10/10/17

# LECTURE 12

- POWER FLOW FORMULATION

## - EXISTENCE AND UNIQUENESS OF SOLUTIONS

POWER FLOW EQUATIONS

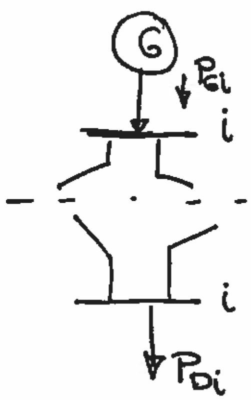
$$\begin{cases} P_i = \sum_K V_i \cdot V_K \cdot \left[ G_{iK} \cos(\theta_i - \theta_K) + B_{iK} \sin(\theta_i - \theta_K) \right] & \forall i = 1,2,..,n \\ Q_i = \sum_K V_i \cdot V_K \left[ G_{iK} \sin(\theta_i - \theta_K) - B_{iK} \cos(\theta_i - \theta_K) \right] & \forall i = 1,2,..,n \end{cases}$$

- As we discuss, we have $2n$ equations, but $4n$ variables: $V_i, \theta_i, P_i, Q_i \quad \forall i = 1,2,..,n$
- We need to fix $2n$ variables and solve for the other $2n$.
  - We are given $P_i$ and $Q_i$ on load buses,
  - We are given $V_i$ and $P_i$ on generator buses.
  - We solve for $V_i, \theta_i$ on load buses, and for $Q_i, \theta_i$ on generator buses.

→ There is a problem with the above set-up. We cannot arbitrarily choose all the $P_i$'s on generator buses.
  - In a lossless system: $\sum P_{Gi} = \sum P_{Di}$
  - In a system with losses, $\sum_i P_{Gi} = \sum_i P_{Di} + \sum_K R_K \cdot |I_K|^2$
    - $I_K$ depends on the flow through the transmission lines, which is a result of the power flow problem.

- Instead of fixing the power on all generator buses, we fix (or we are given) the power on all generator buses but one, which we call the "slack" generator.
- This "slack" generator is then modeled as a voltage source: we fix $V_i$ and $\theta_i$ (usually we make it zero)

- In summary, we will have three different types of buses:
  - A voltage source (slack bus), where we know $V_1$ and $\theta_1$
  - $P, |V|$ sources, which are all other generator buses.
  - $P, Q$ sources. All other load buses.

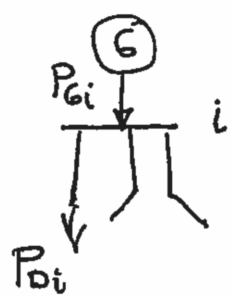



In the formulation of the power flow equations, we deal with power injections.

- Thus for generator buses $P_i = P_{Gi}$
- For load buses: $P_i = -P_{Di}$
- What happens if there is a bus with both generation and load?

It is still a $P, |V|$ bus:

$$P_i \triangleq P_{Gi} - P_{Di}$$




How do we solve the power flow equations?

- No analytical solution
- We need to solve them using numerical methods
  - Gauss (not covered in class)
  - Newton-Rapshon

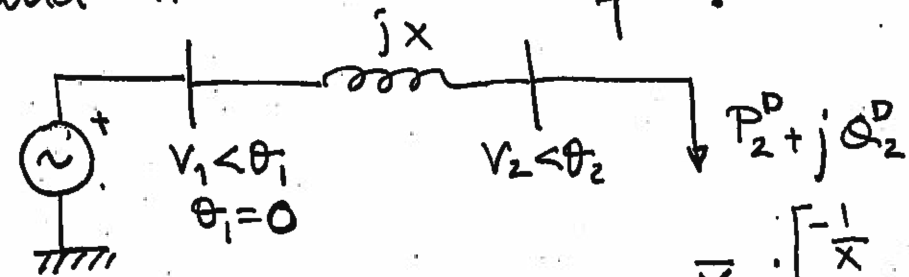
# EXISTANCE AND UNIQUENESS OF SOLUTION

Now, we have same number of unknows and equations, but is there a solution, and if so is unique?

$$Q_{line} = \cancel{\frac{I_{12}^2}{X}}\, I_{12}^2 \cdot X$$

$$\bar{I}_{12} = \frac{\bar{V}_1 - \bar{V}_2}{X}$$

$$\bar{I}_{12}^{\,2} = \frac{1}{X^2}\,(\bar{V}_1 - \ldots)(\bar{V}_1^* - \ldots$$




$$\bar{Y} = j\begin{bmatrix} -\frac{1}{X} & \frac{1}{X} \\ \frac{1}{X} & -\frac{1}{X} \end{bmatrix}$$

$$-P_2^D = \frac{V_1 V_2}{X}\sin\theta_2$$

$$-Q_2^D = +\frac{V_2^2}{X} - \frac{V_1 V_2}{X}\cos\theta_2$$

$\rightarrow$ This checks with what we did the other day

$P_2, Q_2, V_1, \theta_1 = 0$ given!

unknowns $\theta_2, V_2$

$$P_1^G = \frac{+V_1 V_2}{X}\sin(-\theta_2)$$

$$Q_1^G = \frac{+V_1^2}{X} - \frac{V_1 V_2}{X}\cos(-\theta_2)$$

$$Q_1^G = +Q_2^D + Q_{line}$$

$$Q_{line} = Q_1^G - Q_2 = \frac{V_1^2}{X} - \cancel{\frac{V_1 V_2}{X}\cos(-\theta_2)} + \frac{V_2^2}{X} - \cancel{\frac{V_1 V_2}{X}\cos}$$

$$= \frac{V_1^2}{X} + \frac{V_2^2}{X} - 2\cdot\frac{V_1 V_2}{X}\cos\theta_2$$

$$\frac{V_1^2}{X} - \frac{V_1 V_2}{X}\cos(-\theta_2) = \frac{-V_2^2}{X} + \frac{V_1 V_2}{X}\cos\theta_2 + Q_{line}$$

$$\boxed{Q_{line} = \frac{V_1^2 + V_2^2}{X} - 2\frac{V_1 V_2}{X}\cos\theta_2}$$

Is there a solution for every value of $P_2$ an

$$\left.\begin{aligned} P_2^2 &= \left(\frac{V_1 V_2}{X}\right)^2 \sin^2\theta_2 \\ \left(Q_2 + \frac{V_2^2}{X}\right)^2 &= \left(\frac{V_1 V_2}{X}\right)^2 \sin^2\theta_2 \end{aligned}\right\} \longrightarrow$$

$$\longrightarrow \quad P_2^2 + \left(Q_2 + \frac{V^2}{X}\right)^2 = \left(\frac{V_1 V_2}{X}\right)^2$$

$$P_2^2 + Q_2^2 + \left(\frac{V^2}{X}\right)^2 + 2\,\frac{Q}{X}\,V^2 = \left(\frac{V_1 V_2}{X}\right)^2$$

$$\underbrace{(V^2)^2}_{x^2} + \underbrace{\left[2Q_2X - V_1^2\right]}_{b}\,\underbrace{V_2^2}_{x} + \underbrace{X^2\left(P_2^2 + Q_2^2\right)}_{c} = 0$$

Quadratic equation in $V^2$: $ax^2 + bx + c$

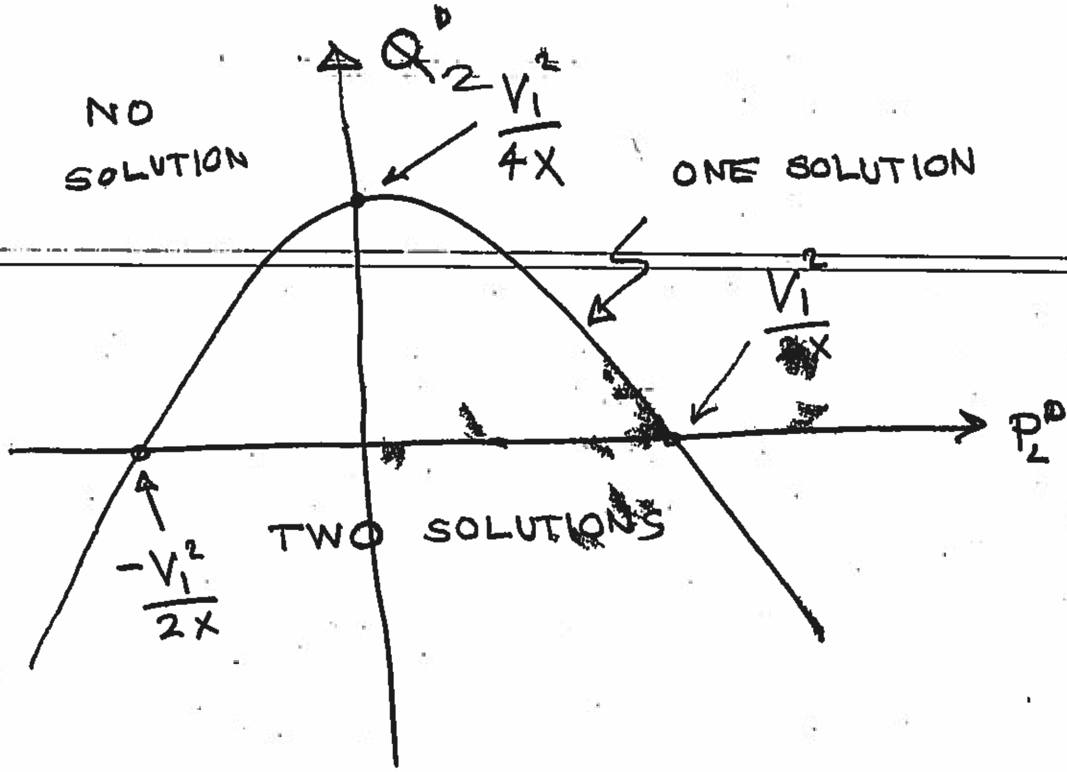
$$b^2 - 4ac \geq 0$$

$$(2Q_2X - V_1)^2 - 4X^2\left(P_2^2 + Q_2^2\right) \geq 0$$

$$\downarrow$$

$$P^2 + \frac{V_1^2}{X}\,Q - \left(\frac{V_1^2}{2X}\right)^2 \cancel{\geq} 0 \longrightarrow$$

$$\longrightarrow Q \leq -\frac{X}{V_1^2}\,P_2^2 + \frac{V_1^2}{4X} \longleftarrow \text{parabola.} \qquad y = ax^2 + bx + c$$




There is no limit to $P_2^D$ as long as you produce reactive power at bus 2; However, there is a limit to the amount of $Q_2^D$ that you can have