_{Gi} + P_{Di} = 0$$

CASE I

Given, $V_1, \theta_1, \bar{S}_2, \bar{S}_3, \ldots, \bar{S}_n$

find, $\bar{S}_1, V_2, \theta_2, \ldots, V_n, \theta_n$.

As before, if we focus on the nodes $2, \ldots, n$ we can solve the problem.

Define $X = [\theta_2, \ldots \theta_n, V_2, \ldots, V_n]$, the problem is to find a solution to

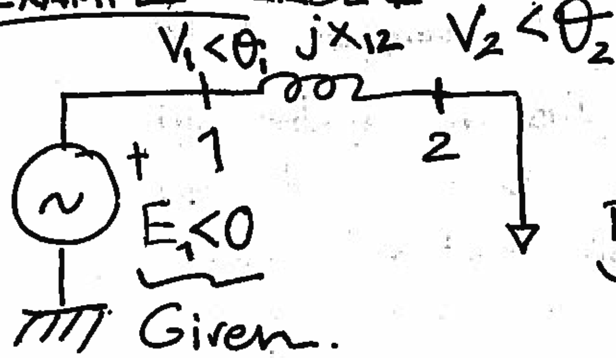
$P_2(X) - P_{G2} + P_{D2} = 0$, where $P_2(X) = \sum_K V_2 V_K \cdot (G_{iK} \cdots$

$$\left.\begin{array}{l} P_3(X) - P_{G3} + P_{D3} = 0 \\ \vdots \\ P_n(X) - P_{Gn} + P_{Dn} = 0 \\ Q_2(X) - Q_{G2} + Q_{D2} = 0 \\ \vdots \\ Q_n(X) - Q_{Gn} + Q_{Dn} = 0 \end{array}\right\} \rightarrow \underbrace{\begin{bmatrix} P_2(X) - P_{G2} + P_{D2} \\ P_3(X) - P_{G3} + P_{D3} \\ \vdots \\ P_n(X) - P_{Gn} + P_{Dn} \\ Q_2(X) - Q_{G2} + Q_{D2} \\ \vdots \\ Q_n(X) - Q_{Gn} + Q_{Dn} \end{bmatrix}}_{f(X)} = 0$$

$$x^{\nu+1} = x^{\nu} - J^{-1}(x^{\nu}) \cdot f(x^{\nu})$$

$$J(x^{\nu}) = \begin{bmatrix} \frac{\partial P_2}{\partial x_2}\Big|_{x^\nu} & \frac{\partial P_2}{\partial x_3}\Big|_{x^\nu} & \cdots & \frac{\partial P_2}{\partial x_n}\Big|_{x^\nu} \\ \vdots & & & \\ \frac{\partial P_n}{\partial x_2}\Big|_{x^\nu} & \frac{\partial P_n}{\partial x_3}\Big|_{x^\nu} & \cdots & \frac{\partial P_n}{\partial x_n}\Big|_{x^\nu} \\ \frac{\partial Q_2}{\partial x_2}\Big|_{x^\nu} & \frac{\partial Q_2}{\partial x_3}\Big|_{x^\nu} & \cdots & \frac{\partial Q_2}{\partial x_n}\Big|_{x^\nu} \\ \vdots & & & \\ \frac{\partial Q_n}{\partial x_2}\Big|_{x^\nu} & \frac{\partial Q_n}{\partial x_3}\Big|_{x^\nu} & \cdots & \frac{\partial Q_n}{\partial x_n}\Big|_{x^\nu} \end{bmatrix}$$

EXAMPLE CASE I



$$B = j \cdot \begin{bmatrix} -\frac{1}{X_{12}} & \frac{1}{X_{12}} \\ \frac{1}{X_{12}} & -\frac{1}{X_{12}} \end{bmatrix}$$

Given: $V_1 = E_1$, $\theta_1 = 0$, P_2^D, Q_2^D

To compute: P_1, Q_1, V_2, θ_2

- We want to compute P_1, Q_1 and V_2, θ_2.

- By stripping off the P & Q eqns. for the slack bus we are left with:

$$-P_2^D = E_1 V_2 \cdot B_{21} \cdot \sin(\theta_2)$$

$$-Q_2^D = -E_1^2 \cdot B_{11} - E_1 V_2 B_{12} \cos\theta_2$$

$$-P_2^D = \frac{E_1 V_2}{X_{12}} \sin\theta_2$$

$$-Q_2^D = \frac{E_1^2}{X_{12}} - \frac{E_1 V_2}{X_{12}} \cos\theta_2$$

In this case $X = [\theta_2, V_2]^T$ and

$$P_2(x) = \frac{E_1 \cdot V_2}{X_{12}} \sin\theta_2$$

$$Q_2(x) = \frac{V_2^2}{X_{12}} - \frac{E_1 \cdot V_2}{X_{12}} \cos\theta_2$$

$$f(x) = \begin{bmatrix} P_2(x) + P_2^D \\ Q_2(x) + Q_2^D \end{bmatrix}$$

Newton iteration

$$x^{\nu+1} = x^{\nu} - \left(J(x^{\nu})\right)^{-1} \cdot f(x).$$

In this case:

$$J(x^{\nu}) = \begin{bmatrix} \frac{\partial P_2(x)}{\partial \theta_2} & \frac{\partial P_2(x)}{\partial V_2} \\ \frac{\partial Q_2(x)}{\partial \theta_2} & \frac{\partial Q_2(x)}{\partial V_2} \end{bmatrix}$$

$$\frac{\partial P_2}{\partial \theta_2} = \frac{E_1 V_2}{X_{12}} \cos\theta_2 \qquad \frac{\partial Q_2}{\partial \theta_2} = \frac{E_1 V_2}{X_{12}} \sin\theta_2$$

$$\frac{\partial P_2}{\partial V_2} = \frac{E_1}{X_{12}} \sin\theta_2 \qquad \frac{\partial Q_2}{\partial V_2} = 2 \cdot \frac{V_2}{X_{12}} - \frac{E_1}{X_{12}} \cos$$

$$\begin{bmatrix} \theta_2^{\nu+1} \\ V_2^{\nu+1} \end{bmatrix} = \begin{bmatrix} \theta_2^{\nu} \\ V_2^{\nu} \end{bmatrix} - \begin{bmatrix} \frac{E_1 V_2^{\nu}}{X_{12}} \cos\theta_2^{\nu} & \frac{E_1}{X_{12}} \sin\theta_2^{\nu} \\ \frac{E_1 V_2^{\nu}}{X_{12}} \sin\theta_2^{\nu} & 2\frac{V_2^{\nu}}{X_{12}} - \frac{E_1}{X_{12}} \cos\theta_2^{\nu} \end{bmatrix}^{-1} \times \begin{bmatrix} \frac{E_1 V_2^{\nu}}{X_{12}} \sin\theta_2^{\nu} + P_2^D \\ \frac{(V_2^{\nu})^2}{X_{12}} - \frac{E_1 V_2^{\nu}}{X_{12}} \cos\theta_2^{\nu} + Q_2^D \end{bmatrix}$$

Once the value of θ_2 and V_2 has been obtained, we can plug those into the power balance equations for bus 1 to obtain the two other unknows, P_1, Q_1.