

ECE 476 – Power System Analysis Fall 2017

Homework 5

Reading: Chapter 3.

In-class quiz: Tuesday October 10, 2017

Problem 1. The following data are obtained when open-circuit and short-circuit tests are performed on a single-phase, 50-kVA, 2400/240-volt, 60-Hz distribution transformer

- **Measurement on low-voltage side with high-voltage winding open.**
Voltage: 240 V. Current: 5.97 A. Power: 213 W.
- **Measurements on high-voltage side with low-voltage winding shorted.**
Voltage: 60 V. Current: 20.8 A. Power: 750 W.

(a) Neglecting the series impedance, determine the exciting admittance referred to the high-voltage side.

The open circuit test data can be used to find the exciting admittance by neglecting the series impedance. First, we compute the turns ratio as

$$a = \frac{V_{1, \text{rated}}}{V_{2, \text{rated}}} = \frac{2400}{240} = 10.$$

G_c , Y_m , and B_m are then determined as follows:

$$\begin{aligned} G_c &= \frac{P_2}{V_1^2} = \frac{213}{2400^2} = 3.698 \times 10^{-5} \text{ S}, \\ |Y_m| &= \frac{I_1}{V_1} = \frac{\frac{1}{a} I_2}{V_1} = \frac{0.1 \cdot 5.97}{2400} = 2.488 \times 10^{-4} \text{ S}, \\ B_m &= \sqrt{|Y_m|^2 - G_c^2} = \sqrt{(2.488 \times 10^{-4})^2 - (3.698 \times 10^{-5})^2} = 2.460 \times 10^{-4} \text{ S}, \\ Y_m &= G_c - jB_m = 3.698 \times 10^{-5} - j2.460 \times 10^{-4} = 2.488 \angle -81.45^\circ \text{ S}. \end{aligned}$$

(b) Neglecting the exciting admittance, determine the equivalent series impedance referred to the high-voltage side.

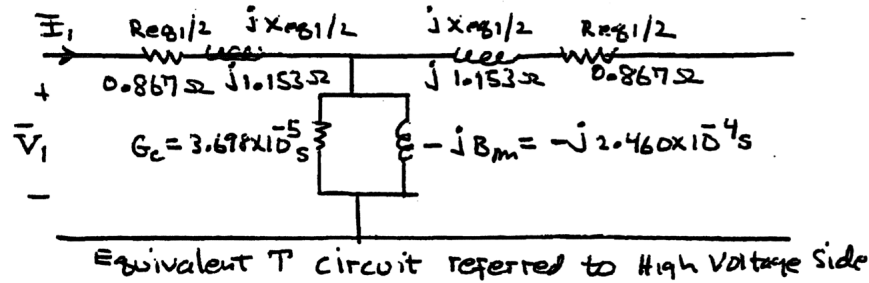
The short circuit test data can be used to find the equivalent series impedance referred to the high-voltage side. The rated current for the high-voltage side is

$$I_{1, \text{rated}} = \frac{S_{\text{rated}}}{V_{1, \text{rated}}} = \frac{50000}{2400} = 20.83 \text{ A}.$$

R_{eq1} , Z_{eq1} , and X_{eq1} are then determined as follows:

$$\begin{aligned} R_{eq1} &= \frac{P_1}{I_{1, \text{rated}}^2} = \frac{750}{20.83^2} = 1.728 \ \Omega \\ |Z_{eq1}| &= \frac{V_1}{I_{1, \text{rated}}} = \frac{60}{20.83} = 2.880 \ \Omega \\ X_{eq1} &= \sqrt{|Z_{eq1}|^2 - R_{eq1}^2} = \sqrt{2.880^2 - 1.728^2} = 2.305 \ \Omega \\ Z_{eq1} &= R_{eq1} + jX_{eq1} = 1.728 + j2.305 = 2.880 \angle 53.14^\circ \ \Omega \end{aligned}$$

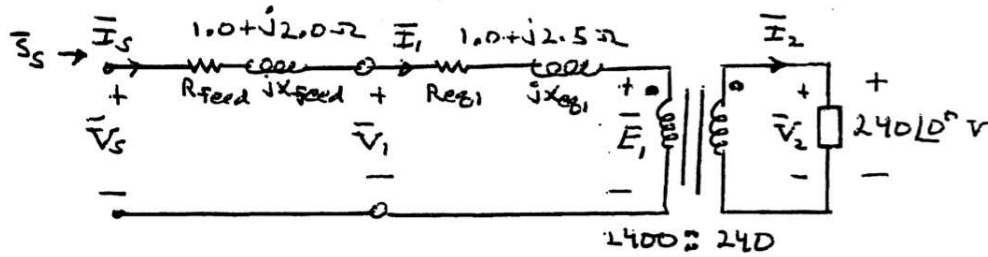
- (c) Assuming equal series impedances for the primary and referred secondary, obtain an equivalent T-circuit referred to the high-voltage side.



Problem 2. A single-phase 50-kVA, 2400/240-volt, 60-Hz distribution transformer is used as a step-down transformer at the load end of a 2400-volt feeder whose series impedance is $(1.0 + j2.0) \Omega$. The equivalent series impedance of the transformer is $(1.0 + j2.5) \Omega$ referred to the high-voltage (primary) side. The transformer is delivering rated load at 0.8 power factor lagging and at rated secondary voltage. Neglecting the transformer exciting current, determine:

- (a) The voltage at the transformer primary terminals.

A circuit representation of the feeder and transformer is as follows:



The voltage at the transformer primary terminals is denoted by \bar{V}_1 . Let \bar{V}_2 denote the voltage at the secondary terminals and set \bar{V}_2 with reference angle 0. Then, the secondary-side current is

$$\bar{I}_2 = \left(\frac{\bar{S}_{rated}}{\bar{V}_2} \right)^* = \left(\frac{50000 \angle \cos^{-1} 0.8}{240 \angle 0^\circ} \right)^* = 208.3 \angle -36.87^\circ \text{ A.}$$

The current referred to the primary side is

$$\bar{I}_1 = \frac{\bar{I}_2}{a} = \frac{208.3 \angle -36.87^\circ}{10} = 20.83 \angle -36.87^\circ \text{ A.}$$

The voltage at the primary side of the ideal transformer is $\bar{E}_1 = a\bar{V}_2 = 10 \cdot 240 \angle 0^\circ = 2400 \angle 0^\circ \text{ V}$. Then, we get

$$\bar{V}_1 = \bar{E}_1 + \bar{Z}_{eq} \bar{I}_1 = 2400 \angle 0^\circ + (1.0 + j2.5)(20.83 \angle -36.87^\circ) = 2448 \angle 0.683^\circ \text{ V.}$$

- (b) The voltage at the sending end of the feeder.

$$\bar{V}_s = \bar{V}_1 + \bar{Z}_{feed} \bar{I}_1 = 2448 \angle 0.683^\circ + (1.0 + j2.0)(20.83 \angle -36.87^\circ) = 2490 \angle 1.150^\circ \text{ V}$$

- (c) The real and reactive power delivered to the sending end of the feeder. The complex power at the sending end of the feeder is

$$\bar{S}_s = \bar{V}_s \bar{I}_1^* = (2490 \angle 1.150^\circ)(20.83 \angle -36.87^\circ)^* = 40.861 + j31.948 \text{ kVA.}$$

Therefore, the real and reactive power delivered at the sending end are

$$P_s = 40.861 \text{ kW,}$$

$$Q_s = 31.948 \text{ kVar.}$$

Problem 3. For a bank of three single-phase, two-winding, transformers whose high-voltage terminals are connected to a three-phase, 13.8 kV feeder (line-line), and the low-voltage terminals connected to a three-phase substation load rated 2.1 MVA and 2.3 kV, determine the required voltage, current, and MVA ratings of both windings of *each* transformer, when the high-voltage/low-voltage windings are connected

1. Wye-Delta.

	High-voltage side	Low-voltage side
MVA rating	$\frac{2.1}{3} = 0.7 \text{ MVA}$	$\frac{2.1}{3} = 0.7 \text{ MVA}$
Voltage	$\frac{13.8}{\sqrt{3}} = 7.97 \text{ kV}$	2.3 kV
Current	$\frac{0.7 \times 10^6}{7.97 \times 10^3} = 87.83 \text{ A}$	$\frac{0.7 \times 10^6}{2.3 \times 10^3} = 304.3 \text{ A}$

2. Delta-Wye.

	High-voltage side	Low-voltage side
MVA rating	$\frac{2.1}{3} = 0.7 \text{ MVA}$	$\frac{2.1}{3} = 0.7 \text{ MVA}$
Voltage	13.8 kV	$\frac{2.3}{\sqrt{3}} = 1.328 \text{ kV}$
Current	$\frac{0.7 \times 10^6}{13.8 \times 10^3} = 50.72 \text{ A}$	$\frac{0.7 \times 10^6}{1.328 \times 10^3} = 527.1 \text{ A}$

3. Wye-Wye.

	High-voltage side	Low-voltage side
MVA rating	$\frac{2.1}{3} = 0.7 \text{ MVA}$	$\frac{2.1}{3} = 0.7 \text{ MVA}$
Voltage	$\frac{13.8}{\sqrt{3}} = 7.97 \text{ kV}$	$\frac{2.3}{\sqrt{3}} = 1.328 \text{ kV}$
Current	$\frac{0.7 \times 10^6}{7.97 \times 10^3} = 87.83 \text{ A}$	$\frac{0.7 \times 10^6}{1.328 \times 10^3} = 527.1 \text{ A}$

4. Delta-Delta.

	High-voltage side	Low-voltage side
MVA rating	$\frac{2.1}{3} = 0.7 \text{ MVA}$	$\frac{2.1}{3} = 0.7 \text{ MVA}$
Voltage	13.8 kV	2.3 kV
Current	$\frac{0.7 \times 10^6}{13.8 \times 10^3} = 50.72 \text{ A}$	$\frac{0.7 \times 10^6}{2.3 \times 10^3} = 304.3 \text{ A}$

Problem 4. A three-phase generator rated 300 MVA, 23 kV, is supplying a system load of 240 MVA and 0.9 power factor lagging at 230 kV through a 330 MVA, 23 kV Delta-230 kV Wye step-up transformer with a leakage reactance of 0.11 p.u. Use $\bar{V}_A = 1.0 \angle 0^\circ$ as reference.

1. Neglecting the exciting current and choosing base values at the load of 100 MVA and 230 kV, find the phasor currents I_A , I_B , and I_C supplied to the load in per unit (magnitude and angle).

The current drawn by each phase of the load is

$$\begin{aligned}\bar{I}_A &= \frac{\bar{S}_{load1\phi}}{\bar{V}_A} = \left(\frac{(240/3)\angle \cos^{-1} 0.9}{(230/\sqrt{3})\angle 0^\circ} \right)^* = 602.45\angle -25.84^\circ \text{ A}, \\ \bar{I}_B &= \frac{\bar{S}_{load1\phi}}{\bar{V}_B} = \left(\frac{(240/3)\angle \cos^{-1} 0.9}{(230/\sqrt{3})\angle -120^\circ} \right)^* = 602.45\angle -145.84^\circ \text{ A}, \\ \bar{I}_C &= \frac{\bar{S}_{load1\phi}}{\bar{V}_C} = \left(\frac{(240/3)\angle \cos^{-1} 0.9}{(230/\sqrt{3})\angle 120^\circ} \right)^* = 602.45\angle 94.16^\circ \text{ A},\end{aligned}$$

The base quantities are

$$\begin{aligned}S_{base,3\phi} &= 100 \text{ MVA}, \\ V_{base,ll} &= 230 \text{ kV}, \\ I_{base} &= \frac{S_{base1\phi}}{V_{base,ln}} = \frac{100/3}{230/\sqrt{3}} = 251.02 \text{ A}.\end{aligned}$$

Therefore, the current drawn by each phase of the load, in per-unit, is

$$\begin{aligned}\bar{I}_{A,pu} &= \frac{\bar{I}_A}{I_{base}} = \frac{602.45\angle -25.84^\circ}{251.02} = 2.4\angle -25.84^\circ \text{ p.u.}, \\ \bar{I}_{B,pu} &= \frac{\bar{I}_B}{I_{base}} = \frac{602.45\angle -145.84^\circ}{251.02} = 2.4\angle -145.84^\circ \text{ p.u.}, \\ \bar{I}_{C,pu} &= \frac{\bar{I}_C}{I_{base}} = \frac{602.45\angle 94.16^\circ}{251.02} = 2.4\angle 94.16^\circ \text{ p.u.}.\end{aligned}$$

- Draw the per phase equivalent circuit and compute the phasor currents I_a , I_b , and I_c , from the generator in per unit. (Note: Take into account the phase shift of the transformer.)

The high-voltage side leads low-voltage side by 30° . So on the low-voltage side, in per-unit,

$$\begin{aligned}\bar{I}_{a,pu} &= 2.4\angle -55.84^\circ \text{ p.u.}, \\ \bar{I}_{b,pu} &= 2.4\angle -175.84^\circ \text{ p.u.}, \\ \bar{I}_{c,pu} &= 2.4\angle 64.16^\circ \text{ p.u.}.\end{aligned}$$

- Find the generator terminal voltage magnitude in kV and the total three-phase real power supplied by generator in MW.

Even though the leakage reactance X_l is given in per unit, this is per unit with respect to the low-voltage side transformer. So we convert this to per-unit value in the system base.

$$X_{l,p.u.} = X_l \left(\frac{Z_{base,sys}}{Z_{base,low}} \right) = X_l \left(\frac{\frac{V_{base,low}^2}{S_{base,low}}}{\frac{V_{base,sys}^2}{S_{base,sys}}} \right) = 0.11 \left(\frac{\frac{23^2}{330}}{\frac{23^2}{100}} \right) = 0.11 \left(\frac{100}{330} \right) = 0.0333 \text{ p.u.}$$

The high-voltage side leads low-voltage side by 30° . So on the low-voltage side,

$$\bar{V}_{a,pu} = 1\angle -30^\circ \text{ p.u.}$$

Therefore,

$$\bar{V}_{G,pu} = \bar{I}_{a,pu}(jX_{l,pu}) + \bar{V}_{a,pu} = (2.4\angle -55.84^\circ)(j0.033) + 1\angle -30^\circ = 1.037\angle -26.02^\circ \text{ p.u.}$$

Then,

$$|\bar{V}_G| = V_{base,low}|\bar{V}_{G,pu}| = 23(1.037) = 23.86 \text{ kV}.$$

The complex power supplied by the generator is

$$\bar{S}_G = \bar{V}_G \bar{I}_a^* = (1.037\angle -26.02^\circ)(2.4\angle -55.84^\circ)^* = 2.16 + j1.24 \text{ p.u.},$$

which corresponds to 2.16 p.u. or 216 MW real power supplied. This matches the real power absorbed since there are no I^2R losses.

4. By omitting the transformer phase shift altogether, check to see whether you get the same magnitude of generator terminal voltage and real power delivered by the generator (must show work).

By omitting the transformer phase shift, the high-voltage side and low-voltage side have the same phase shift. So on the low-voltage side,

$$\bar{V}_{a,pu} = 1\angle 0^\circ \text{ p.u.}$$

Therefore,

$$\bar{V}_{G,pu} = \bar{I}_{a,pu}(jX_{l,pu}) + \bar{V}_{a,pu} = (2.4\angle -25.84^\circ)(j0.033) + 1\angle 0^\circ = 1.037\angle 3.98^\circ \text{ p.u.}$$

Then,

$$|\bar{V}_G| = V_{base,low}|\bar{V}_{G,pu}| = 23(1.037) = 23.86 \text{ kV.}$$