

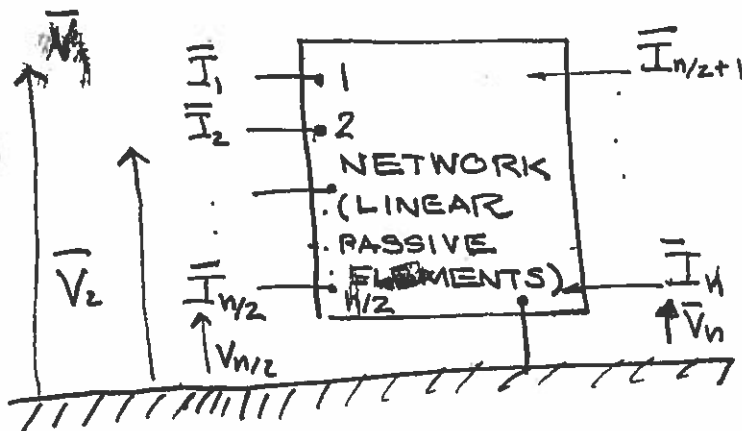
10/03/17

LECTURE 11

- ADMITTANCE MATRIX REVIEW
- THE POWER FLOW PROBLEM
- EXISTENCE AND UNIQUENESS OF SOLUTIONS

ADMITTANCE MATRIX REVIEW

- By applying KCL to all the nodes of a network, we can obtain a relation between current injections in the network and the node voltages



$$\bar{I} = [\bar{I}_1, \bar{I}_2, \dots, \bar{I}_n]^T$$

$$\bar{V} = [\bar{V}_1, \bar{V}_2, \dots, \bar{V}_n]^T$$

$$\bar{I} = \bar{Y} \cdot \bar{V}$$

$\bar{Y} \equiv$ Network admittance matrix.

$\bar{Y}[i, j] \equiv$ Negative of the admittance of the transmission line connected between nodes i and j

$\bar{Y}[i, i] \equiv$ Sum of the admittances of all the elements connected to node i :

- Transmission lines series admittances
- Shunt admittances
- Transmission lines shunt admittance

THE POWER FLOW PROBLEM FORMULATION

- We will look for a relation between the powers (active and reactive) injected in the network and the voltages.

~~$$P_i = f_i(V) \quad P = [P_1, P_2, \dots, P_n]^T, \quad f_i: \mathbb{R}^n \rightarrow \mathbb{R}$$~~

~~$$Q_i = g_i(V) \quad Q = [Q_1, Q_2, \dots, Q_n]^T, \quad g_i: \mathbb{R}^n \rightarrow \mathbb{R}$$~~

~~$$P = f(V), \quad f: \mathbb{R}^n \rightarrow \mathbb{R}^n$$~~

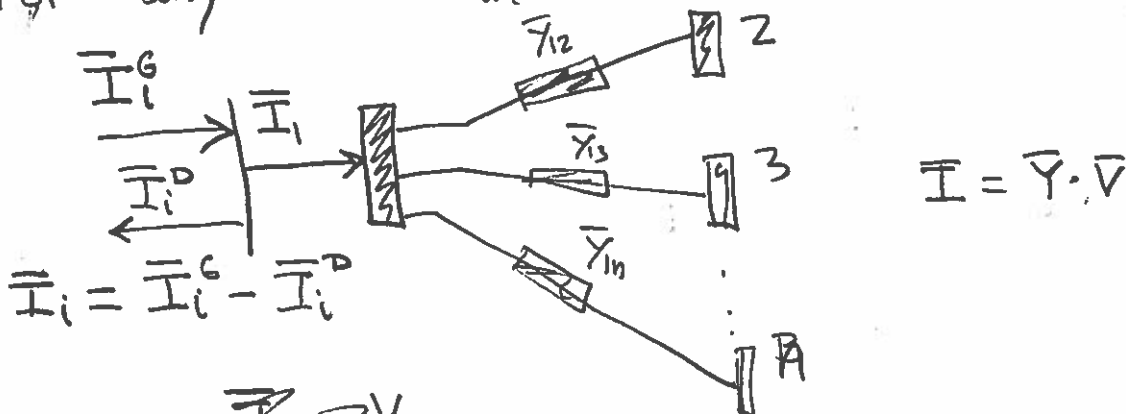
~~$$P_i \in f_i(\theta, V)$$~~

$$P_i = f_i(\theta, V), \quad \theta = [\theta_1, \theta_2, \dots, \theta_n]^T$$

$$Q_i = g_i(\theta, V), \quad V = [V_1, V_2, \dots, V_n]^T$$

$$f_i: \mathbb{R}^{n \times n} \rightarrow \mathbb{R}, \quad g_i: \mathbb{R}^{n \times n} \rightarrow \mathbb{R}$$

For any node in the network:



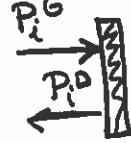
$$\bar{I}_i = \sum_k \bar{Y}_{ik} \bar{V}_k$$

$$\bar{I}_i = \underbrace{\bar{Y}_{i1} \bar{V}_1 + \bar{Y}_{i2} \bar{V}_2 + \dots + \bar{Y}_{in} \bar{V}_n}_{\text{entries of row } i}$$

$\bar{S}_i = \bar{V}_i \cdot \bar{I}_i^*$ ← complex power "entering" the network through port i (think of a 1-port network)

$$\bar{S}_i = \bar{V}_i \cdot \left(\sum_{k=1}^n \bar{Y}_{ik} \bar{V}_k \right)^*, \quad \forall i$$

Let's rewrite the equations above slightly differently.



$$P_i = P_i^G - P_i^D$$

$$\bar{S}_i = P_i + jQ_i$$

$$\bar{V}_i = V_i e^{j\theta_i} \quad \bar{Y}_{ik} = G_{ik} + jB_{ik}$$

$$P_i + jQ_i = V_i e^{j\theta_i} \cdot \left[\sum_{k=1}^n (G_{ik} + jB_{ik}) V_k e^{j\theta_k} \right]^*$$

$$= V_i e^{j\theta_i} \left[\sum_{k=1}^n (G_{ik} - jB_{ik}) V_k e^{-j\theta_k} \right]$$

$$= V_i \sum_{k=1}^n (G_{ik} - jB_{ik}) V_k e^{j(\theta_i - \theta_k)}$$

$$= V_i \cdot \sum_{k=1}^n (G_{ik} - jB_{ik}) V_k \cdot [\cos(\theta_i - \theta_k) + j \sin(\theta_i - \theta_k)]$$

$$= V_i \sum_{k=1}^n V_k \cdot (G_{ik} \cos(\theta_i - \theta_k) + j G_{ik} \sin(\theta_i - \theta_k) - j B_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k))$$

$$= V_i \cdot \sum_{k=1}^n V_k [G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)]$$

$$+ j V_i \sum_{k=1}^n V_k [G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)]$$

$$P_i = V_i \sum_{k=1}^n V_k$$

$$P_i = \sum_{k=1}^n V_i V_k [G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)]$$

$$Q_i = \sum_{k=1}^n V_i V_k [G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)]$$

$$f_i(\theta, V) = \sum_{k=1}^n V_i V_k [G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)]$$

$$g_i(\theta, V) = \sum_{k=1}^n V_i V_k [G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)]$$

How many variables do we have?

$$\theta_1, \theta_2, \dots, \theta_n \rightarrow n$$

$$V_1, V_2, \dots, V_n \rightarrow n$$

$$P_1, P_2, \dots, P_n \rightarrow n$$

$$Q_1, Q_2, \dots, Q_n \rightarrow n$$

4n variables.

Each bus

$$V_i, \theta_i$$

$$P_i, Q_i$$

How many equations do we have: $2n$

- We need to fix $2n$ variables and solve for the other $2n$ variables.
- In reality, the $2n$ variables that we fix are (i) things that we can control (generators output voltage and power generation) and (ii) things that we can estimate (load)

• In general we cannot specify all the generator powers independently.

- If a system is lossless

$$\sum_i P_i^G = \sum_i P_i^D$$

- In a ~~lossless~~ lossy system: $\sum_i P_i^G = \sum_i P_i^D + \underbrace{\sum_{e,m} R_{em} I_{em}^2}_{\text{losses}}$

- In order to obtain I_{em} , we need to know \bar{V}_e, \bar{V}_m , but that's what we are trying to solve for!

- Instead of fixing all the power generations we fix all but one and we make that one an ideal voltage source. We call that generator the "swing generator" or "slack bus"

• In summary three types of buses:

- (P, Q) \rightarrow loads

- P, V \rightarrow all generators but 1

- $V_1, \theta_1 = 0 \rightarrow$ slack bus.

If we fix 2 variables per bus, we reduce the number of unknown variables to $2n$! now we can solve for those as we also have $2n$ equations

