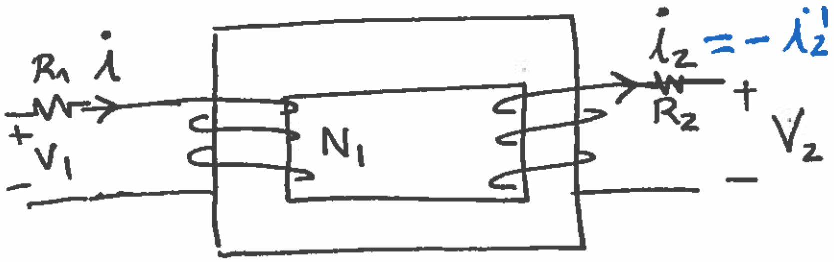
LECTURE 8 09/21/17

- 1ϕ NON-IDEAL TRANSFORMER MODEL

Assumptions:

- losses: $R_1 \neq 0$, $R_2 \neq 0$
- Flux leakage, $\phi_{e1}, \phi_{e2} \neq 0$
- Finite permeability, $\mu < \infty$.



$$V_1 = R_1 i_1 + \frac{d\lambda_1}{dt}$$

$$V_2 = -R_2 i_2 + \frac{d\lambda_2}{dt}$$

$$\lambda_1 = N_1(\underbrace{\phi_{11} + \phi_{12}}_{\phi_m} + \phi_{e1}) = N_1\phi_m + \overbrace{\phi_{e1}N_1}^{\lambda_{e1}}$$

$$\lambda_2 = N_2(\underbrace{\phi_{22} + \phi_{21}}_{\phi_m} + \phi_{e2}) = N_2\phi_m + \underbrace{\phi_{e2}N_2}_{\lambda_{e2}}$$

Assumption: (linearity of the magnetic circuit)

$$\lambda_{e1} = N_1\phi_{e1} = L_{e1} \cdot i_1$$

$$\lambda_{e2} = N_2\phi_{e2} = L_{e2} i_2' = -L_{e2} \cdot i_2$$

$$\begin{cases} V_1 = R_1 i_1 + L_{e1} \cdot \dfrac{di_1}{dt} + N_1 \dfrac{d\phi_m}{dt} \\ V_2 = -R_2 i_2 - L_{e2} \dfrac{di_2}{dt} + N_2 \dfrac{d\phi_m}{dt} \end{cases}$$

$$V_1 = R_1 i_1 + L_{e_1} \frac{di_1}{dt} + \underbrace{N_1 \frac{d\phi_m}{dt}}_{e_1}$$

$$V_2 = -R_2 i_2 - L_{e_2} \frac{di_2}{dt} + \underbrace{N_2 \frac{d\phi_m}{dt}}_{e_2}$$

$$N_1 i_1 - N_2 i_2 = \mathcal{R} \phi_m$$

with secondary open: $N_1 i_1 = \mathcal{R}\phi_m \rightarrow i_m = \frac{\mathcal{R}\phi_m}{N_1}$

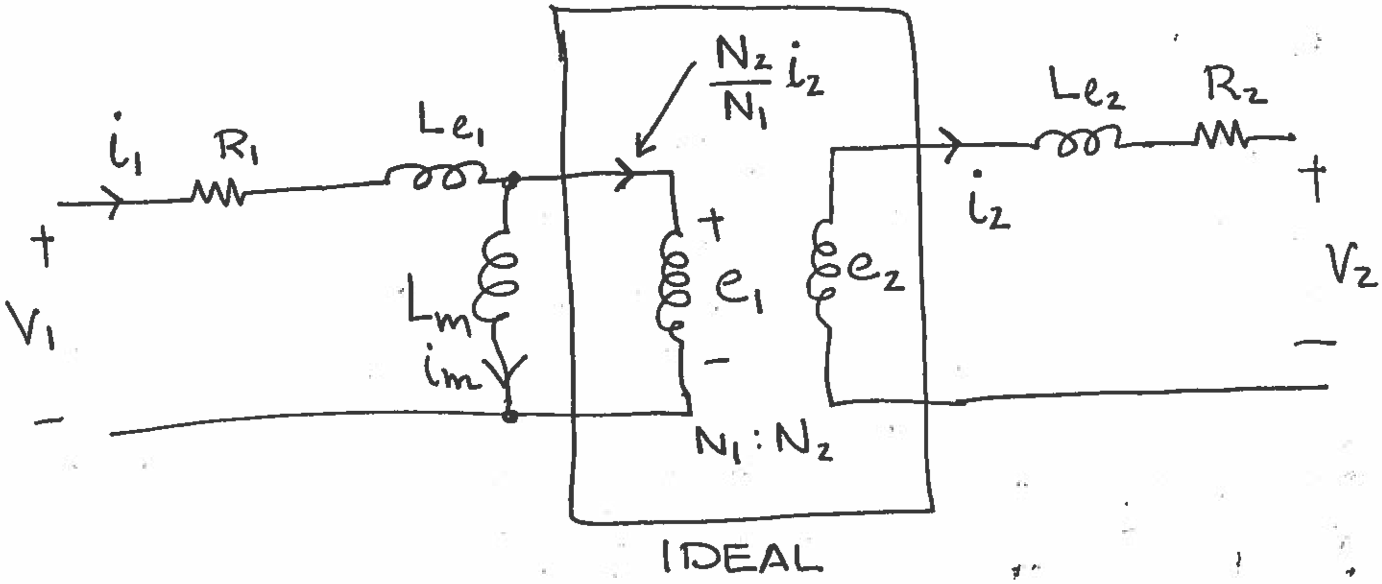
$$i_1 = \frac{\mathcal{R}}{N_1} \phi_m =: i_m$$

Assumption: (linearity of the magnetic circuit)

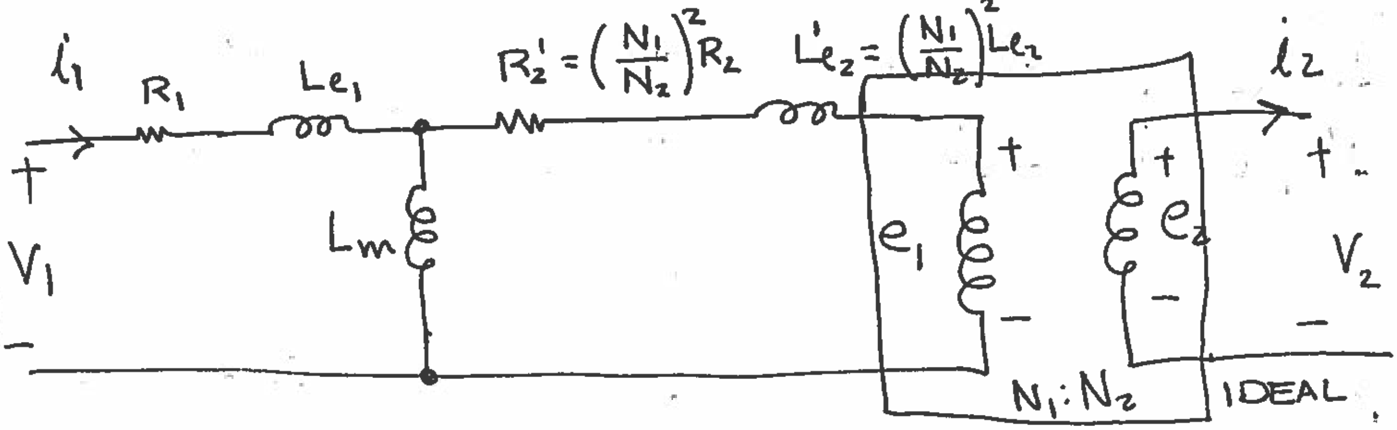
$$\lambda_m = N_1 \phi_m = L_m i_m$$

$$e_1 = \frac{d\lambda_m}{dt} = L_m \frac{di_m}{dt}$$

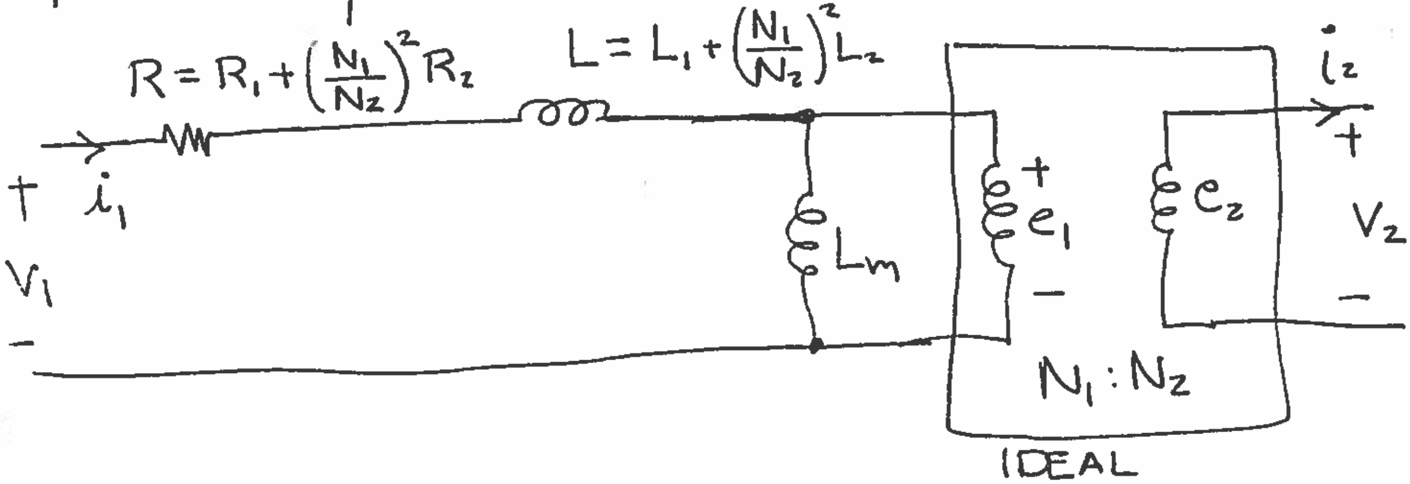
- $V_1 = R_1 i_1 + L_{e_1} \frac{di_1}{dt} + e_1$
- $V_2 = -R_2 i_2 - L_{e_2} \frac{di_2}{dt} + e_2$
- $\frac{e_1}{e_2} = \frac{N_1}{N_2}$
- $e_1 = L_m \frac{di_m}{dt}$
- $i_1 = \frac{N_2}{N_1} i_2 + i_m$



We can refer L_{e_2} and R_2 to the primary



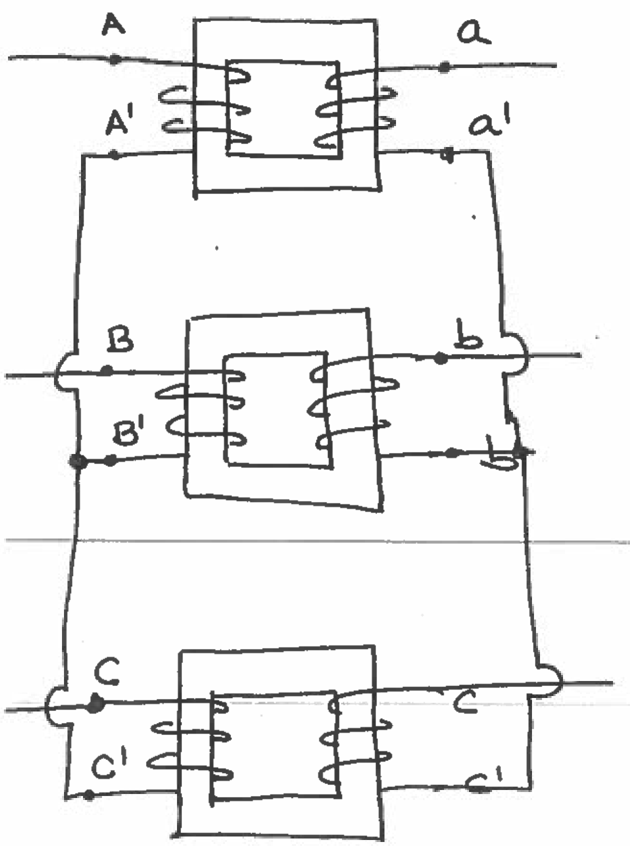
L_m is very large, whereas R_2' and L_{e_2}' are very small in comparison, thus we can swap them to get:



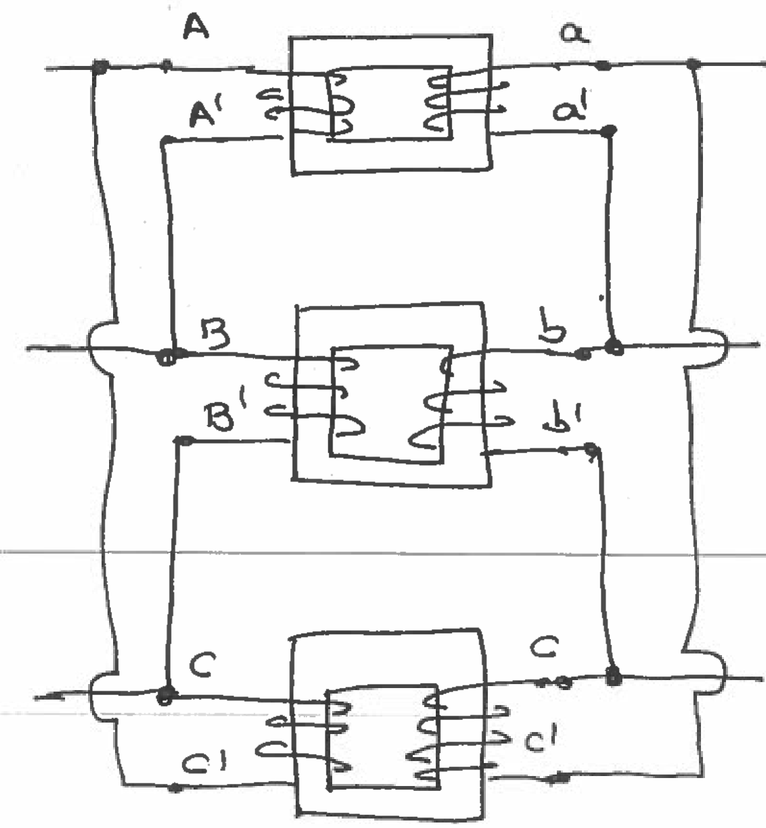
SEE (2b)

THREE-PHASE TRANSFORMERS

We can construct a 3ø transformer by interconnecting three 1ø transformers



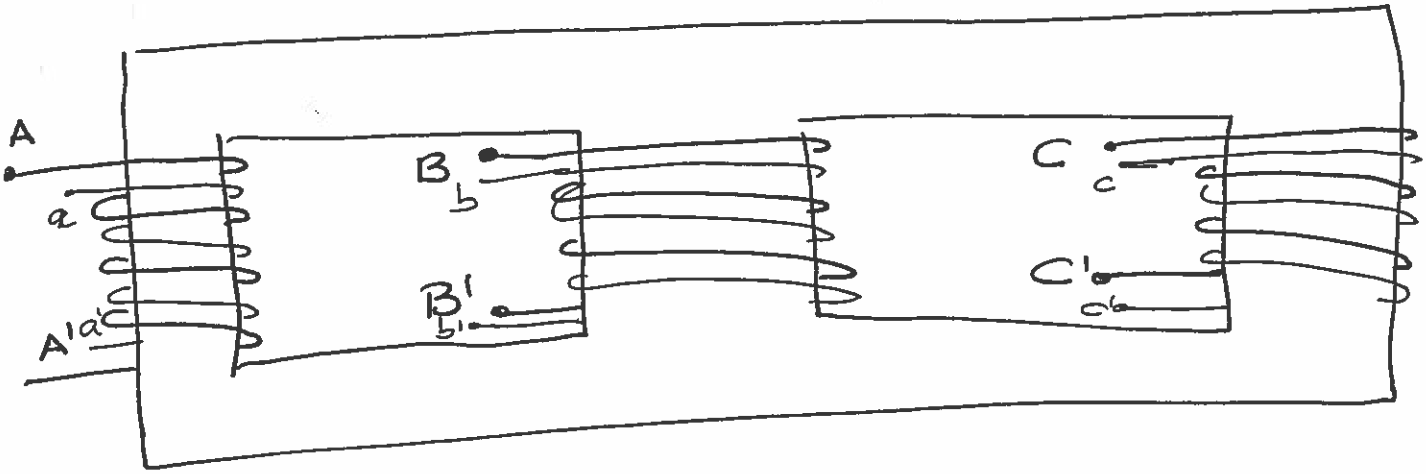
Y-Y CONNECTION



Δ-Δ CONNECTION

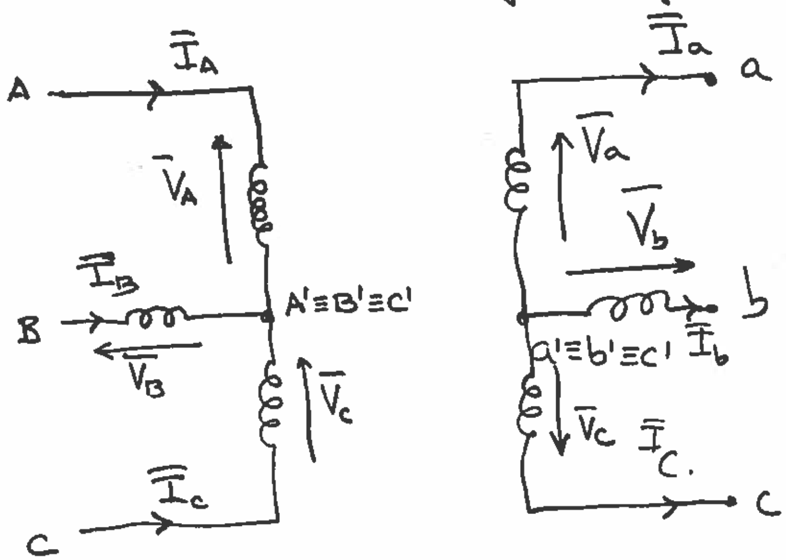
- The type of connection, i.e., Y-Y, Δ-Δ, Δ-Y, or Y-Δ depends on the application e.g., a step-up transformer for connecting a generator to the grid vs a step-down transformer used in distribution.

- It is very common (specially for small- and medium-size) transformers to use a single core

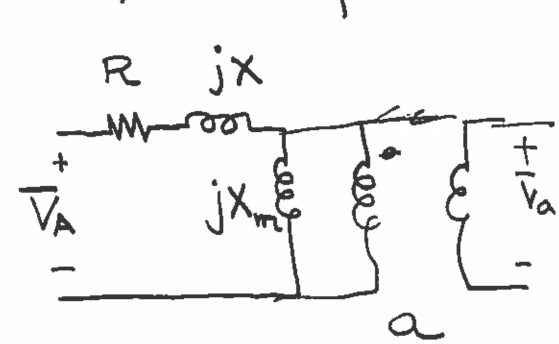


Y–Y TRANSFORMER

- Turn ratio of individual transformers is a
- No phase-shifting
- Possibility of grounding both sides of the transformer



Per-phase equivalent

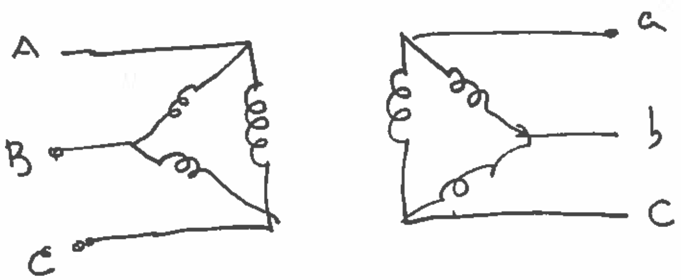


$$\frac{\bar{V}_A}{\bar{V}_a} = \frac{\bar{V}_B}{\bar{V}_b} = \frac{\bar{V}_C}{\bar{V}_c} = a$$

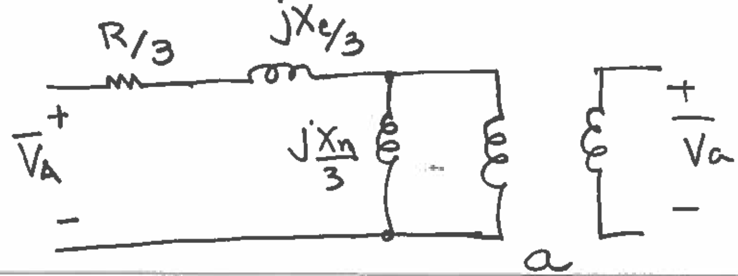
$$\frac{\bar{I}_A}{\bar{I}_a} = \frac{\bar{I}_B}{\bar{I}_b} = \frac{\bar{I}_C}{\bar{I}_c} = \frac{1}{a}$$

$\Delta - \Delta$ TRANSFORMER

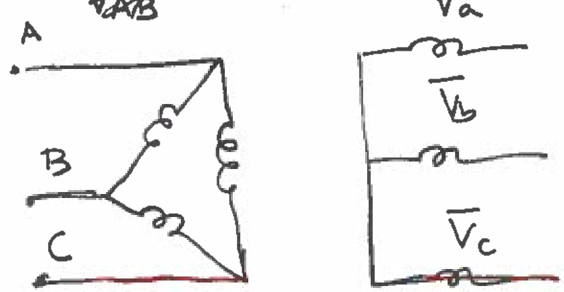
Similar to the Y-Y connection but all the parameter of the per-phase equivalent are divided by 3:



$$\frac{\overline{V}_{AB}}{\overline{V}_{ab}} = a \qquad \frac{\overline{V}_{BC}}{\overline{V}_{bc}} = a \qquad \frac{\overline{V}_{CA}}{\overline{V}_{ca}} = a$$



$\Delta - Y$ CONNECTION



$$\frac{\overline{V}_{AB}}{\overline{V}_a} = a$$

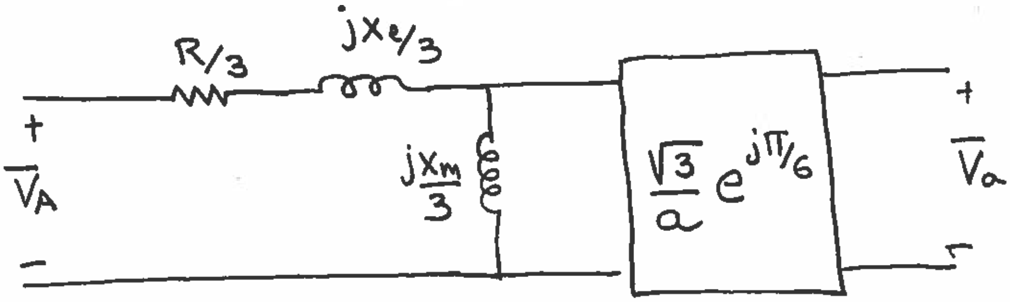
$$\frac{\overline{V}_{BC}}{\overline{V}_b} = a$$

$$\frac{\overline{V}_{CA}}{\overline{V}_c} = a$$

$$\overline{V}_a = \frac{1}{\sqrt{3}} \overline{V}_{ab} \, e^{-j\pi/6}$$

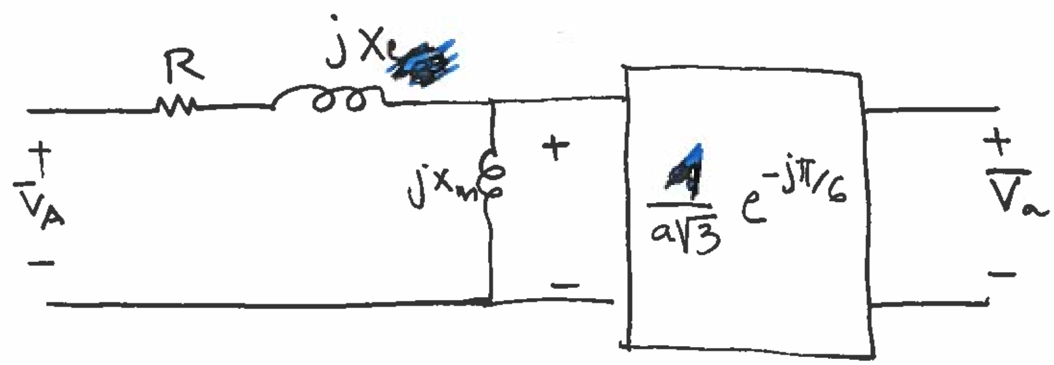
$$\overline{V}_{AB} = a \cdot \frac{1}{\sqrt{3}} \overline{V}_{ab} \, e^{-j\pi/6} \longrightarrow \overline{V}_{ab} = \frac{\sqrt{3}}{a} e^{+j\pi/6} \cdot \overline{V}_{AB}$$

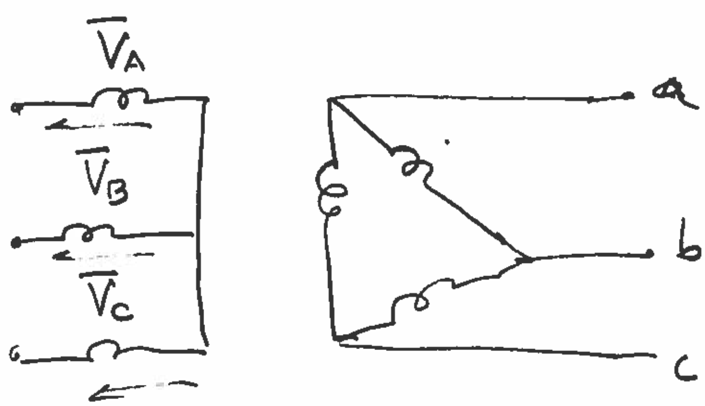
$$\boxed{\overline{V}_a = \frac{\sqrt{3}}{a} e^{j\pi/6} \overline{V}_A}$$



- Δ−Y CONNECTION introduces a 30° phase shift
- Common in distribution step-down since there is a neutral on the low voltage side.

Y−Δ CONNECTION





$$\overline{V}_A = a\overline{V}_{ab}$$
$$= a\sqrt{3}e^{j\pi/6}\,\overline{V}_a$$

$$\overline{V}_A = a\sqrt{3}e^{j\pi/6}\,\overline{V}_a$$

$$\overline{V}_a = \cancel{\emptyset}\,\frac{1}{a\sqrt{3}}\,e^{-j\pi/6}\,\overline{V}_a$$

$$\frac{\overline{V}_A}{\overline{V}_{ab}} = a \qquad \overline{V}_A = \frac{1}{\sqrt{3}}\,\overline{V}_{AB}\,e^{-j\pi/6}$$

$$\frac{1}{\sqrt{3}}\,\overline{V}_{AB}\,e^{-j\pi/6} = a\,\overline{V}_{ab} \rightarrow \overline{V}_{ab} = \frac{1}{a\sqrt{3}}\,e^{-j\pi/6}\,\overline{V}_{AB}$$

$$\overline{V}_{AB} = \sqrt{3}\,a\,\overline{V}_{ab}\,e^{j\pi/6} \rightarrow \boxed{\overline{V}_a = \frac{1}{\sqrt{3}\cdot a}\cdot e^{-j\pi/6}\,\overline{V}_A}$$